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Methods SIM&M and M&SIM to Evaluate Double Integrals with Continuous Integrands on the Two Dimensions are not Equal

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Abstract: The main objective of this study is to derive two new rules numerically for finding the values of the continuous double integrals in region of the integration by using mid-point rule and Simpson's rule on the x and y dimensions and how to find the correction terms (error formula) and we use Romberg acceleration to improve the results when the number of subintervals of on the two dimensions are not equal.

Key words: Region, integrals, dimensions, objective, subintervals, finding

INTRODUCTION

Numerical analysis is characterized by the creation of diverse methods for finding approximate solutions to certain mathematical problems in an effective manner. The efficiency of these methods depends on both the accuracy and the ease which they can be performed. Modern numerical analysis is the numerical interface of the field of applied analysis.

The study of error is central to numerical analysis because the results obtained from applying most numerical methods are only approximations of the real solution to be found. It is important to know the resulting error and how to estimate it across a set of calculations.

The numerical analysis of integrals is an important part of this topic as it is more evident in the practical applications of engineers and physicists. The process of finding value for double integration is a much more complex issue than finding the value of singular integration because the integrand here depends on two variables. The question of continuity and single arty in integrand and derivatives in the partial derivatives of the integrand is a major difficulties.

The importance of double integrals lies in (finding) the surface area (finding) intermediate centers, the introspection of flat surfaces and finding the volume under the surface of double integration (Ayers, 1988). Schjar-Jacobsen (1973) highlighted the computation of double integrals with continues integrands of the expression $f(x, y) = f_1(x)f_2(y)$ whereas, Davis and Robinowitz (1975), worked with integrals that including improper integrands. Others also studied double integrations such as Mohammed (2002), Mahdi (2011) and Hilal (2003). In this study, two numerical methods are derived using one of the Newton cuts formulas (the mid-point rule's and the Simpson rule's) to calculate the values of double integrals. When the integration function (integrand) is continuous on unequal periods, we shall divided the integration period on the internal dimension x for a number of partial periods on the external dimension y for a number of partial periods in a particular case we will take (m) in a particular case we will take (m = 2n), i.e., ($h = 1/2h_1$)(where h is the distance between the x-coordinates and h_1 is the distance between the y-coordinates (h_1 = d-c/m, h = b-a/n) and Romberg is used to improve the results on the two bases SIM&M and M&SIM taking advantage of the correction limits obtained and symbolizing the application of Romberg's acceleration with the two rules indicated by the symbol RSIM&M and RM&SIM. We obtained a good results with respect to the accuracy and speed of the approach with subintervals of relativity scarce.

MATERIALS AND METHODS

Evaluation of double integrals for continuous integrands numerically

Mid-point rule on the outer dimension x and simpson's rule on the inner dimension y

Theorem: Let the function f(x, y) is continuous and differentiable at each point of the region and [a, b]×[c, d] then approximate value of the integration $I=\int_c^d\int_a^b f(x,y)$ can be calculated from it form the following rule:

$$\int_a^b\int_c^d f(x,y)\,dy\,dx = \frac{h\cdot h1}{3}\sum_{j=1}^{2n}\left[\begin{array}{l} f(a, y_j)+f(b, y_j) \\ +4\sum_{i=1}^{n} f(x_{(2i-1)}, y_j) \\ +2\sum_{i=1}^{n-1} f(x_{2i}, y_j) \end{array}\right]$$

The formula for the correction limits (error formula) is:

$$\text{I-M \& SIM}(h, h_1) = A_{M\&SIM}h^2+B_{M\&SIM}h^4+C_{M\&SIM}h^6+, ...$$

where, $A_{M\&SIM}$, $B_{M\&SIM}$ $B_{M\&SIM}$are constants.

Proof: Double integration can be written in general with the following image:

$$I = \int_c^d\int_a^b f(x,y)\,dxdy = \text{M \& SIM}(h, h_1)+E(h, h_1) \quad (1)$$

The M&SIM (h, h_1) is the value of integration numerically with using two rules of the Simpson's rule on the dimension x and mid-point rules on the dimensions y, and E (h, h_1) is a series of the correction that can beaded to the values and M&SIM (h, h_1) and d-c/m = b-a/n such that m = 2n. The error formula for the single integrals with a continuous integrands by using the mid-point rule is:

$$E_M(h) = \frac{1}{6}h^2(f'_{2n}-f'_0)-\frac{7}{360}h^4(f^{(3)}_{2n}-f^{(3)}_0)+ \frac{31}{15120}h^6(f^{(5)}_{2n}-f^{(5)}_0)-, ... \quad (2)$$

And by using Simpson' s rule is:

$$E_{SIM}(h) = -\frac{1}{180}h^4(f^{(3)}_{2n}-f^{(3)}_0)+\frac{1}{1512}h^6(f^{(5)}_{2n}-f^{(5)}_0)-, ... \quad (3)$$

And by using the mean-value theorem for derivatives with the Eq. 2 and 3 we are obtaining:

$$E_{SIM}(h) = \frac{-(x_{2n}-x_0)}{180}h^4f^{(4)}(\mu_1)+\frac{(x_{2n}-x_0)}{1512}h^6f^{(6)}(\mu_2)+, ... \quad (4)$$

$$E_M(h) = \frac{(x_{2n}-x_0)}{6}h^2f^{(2)}(\eta_1)-\frac{7(x_{2n}-x_0)}{360}h^4f^{(4)}(\eta_2)+ \frac{31(x_{2n}-x_0)}{15120}h^6f^{(6)}(\eta_3)-, ... \quad (5)$$

Such that I = 1, 2, 3, ..., μ_i, η_i (Mahdi, 2011). With respect to the single integration $\int_c^d f(x,y)dy$, we can calculate it numerically by mid-point rule on the dimension y and (dealing with x constant) and it's valued:

$$\int_c^d f(x,y)\,dy = h_1\sum_{j=1}^{2n} f(x,y_j)+\frac{(d-c)}{6}h_1^2\frac{\partial^2 f(x,\mu_1)}{\partial y^2}- \frac{7(d-c)}{360}h_1^4\frac{\partial^4 f(x,\mu_2)}{\partial y^4}+\frac{31(d-c)}{15120}h_1^6\frac{\partial^6 f(x,\mu_3)}{\partial y^6}-, \cdots \quad (6)$$

Such that $\mu_1, \mu_2, ..., \in(c, d)$, j = 1, 2, ..., 2n, y = c+2j-1/2 h1. So, by integrated the Eq. 6 numerically on the interval [a, b] also by using Simpson's rule on the dimension x we are obtaining:

$$\int_a^b\int_c^d f(x, y)dydx = \frac{h\cdot h1}{3}\sum_{j=1}^{2n}\left[\begin{array}{l} f(a, y_j)+f(b, y_j)+ \\ 4\sum_{i=1}^{n} f(x_{(2i-1)}, y_j)+2\sum_{i=1}^{n-1} f(x_{2i}, y_j)\end{array}\right] +h\sum_{j=1}^{2n}\left(\frac{-(b-a)}{180}h^4\frac{\partial^4 f(\xi_{1j}, y_j)}{\partial x^4}+\frac{(b-a)}{1512}h^6\frac{\partial^6 f(\xi_{2j}, y_j)}{\partial x^6}-, \cdots\right) +\int_a^b\left[\frac{(d-c)}{6}h_1^2\frac{\partial^2 f(x,\mu_1)}{\partial y^2}-\frac{7(d-c)}{360}h_1^4\frac{\partial^4 f(x,\mu_2)}{\partial y^4} +\frac{31(d-c)}{15120}h_1^6\frac{\partial^6 f(x,\mu_3)}{\partial y^6}-\cdots\right]dx$$

Such that, $\zeta_{1j}, \zeta_{2j}, ..., \in(a, b)$, $\mu_1, \mu_2, x_i, ..., \in(c, d)$, y_j = c+ (2j-1/2) h1, x_i = a+(2i-1)h, I = 1, 2 , ..., m j = 1, 2, ..., 2n.

Since, $\partial^4 f/\partial x^4$, ... and $\partial^2 f/\partial y^2$ $\partial^4 f/\partial y^4$ are continuous in each point from the region [a, b]×[c, d], so, a formula of the correction term to double integrals I by M&SIM rule becomes:

$$E_{M\&SIM}(h) = (d-c)(b-a)\left[\frac{-h^4}{180}\frac{\eta^4 f(\overline{n_1},\overline{\mu_1})}{\eta x^4}+ \frac{h^6}{1512}\frac{\eta^6 f(\overline{n_2},\overline{\mu_2})}{\eta x^6}-L\right]+(d-c)(b-a) \left[\frac{h_1^2}{6}\frac{\eta^2 f(\overline{\lambda_1},\overline{\mu_1})}{\eta y^2}-\frac{7h_1^4}{360}\frac{\eta^4 f(\overline{\lambda_2},\overline{\mu_2})}{\eta y_1^4}+\frac{31h_1^6}{15120}\frac{\eta^6 f(\overline{\lambda_3},\overline{\mu_3})}{\eta y_1^6}-L\right]$$

$$E_{M\&SIM}(h) = (d-c)(b-a)\frac{h_1^2}{6}\frac{\partial^2 f(\overline{\lambda_2},\overline{\mu_2})}{\partial y^2}-(d-c)(b-a)\frac{h^4}{180} \left(-\frac{\partial^4 f(\overline{\lambda_1},\overline{\mu_1})}{\partial x^4}-7/2\frac{\partial^4 f(\hat{\lambda}_1,\hat{\mu}_1)}{\partial y^4}-\cdots\right)+\cdots \quad (7)$$

Such that $(\hat{\lambda}_1, \hat{\mu}_1)(\hat{\lambda}_2, \hat{\mu}_2), ..., (\overline{\lambda_1},\overline{\mu_1}), (\overline{\lambda_2},\overline{\mu_2}), ..., \in[a,b]\times[c,d]$.

So, if the integrand was continuous function and it's partial derivatives exist ineach point from the integral region [a, b]×[c, d] we can write the error formula to the rule stated as :

$$E(h, h_1) = \text{I-M \& SIM}(h, h_1) = (A_1h^2+B_1h^4+ C_1h^6+,...)+(A_1h_1^4+B_1h_1^6+C_1h_1^8+,...) \quad (8)$$

Such that $A_1, A_2, ..., B_1, B_2, ..., C_1, C_2, ...$ are constants depend on the partial derivatives for the function in the integral region, thus, the proof is completed.

The Simpson's rule on the outer dimension y and the mid-point rule on the internal dimension x

Theorem: Let the function (x, y) is continuous and differentiable at each point of the region and [a, b]×[c, d] then approximate value of the integration can be $I=\int_c^d\int_a^b f(x,y)$ calculated from it form the following rule:

$$\int_a^b\int_c^d f(x,y)\,dydx = \frac{h\cdot h1}{3}\sum_{j=1}^{2n}\left[\begin{array}{l} f(x_j,c)+f(x_j,d)+ \\ 4\sum_{i=1}^{n} f(x_j,y_{(2i-1)})+2\sum_{i=1}^{n-1} f(x_j,y_{2i})\end{array}\right]$$

The formula for the correction limits (error formula) is:

$$\text{I-SIM \& M}(h_1, h) = A_{SIM\&M}h^2+B_{SIM\&M}h^4+C_{SIM\&M}h^6+,\cdots$$

where, $A_{SIM\&M}$ $B_{SIM\&M}$ $C_{SIM\&M}$ A, B, C, ... are constants.

Proof: By applying the Simpson's rule on unilateral integration $\int_c^d f(x,y)dy$ we get :

$$\int_c^d f(x,y)dy = \frac{h}{3}\left(f(x,c)+f(x,d)+4\sum_{j=1}^{n} f(x,y_{(2j-1)})+2\sum_{j=1}^{n-1} f(x,y_{(2j)})\right)$$
$$-\frac{(d\text{-}c)h^4}{180}\frac{\partial^4 f(x,\mu_1)}{\partial y^4}+\frac{(d\text{-}c)h^6}{1512}\frac{\partial^6 f(x,\mu_2)}{\partial y^6}-,\cdots \tag{9}$$

Such that, $\mu_1,\mu_2,\ldots,\in(c,d)$ $j=1, 2, ..., n,$ $y_{(2j\text{-}1)}=c+(2j\text{-}1)h$
$j=1, 2, ..., n\text{-}1$, $y_{2j}=c+2jh$

So, by integrated the Eq. 9 numerically on the interval [c, d] also by using mid-point rule on the dimension x we are obtaining:

$$\int_a^b\int_c^d f(x,y)\,dydx = \frac{h_1\cdot h}{3}\sum_{i=1}^{2n}\left[\begin{array}{l} f(x_i,c)+f(x_i,d)+ \\ 4\sum_{j=1}^{n} f(x_i,y_{(2j-1)})+2\sum_{j=1}^{n-1} f(x_i,y_{2j})\end{array}\right]$$

$$+\frac{h}{3}\left[\frac{(b\text{-}a)}{6}h_1^2\frac{\partial^2 f(\xi_1,c)}{\partial x^2}-\frac{7(b-a)}{360}h_1^4\frac{\partial^4 f(\xi_2,c)}{\partial x^4}+\right.$$
$$\frac{31(b-a)}{15120}h_1^6\frac{\partial^6 f(\xi_3,c)}{\partial x^6}-,\cdots,+\frac{(b\text{-}a)}{6}h_1^2\frac{\partial^2 f(\xi_1,d)}{\partial x^2}-$$
$$\frac{7(b\text{-}a)}{360}h_1^4\frac{\partial^4 f(\xi_2,d)}{\partial x^4}+\frac{31(b\text{-}a)}{15120}h_1^6\frac{\partial^6 f(\xi_3,d)}{\partial x^6}-\cdots$$

$$+4\sum_{j=1}^{n}\left(\frac{(b\text{-}a)}{6}h_1^2\frac{\partial^2 f(\xi_{1j},y_{(2j-1)})}{\partial x^2}-\frac{7(b\text{-}a)}{360}h_1^4\frac{\partial^4 f(\xi_{2j},y_{(2j-1)})}{\partial x^4}\right.$$
$$\left.+\frac{31(b\text{-}a)}{15120}h_1^6\frac{\partial^6 f(\xi_{3j},y_{(2j-1)})}{\partial x^6}-\cdots\right)+2\sum_{j=1}^{n-1}\left(\frac{(b\text{-}a)}{6}h_1^2\frac{\partial^2 f(\xi_{1j},y_{2j})}{\partial x^2}\right.$$
$$+\int_a^b\left[-\frac{(d-c)h^4}{180}\frac{\partial^4 f(x,\mu_1)}{\partial y^4}+\frac{(d-c)h^6}{1512}\frac{\partial^6 f(x,\mu_2)}{\partial y^6}-\cdots\right]dx$$

$$\zeta_{1j},\zeta_{2j},\ldots,\in(a,b),\quad \mu_1,\mu_2,\ldots,\in(c,d)$$
$$\left.\left.-\frac{7(b\text{-}a)}{360}h_1^4\frac{\partial^4 f(\xi_{2j},y_{2j})}{\partial x^4}+\frac{31(b\text{-}a)}{15120}h_1^6\frac{\partial^6 f(\xi_{3j},y_{2j})}{\partial x^6}-,\cdots\right)\right]$$

$$j=1, 2, ...,n,\ y_{(2j-1)}=c+(2j\text{-}1)hx_i=a+\frac{2i\text{-}1}{2}h_1 i=1, 2, ..., 2n$$

$$j=1, 2, ..., n\text{-}1\ y_{2j}=c+2jh$$

Since, $\partial^4 f/\partial x^4$ $\partial^6 f/\partial x^6$ and $\partial^2 f/\partial y^2$, $\partial^4 f/\partial y^4$ are continuous in each point from the region [a, b]×[c, d] So, a formula of the correction term to double integrals I by SIM&M rule becomes:

$$E_{SIM\&M}(h)=(d\text{-}c)(b\text{-}a)\left[\frac{h_1^2}{6}\frac{\partial^2 f(\bar{n}_1,\bar{\mu}_1)}{\partial x^2}-\frac{7h_1^4}{360}\right.$$
$$\left.\frac{\partial^4 f(\bar{n}_2,\bar{\mu}_2)}{\partial x^4}+\frac{31h_1^6}{15120}\frac{\partial^6 f(\bar{n}_3,\bar{\mu}_3)}{\partial x^6}-,\cdots\right]+(d\text{-}c)$$
$$(b\text{-}a)\left[\frac{-h^4}{180}\frac{\partial^4 f(\hat{n}_1,\hat{\mu}_1)}{\partial y^4}+\frac{h^6}{1512}\frac{\partial^6 f(\hat{n}_2,\hat{\mu}_2)}{\partial y^6}-,\cdots\right]$$

$$E_{SIM\&M}(h)=(d\text{-}c)(b\text{-}a)\frac{h_1^2}{6}\frac{\partial^2 f(\bar{n}_1,\bar{\mu}_1)}{\partial x^2}-(d\text{-}c)(b\text{-}a)\frac{h^4}{180}$$
$$\left(\frac{7}{2}\frac{\partial^4 f(\bar{n}_2,\bar{\mu}_2)}{\partial x^4}+\frac{\partial^4 f(\hat{n}_1,\hat{\mu}_1)}{\partial y^4}\right)$$

$$\bar{n}_1\in(c,d), ..., \hat{\mu}_2, \hat{\mu}_1, ..., \in(a,b)\hat{n}_2, \hat{n}_1, ..., \bar{n}_2$$

Therefore, we provide the full range of services and services that are available in all areas of the region [a, b]×[c, d] than we can write the two rule:

$$E_{M\&SIM}(h,h_1) = \text{I-M \& SIM}(h,h_1) = (A_1h^2+B_1h^4+C_1h^6+, ...) +(A_1h_1^4+B_1h_1^6+C_1h_1^8+, ...)$$

$$E_{SIM\&M}(h_1,h) = \text{I-SIM \& M}(h_1,h) = (A_1h_1^4+B_1h_1^6+C_1h_1^8+, ...) +(A_1h^2+B_1h^4+C_1h^6+, ...)$$

Such that $A_{SIM\&M}$, $B_{SIM\&M}$, $A_{M\&SIM}$, $B_{M\&SIM}$ and A_1, A_2, ..., B_1, B_2, ..., C_1, C_2, ... are constants depend on the partial derivatives for the function in the integral region, thus the proof is completed:

$$\int_2^3\int_2^3 \log((x+y)/2)dxdy$$

Examples: Its real value is 0.91293034365166 [approach to fourteen decimals]:

$$\int_1^2\int_1^2 x\,e\^{}-(x+y)dxdy$$

Its real value is 0.076682141300108 [approach to fifteen decimals]:

$$\int_0^1\int_0^1 \cos((xy)/2)dxdy$$

Its real value is 0.98621483608613 [approach to fourteen decimals]:

$$\int_0^1\int_0^1 (x/\sqrt{3-xy})dxdy$$

Its real value is 0.3071345511905 [approach to thirteen decimals].

RESULTS AND DISCUSSION

The integral I = h as defined integrand for all and that the formula of the correction limits for this integration is the following:

$$\int_1^2\int_1^2 x\,e\^{}-(x+y)dxdy$$

$$E(h, h_1) = \text{I-M \& SIM}(h, h_1) = (A_1h^2+B_1h^4+C_1h^6+, ...) +(A_1h_1^4+B_1h_1^6+C_1h_1^8+, ...)$$

Note from Table 1, when n = 64 and m = 128, the integrity value is valid to at least fifteen decimal places using both SIM&M and M&SIM with speed up your Romberg accelerations with 2^{10} partial period while using the method without acceleration mentioned the value is true to six degrees. "The time it takes to The MATLAB program for the account was (4.70 sec):

$$\int_2^3\int_2^3 \log((x+y)/2)dxdy$$

The integral I = ha s defined integrand for alland that the formula of the correction limits for this integration is the following:

$$E(h, h_1) = \text{I-M \& SIM}(h, h_1) = (A_1h^2+B_1h^4+C_1h^6+, ...) +(A_1h_1^4+B_1h_1^6+C_1h_1^8+, ...)$$

Note from Table 2 when n = 64 and m = 128 The integrity value is valid to at least fourteen decimal places using both SIM&M and M&SIM with acceleration by Romberg accelerations 2^{10} partial period while using the method without acceleration mentioned, the value is true to six degrees, "the time it takes in Bern The MATLAB program for the account was (4 sec):

Table 1: Valid integrity at least on 15 values

N	M	M&SIM/ SIM&M	D = 2	D = 4	D = 6	D = 8	D = 10
2	4	0.077025446975611					
4	`8	0.076765090815471	0.076678305428757				
8	16	0.076702692957063	0.076681893670928	0.076682132887072			
16	32	0.076687267512235	0.076682125697292	0.076682141165716	0.076682141297123		
32	64	0.076683422120273	0.076682140322953	0.076682141297997	0.076682141300096	0.076682141300108	
64	128	0.076682461459322	0.076682141239005	0.076682141300076	0.076682141300109	0.076682141300109	0.076682141300109

Table 2: Valid in tegrity at least on 15 values

n	M	M&SIM/ SIM&M	D = 2	D = 4	D = 6	D = 10
2	4	0.91335134535639				
4	8	0.91303638107299	0.91293139297853			
8	16	0.91295690351266	0.91293041099255	0.91293034552682		
16	32	0.91293698679514	0.91293034788930	0.91293034368242	0.91293034365314	
32	64	0.91293200463651	0.91293034391697	0.91293034365215	0.91293034365167	0.91293034365166
64	128	0.91293075891032	0.91293034366825	0.91293034365167	0.91293034365166	0.91293034365166

Table 3: Integrity valid on 14 values when n = 64

N	M	M&IM/ SIM&M	D = 2	D = 4	D = 6	D = 8	D = 10
2	4	0.98706571536226					
4	8	0.98642678090135	0.98621380274771			COS ((xy)/2) dxdy	
8	16	0.98626776512858	0.98621475987100	0.98621482367922			
16	32	0.98622806463377	0.98621483113550	0.98621483588647	0.98621483608024		
32	64	0.98621814298877	0.98621483577377	0.98621483608299	0.98621483608611	0.98621483608613	
64	128	0.98621566279712	0.98621483606656	0.98621483608608	0.98621483608613	0.98621483608613	0.98621483608613

Table 4: Integrity valid on 13 vlues

sn	m	M&SIM/ SIM&M	D = 2	D = 4	D = 6	D = 8	D = 10
2	4	0.3069709837298			$\int_0^1\int_0^1 (x/\sqrt{3-xy}$ dxdy		
4	8	0.3070895867786	0.3071291211282				
8	16	0.3071229814427	0.3071341129974	0.3071344457887			
16	32	0.3071316367898	0.3071345219055	0.3071345491660	0.3071345508070		
32	64	0.3071338211939	0.3071345493286	0.3071345511568	0.3071345511884	0.3071345511899	
64	128	0.3071343686037	0.3071345510736	0.3071345511900	0.3071345511905	0.3071345511905	0.3071345511905

$$\int_0^1\int_0^1 \cos((xy)/2)dxdy$$

The integral I = has defined integrand for all and taht formula of the correction limits for this integration is the following:

$$E(h, h_1) = I\text{-}M \,\&\, SIM(h, h_1) = (A_1h^2+B_1h^4+C_1h^6+, ...)+ (A_1h_1^4+B_1h_1^6+C_1h_1^8+, ...)$$

Note from Table 3, when n = 64 and m = 128, the integrity value is valid to at least fourteen decimal places using both SIM&M and M&SIM with speed up your with Romberg accelerations 2^{10} partial period while using the method without acceleration mentioned the value is right to five degrees. "The time it takes to The MATLAB program for the account was (4.70 sec):

$$\int_0^1\int_0^1 (x/\sqrt{3-xy})dxdy$$

The integral I = has defined integrand for all:

$$E(h, h_1) = I\text{-}M \,\&\, SIM(h, h_1) = (A_1h^2+B_1h^4+C_1h^6+, ...) +(A_1h_1^4+B_1h_1^6+C_1h_1^8+, ...)$$

Note from Table 4 when n = 64 and m = 128, the integrity value is valid to at least thirteen decimal places using both SIM&M and M&SIM with speed up your Romberg accelerations with 2^{10} partial period while using the method without acceleration mentioned the value is true to six levels. "The time it takes to The MATLAB account was (4.79 sec).

It is clear from the results of this research that when calculating the approximate values of the continuous integrations of the continuous inversions of the two bases from the two mid-point bases and the Simpson base on the dimensions x and when the number of partial periods divided by the period on the internal dimension is not equal to the number of partial periods divided by the period on the outer dimension. These two rules give correct values (for several decimal places) in comparison with the true values of integrations and using a n umber of partial periods without using Romberg acceleration. This is also more evident in the calculation of binary integrations with continuous calls at each point of the integration area. For example, in integrations I, II and IV, there is a correct value of six decimal places. In the third integration, the value was valid for five decimal places using the SIM&M and M&SIM rules.

CONCLUSION

RSIM&M and RM&SIM gave better results in terms of the speed of approaching a few relatively partial periods to real integrals values as they corresponded to the real value in the four integrations. Thus, RSIM&M with continuous calls giving high accuracy in the results in relatively few partial periods and in a very short time.. we can depend about the two method in a calculation the double integrals with continuous integrand.

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