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An Asymptotic Solution to the Blasius Equation and Nonexistence of Periodic Orbits of the Blasius System

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Abstract: In this study, we find a Blasius solution using Neumann series for big values of the independent variable and we also prove that the Blasius dynamical system on the three dimensional space does not have periodic orbits by mean of an auxiliary function and Poincare's method of tangential curves. Also, we use finite differences method to find a numerical solution of the Blasius equation, for this porpose we write a code in MATLAB which gives values of the solution, first and second derivatives and its respective plot on the plane.

Key words: Boundary layer, Blasius equation, numerical solution, dynamical systems, periodic orbits, plane

INTRODUCTION

Using Poincare's method of tangential curves some non periodic orbits of fluids in three dimensional space was treated (Osuna *et al.*, 2014). Something about thermal conductivity with some velocity distribution and fluid flow can be found (Diaz-Salgadu *et al.*, 2014). Using Poincare transformation was proved that there are not periodic orbits in a Duffing equation (Bush, 1992) and in a Lienard system (Jimenez-Sarmiento *et al.*, 2014). Neumann series was used for the combined Sinh-Cosh-Gordon equation (Marin *et al.*, 2014, 2015). A dynamical system was taken to study the dissipative Klein-Gordon equation (Almanza-Vasquetz *et al.*, 2015). Blasius fluid dynamics can be found in (Tritton, 1989). It was found a method for the nonexistence of periodic orbits (Busenberg and Vandenriessche, 1993). The Oreganetor system has no periodic orbits (Murray, 1989; Osuna *et al.*, 2014). By Ahmad and Al-Barakat (2009) an approximate analytic solution of the Blasius problem was found. In this study, we look for a new Blasius boundary layer solution and we show that the solution does not have periodic orbits.

MATERIALS AND METHODS

The governing equations are:

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=V\frac{\partial^2 U}{\partial y^2} \quad (1)$$

$$U\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \quad (2)$$

where, v = 1/Re<<1 and Re is the Reynolds number. Consider the region x>0, y>0 with the following conditions:

$$U(x,0)=V(x,0)=0,\ x>0$$
$$U(0,y)=1,\ V(0,y)=0,\ y>0$$
$$U(x,y)=1,\ x>0,\ y\to\infty$$

We define:

$$U=\frac{\partial F}{\partial \eta} \quad (3)$$

Where:

$$\xi=x,\eta=\frac{ay}{\sqrt{x}}\text{ with } a=\sqrt{\frac{1}{2y}}\text{ leads to}$$

The Blasius equation:

$$F'''+FF''=0 \quad (4)$$

with boundary conditions:

$$F(0)=0,\ \frac{\partial F}{\partial \eta}(0)=0$$
$$\frac{\partial F}{\partial \eta(\eta)}=1 \quad \text{as} \quad \eta\to\infty$$

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RESULTS AND DISCUSSION

Theorem 2.1: The solution of the Blasius Eq. 4 is given by:

$$F(x) = x - \sigma\left(-\sqrt{\frac{\pi}{2}}\, x\,\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right) - e^{-\frac{x^2}{2}} + \sqrt{\frac{\pi}{2}} + 1\right)$$
as $x \to \infty$

where, T is defined by:

$$T(F)(x) = \int_0^x \int_x^\infty e^{-\int_0^\eta F(y)dy} d\eta dx$$

σ is defined by $\sigma = F''$ (0 = 0.47).

Proof: Taking:

$$F' = 0 \qquad \frac{F'''}{F''} = -F \tag{5}$$

and integrating with respect to y, we get:

$$\mathrm{In}F''(y) = -\int F(y)d + C$$

where, C is a constant:

$$F''(\eta) = Ke^{-\int_0^\eta F(y)dy} \tag{6}$$

where, $K = e^C$ with e is the Euler number. Integrating with respect to η with η between x and ∞, we obtain:

$$\int_x^\infty F''(\eta)d\eta\ K\int_x^\infty e^{-\int_0^\eta F(y)dy} d\eta$$

Integrating with respect to η on the left hand side:

$$1 - F'(x) = K\int_x^\infty e^{-\int_0^\eta F(y)dy} d\eta$$

because $F'(\infty) = 1$, then:

$$1 - K\int_x^\infty e^{-\int_0^\eta F(y)dy} d\eta = F'(x)$$

We have that if $x \to \infty$ then:

$$\int_x^\infty e^{-\int_0^\eta F(y)dy} d\eta \to 0 \text{ and } F'(x) \to 1$$

Because $F'(0) = 0$ then:

$$\int_x^\infty e^{-\int_0^\eta F(y)dy} d\eta = 1 \tag{7}$$

By Eq. 6 $-F''(0) = 0$, then:

$$1 - F''(0)\int_x^\infty e^{-\int_0^\eta F(y)dy} d\eta = F'(x) \tag{8}$$

With F (0) = 0 putting Eq. 8 in the form:

$$1 = F'(x) + F''(0)\int_x^\infty e^{-\int_0^\eta F(y)dy} d\eta \tag{9}$$

Then, we can write:

$$1 - \frac{\int_x^\infty e^{-\int_0^\eta \int_0^y F(y)dtdy} d\eta}{\int_x^\infty e^{-\int_0^\eta \int_0^y F(y)dtdy} d\eta} = \left[\left(1-\tilde{T}\right)F'\right](x) \tag{10}$$

Where:

$$\left[\tilde{T}(F')\right](x) = \int_x^\infty \frac{e^{-\int_0^\eta \int_0^y F(t)dtdy} d\eta}{\int_x^\infty} = e^{-\int_0^\eta \int_0^y F(t)dtdy}$$

If $F'(x) = 1$ then:

$$\left[\tilde{T}(F')\right](x) = \int_x^\infty e^{-\frac{\eta 2}{2}} d\eta = \sqrt{\left(\frac{\pi}{2}\right)\left(1 - \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right)}$$

$$1 - \sigma e^{-\frac{x2}{2}}\frac{1}{x} = 1 - \sigma\sqrt{\left(\frac{\pi}{2}\right)\left(1 - \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right)} = F'(x)$$

We observe $F'(x) \to 1$ when $x \to \infty$. $F''(x) = \sigma e^{-x2/2}$ for big x. If: $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$ with $f(0) = 0$, $f'(0) = 0$, $f''(0) = \sigma$ then $a_0 = 0$, $a_1 = 0$, $a_2 = \sigma/2$ we obtain for the main term $f(x) \approx \sigma/2\, x^2$ and $f'(x) = \sigma x$, $f''(x) = \sigma$ for small x. By truncated Taylor series and: $f''' = -ff''$ and $\varepsilon = 0.2$:

$$f(x+\varepsilon) \simeq f(x) + \varepsilon f'(x) \tag{13}$$

$$f(x+\varepsilon) \simeq f'(x) + \varepsilon f''(x) \tag{14}$$

$$f''(x+\varepsilon) \simeq f''(x) + \varepsilon f'''(x) = (x) - \varepsilon f(x) f'''(x) \tag{15}$$

and with the initial conditions $f(0) = f'(0) = 0$ and $f'(x) \to 1$ when $x \to$ and $f''(0) = \sigma = 0.47$. We write the following code in MATLAB:

$y0 = 0,\ z0 = 0,\ w0 = 0.47,\ h = 0.2$

$t = 0:h:10,\ k0 = zeros(size(t))$

$k2 = zeros(size(t)),\ k1 = zeros(size(t))$

$k0(1)=y0,\ k2(1) = w0$

For:

$i = 1:(length(t)-1)$

$k0(i+1) = k0(i)+h*k1(i)$

$k1(i+1) = k1(i)+h*k2(i)$

$k2(i+1) = k2(i)-h*k0(i)*k2(i)$

End:

plot(t,k0,t, k1,t,k2)

legend('f,',f1,',f2')

Some values of f, f′ and f′′ are:

$$f(0) = 0, f(1) = 0.1879, f(2) = 0.8289, ..., f(10) = 0.5217$$
$$f'(0) = 0, f'(1) = 0.4682, f'(2) = 0.8716, ..., f'(10) = 0.2057$$
$$f''(0) = 0.47, f''(1) = 0.4525, f''(2) = 0.2938, ..., f''(10)$$

Non existence of periodic orbits: From the Eq. 4 and taking:

$$z = F'',\ Z = F''\text{ and }y = F''\ x'$$

Therefore, we obtain the following Blasius system:

$$\begin{cases} x' = y \\ y' = z \\ z' = -xz \end{cases} \qquad (16)$$

The critical points of the Blasius system are (x, 0, 0) for all x∈R. From Osuna *et al*. (2014) and Zhifen *et al*. (2006), we have:

$$\dot{X} = F(x),\ x \in \Omega \qquad (17)$$

Where:

$$F(x):= (f_1(x_1, ..., x_n), ..., f_n(x_1, ..., x_n)),\ \Omega \subset R^n$$

is an open set and fi: $\Omega \to R$, $1 \le I \le n$ are C^1. We consider the critical points. Z (F): = $\{x \in R^n: F(x) = 0\}$, then.

Theorem 3.1; Osuna *et al.* (2014): Let $\Omega \subseteq R^n$ be an open connected subset. Suppose that there exists H: $\Omega \to R$, C^1, such that for any non-constant curve a: $[0:T] \to \Omega/Z(F)$ the integral:

$$\int_0^T DH(a)F(a)a's \neq 0 \qquad (18)$$

then the Eq. 17 does not have any closed orbit in Ω. Taking the following equation that satisfies the properties of Theorem 3.1:

$$f_1\frac{\partial H}{\partial x}+f_2\frac{\partial H}{\partial y}+f_3\frac{\partial H}{\partial z} = HC \qquad (19)$$

Now, we use the Eq. 19 to prove nonexistence of periodic orbits of Blasius system. We find a different H from (Osuna *et al*., 2014).

Theorem 3.2: The Blasius system (Eq. 16) does not have periodic orbits.

Proof: We suppose that H depends on t = t (x, y, z). Using chain rule in the differential Eq. 19 and $f_1 = y$, $f_2 = -xz$ we obtain:

$$y\frac{\partial H}{\partial t}\frac{\partial t}{\partial x}+z\frac{\partial H}{\partial t}\frac{\partial t}{\partial y}-xz\frac{\partial H}{\partial t}\frac{\partial t}{\partial z} = H(t)C(x,y,z)$$

Then:

$$\left(y\frac{\partial t}{\partial x}+z\frac{\partial t}{\partial y}-xz\frac{\partial t}{\partial z}\right)\frac{\partial H}{\partial t} = H(t)C(x,y,z)$$

$$\text{if }\frac{\partial H}{\partial t} = \text{then } H(t) = e^t$$

$$\left(y\frac{\partial t}{\partial x}+z\frac{\partial t}{\partial y}-xz\frac{\partial t}{\partial z}\right) = (t)C(x,y,z)$$

$$\text{if } t = xy+z \text{ then:}$$

$$\frac{\partial t}{\partial x} = y,\ \frac{\partial t}{\partial y} = x\frac{\partial t}{\partial z} = 1$$

and we obtain C (x, y, z) = $y^2>0$ with $y\neq 0$, this indicates (X, y, z) with $y\neq 0$ is not a critical point, because (x, 0, 0) with x∈R are the critical points of the Blasius system. C is positive outside the set of critical points. Then, we get the integral:

$$\int_0^T (e^{xy+z}y^2)(a(s))ds > 0$$

for any curve a. Hence, by Theorem 3.1 Blasius system does not have periodic orbits.

CONCLUSION

In this reserach was found a new solution of the aproximated Blasius equation using Newmann series and was proved the corresponding Blasius dynamical system does not have periodic orbits in all space using auxiliary functions.

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REFERENCES

Ahmad, F. and W.H. Al-Barakati, 2009. An approximate analytic solution of the Blasius problem. Commun. Nonlinear Sci. Numer. Simul., 14: 1021-1024.

Almanza-Vasquez, E., C. de Indias, A.M. Marin-Ramirez and R.D. Ortiz-Ortiz, 2015. Approximation to the dissipative klein-gordon equation. Int. J. Math. Anal., 9: 1059-1063.

Busenberg, S. and P. Vandendriessche, 1993. A method for proving the non-existence of limit cycles. J. Math. Anal. Appl., 172: 463-479.

Bush, A.W., 1992. Perturbation Methods for Engineers and Scientist. CRC Press, Florida.

Diaz-Salgado, A., C. de Indias, A.M. Marin-Ramirez and R.D. Ortiz-Ortiz, 2014. The fluid of couette and the boundary layer. Int. J. Math. Anal., 8: 2561-2565.

Jimenez-Sarmiento, J.E., C. de Indias, A.M. Marin-Ramirez and R.D. Ortiz-Ortiz, 2014. Poincare transformation in a lienard system. Int. J. Math. Anal., 8: 2421-2425.

Marin-Ramirez, A.M., C. de Indias, J.A. Ortega-Ramos and R.D. Ortiz-Ortiz, 2014. A result of a duffing equation. Int. J. Math. Anal., 8: 2173-2176.

Marin-Ramirez, A.M., C. de Indias, V.P. Jaramillo-Camacho and R.D. Ortiz-Ortiz, 2015. Solutions for the combined sinh-cosh-gordon equation. Int. J. Math. Anal., 9: 1159-1163.

Murray, J.M., 1989. Mathematical Biology. 2nd Edn., Springer-Verlag, Berlin, Germany, ISBN-13: 9780387194608, Pages: 767.

Osuna, O., J. Rodriguez and G. Villasenor, 2014. Some remarks on Poincares method of tangencial curves. Intl. J. Contemp. Math. Sci., 9: 411-415.

Tritton, D., 1989. Physical Fluid Dynamics. Oxford University Press, Oxford, UK.,.

Zhifen, Z., D. Tongren, H. Wenzao and D. Zhenxi, 2006. Qualitative Theory of Differential Equations. Vol. 101, American Mathematical Society, Providence, Rhode Island, ISBN:0-8218-4551-9, Pages: 464.