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Optimization of the CBR of Lateritic Soil Stabilized with Quarry Dust

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Abstract: This study proposes a model for optimizing the California Bearing Ratio (CBR) of loose lateritic soil mixed with quarry dust and water using Scheffe's experimental design techniques on simplex lattice. The developed model uses a polynomial of second degree to express the behavior of the components of the mixture in a simplex lattice. Weights of different components of the mixture generated from Scheffe's theory were used in arriving at the resulting model. A computer program for optimization based on the model is also developed. The optimal mix proportion obtained by the model is 1:2.75:0.135 (Lateritic soil: quarry dust: water) with a CBR value of 18.2%. This represents an increase of about 56.9% in the CBR value of the original lateritic soil. The model predictions were compared with the experimental results and found to be adequate at 5% significance level.

Key words: CBR, simplex lattice, stabilization, optimization, developed, arriving

INTRODUCTION

Quite often engineers are faced with the challenge of very poor soil as the subgrade of the road. A solution commonly proffered to such problems of unsuitable soil is part or total removal of the existing soil and its replacement with imported better soil. In extreme cases, the site is abandoned in favour of a more suitable land. Recently, however, engineers have been compelled to search for methods of improving the quality of the naturally-occurring poor soil without having to remove or abandon it more, so with the evolving scarcity of construction land. This expediency has no doubt, spurred interest in the area of soil improvement through stabilization. The conventional methods of soil stabilization using chemicals have been found to be increasingly expensive and in some cases there is scarcity of chemical stabilizers. Quarry dust which is readily available as an industriall waste has been proven to improve the bearing capacity of the soil (Indiramma and Sudharani, 2016). It therefore, becomes an interesting area of research to ascertain its optimal applicability in the stabilization of lateritic soil. Being that the California Bearning Ratio (CBR) is an indicator of the mechanical strength of natural ground (subgrade) and base courses beneath new carriageway construction, this study therefore, aims at the development of a model for optimizing the CBR of loose lateritic soil stabilized with quarry dust.

Formulation of model: Scheffe (1963) formulated a model for the assessment of the response of a particular characteristic of a mixture to variations in the proportions of its component materials. In his simplex lattice model, he considered experiments with mixtures in which the desired property (in this case, the CBR) depends on the proportion of the constituent materials present as atoms of the mixture. A simplex lattice can be described as a structural representation of lines joining the atoms of a mixture. It can be used as a mathematical space in model experiments involving mixtures by considering the atoms as the constituent components of the mixture (Akhnazarova and Kafarov, 1982).

When studying the components of a q-component mixture which are dependent on the component ratio only, the factor space is a regular (q-1) simplex and for the mixture, the following relationship holds (Scheffe, 1963):

$$\sum_{i=1}^{q} X_i = 1 \quad (1)$$

Where:
$X_i \geq 0$ = The component concentration
q = The number of components

For a 3 component mixture (q = 3), the regular 2 simplex is an equilateral triangle, each with its interior. Each point in the triangle corresponds to a certain composition of the ternary system and conversely each composition is represented by one distinct point. The vertices of the triangle represent pure substances and the side's binary systems.

To describe the response surfaces in multi component systems adequately, high degree polynomials are required

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and hence, a great many experimental trials. The response is the property of mixture sought and in this case it is the CBR of the mixture of lateritic soil, quarry dust and water. A polynomial of degree n in q variables has C^{n}_{q+n} coefficients (Scheffe, 1963):

$$\hat{Y} = b_0 + \sum_{1 \le i \le q} b_i x_i + \sum b_{ij} x_i x_j + \sum b_{ijk} x_i x_j x_k + ,...,+ \sum b_{i1i2}, ..., \text{ in } x_{i1}\ x_{i2}\ ,..., x_{in} \quad (2)$$

The relationship $\sum^{q} X_{i=1}$ enables the qth component to be eliminated and the number of coefficients reduced to C^{n}_{q+n-1}. Scheffe (1963) suggested to describe mixture properties by reduced polynomials obtainable from Eq. 2 subject to the normalization condition of Eq. 1 for the sum of independent variables. For a ternary system, the polynomial has the general form:

$$\hat{Y} = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 \quad (3)$$

But:

$$X_1 + X_2 + X_3 = 1 \quad (4)$$

Multiplying Eq. 4 by b_0:

$$b_0X_1 + b_0X_2 + b_0X_3 = b_0 \quad (5)$$

Multiplying Eq. 4 by X_1,-X_3 in succession gives:

$$\left.\begin{aligned} X_1^2 &= X_1 - X_1X_2 - X_1X_3 \\ X_2^2 &= X_2 - X_1X_2 - X_2X_3 \\ X_3^2 &= X_3 - X_1X_3 - X_2X_3 \end{aligned}\right\} \quad (6)$$

Substituting Eq. 5 and 6 into Eq. 3, we obtain after necessary transformations:

$$\hat{Y} = (b_0 + b_1 + b_{11})X_1 + (b_0 + b_2 + b_{22})X_2 + (b_0 + b_3 + b_{33})X_3 + (b_{12} - b_{11} - b_{22})X_1X_2 + (b_{13} - b_{11} - b_{33})X_1X_3 + (b_{23} - b_{22} - b_{33})X_2X_3 \quad (7)$$

If, we denote:

$$\left.\begin{aligned} \beta_i &= b_0 + b_i + b_{ii} \\ \beta_{ij} &= b_{ij} - b_{ii} - b_{jj} \end{aligned}\right\} \quad (8)$$

Then, we arrive at the reduced second-degree polynomial in three variables:

$$\hat{Y} = \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{23}X_2X_3 \quad (9)$$

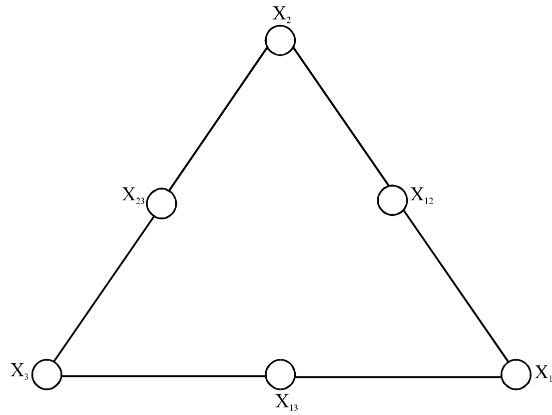

Fig. 1: 3, 2-lattice for a second-degree polynomial; We denote the components as; X_1 = Proportion of lateritic soil in the mixture; X_2 = Proportion of quarry dust in the mixture; X_3 = Proportion of water in the mixture

Thus, the number of coefficients has reduced from ten to six. In a more general form, the reduced second degree polynomial in q variables is:

$$\hat{Y} = \sum_{1 \le i \le q\ 1} \beta_i x_i + \sum \beta_{ij} x_i x_j \quad (10)$$

The simplex lattice design provides a uniform scatter of points over the (q-1) simplex. The points form a (q, n) lattice on the simplex where q is the number of mixture components and n is the degree of the polynomial. For a second degree polynomial, the (3, 2)-lattice is represented schematically in Fig. 1

As can be seen from Fig. 1 at any vertex of the triangle only one component of the mixture is present while at the boundary lines two components exist and the third is absent. Thus, points 1, 2 and 3 of the triangle have coordinates (1, 0, 0) (0, 1, 0) and (0, 0, 1), respectively. If we substitute the above lattice coordinates into Eq. 9, we obtain the coefficients of the second degree polynomial as:

$$\left.\begin{aligned} \beta_1 &= Y_1 \\ \beta_2 &= Y_2 \\ \beta_3 &= Y_3 \end{aligned}\right\} \quad (11)$$

Also:

$$\left.\begin{aligned} \beta_{12} &= 4Y_{12} - 2Y_1 - 2Y_2 \\ \beta_{13} &= 4Y_{13} - 2Y_1 - 2Y_3 \\ \beta_{23} &= 4Y_{23} - 2Y_2 - 2Y_3 \end{aligned}\right\} \quad (12)$$

Generally, the coefficients of the second-degree polynomial for a q-component mixture is given by:

$$\left.\begin{aligned}\beta_i &= Y_i\\ \beta_{ij} &= 4Y_{ij}-2Y_i-2Y_j\end{aligned}\right\} \quad (13)$$

MATERIALS AND METHODS

The relation between the actual components (Z) and the pseudo-components (X) is Ezeh *et al*. (2010):

$$(Z) = (A)(X) \quad (14)$$

From the real components, a Z-matrix is formed whose transpose becomes the conversion of the factor from pseudo to real component. Thus, if we select the first three mix ratios of our mixture components (Lateritic soil: quarry dust: water) then the real component simplex will be as shown in Fig. 2. Thus:

$$(Z) = \begin{bmatrix}1.00 & 2.00 & 0.12\\ 1.00 & 2.50 & 0.13\\ 1.00 & 3.00 & 0.14\end{bmatrix}$$

And:

$$(Z)^T = \begin{bmatrix}1.00 & 1.00 & 1.00\\ 2.00 & 2.50 & 3.00\\ 0.12 & 0.13 & 0.14\end{bmatrix}$$

Table 1 is a matrix table showing pseudo-components and real components for the (3, 2)-lattice. Note that a row in the real component side is obtained by multiplying $[Z]^T$ matrix by the corresponding row in the pseudocomponent side of Table 1. For example for row 4 in Table 1:

$$(Z)^T = \begin{bmatrix}1.00 & 1.00 & 1.00\\ 2.00 & 2.50 & 3.00\\ 0.12 & 0.13 & 0.14\end{bmatrix}\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\\ 0\end{bmatrix} = \begin{bmatrix}1.000\\ 2.250\\ 0.125\end{bmatrix}$$

Laboratory CBR test: The California Beaming Ratio (CBR) test is used for evaluating the suitability of subgrade and the materials used in sub-base and base courses. The test is generally carried out in the laboratory on remoulded samples as per BS 1377.

Lateritic soil for the test was collected from a construction site at the University of Nigeria, Nsukka Campus in Enugu state. The quarry dust was sourced from Ishiagu quarry plant in Ebonyia state. The soil, quarry dust and water were then mixed by weight as per

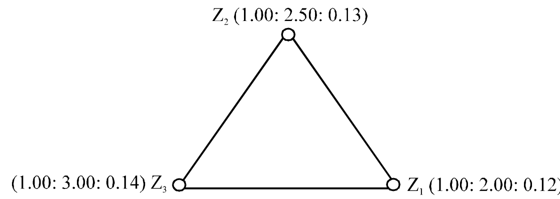

Fig. 2: Real component simplex (only vertices shown)

Table 1: Matrix table of real and pseudo-components for (3.2)-lattice polynomial

Pseudo-components				Real components		
X_1	X_2	X_3	Response	Z_1	Z_2	Z_3
1	0	0	Y_1	1.00	2.00	0.120
0	1	0	Y_2	1.00	2.50	0.130
0	0	1	Y_3	1.00	3.00	0.140
1/2	1/2	0	Y_{12}	1.00	2.25	0.125
1/2	0	1/2	Y_{13}	1.00	2.50	0.130
0	1/2	1/2	Y_{23}	1.00	2.75	0.135
Control points						
1/3	1/3	1/3	C_1	1.00	2.50	0.130
1/3	2/3	0	C_2	1.00	2.33	0.127
0	1/3	2/3	C_3	1.00	2.83	0.137
1/6	1/6	2/3	C_4	1.00	2.75	0.135
1/6	2/3	1/6	C_5	1.00	2.50	0.130
2/3	1/6	1/6	C_6	1.00	2.25	0.125

Z_1 = Lateritic soil, Z_2 = Quarry dust, Z_3 = Water

the actual component ratios in Table 1. The mixture is subjected to CBR test as per BS 1377 Pt. 9: 1990 (Anonymous, 1957). Two replicate experiments were conducted for each test point.

RESULTS AND DISCUSSION

The results and average from the test points are tabulated in columns 6-8 of Table 2. Extra six test points (control points) were provided for validation of the model.

Development of the model: The general form of Scheffe's reduced second-degree polynomial in q-variables is given by Eq. 10:

$$\hat{Y} = \sum_{1\le i\le q}\beta_i x_i + \sum \beta_{ij} x_i x_j \quad (15)$$

Hence:

$$\beta_i = Y_i,\ \beta_{ij} = 4Y_{ij}-2Y_i-2Y_j$$

For a 3-component mixture (i.e. (3, 2)-simplex lattice), q = 3. Therefore:

$$\hat{Y} = \beta_1X_1+\beta_2X_2+\beta_3X_3+\beta_{12}X_1X_2+\beta_{13}X_1X_3+\beta_{23}X_2X_3 \quad (16)$$

Using the average responses from Table 2 (column 8):

Table 2: Responses from experiment and predictions from model

Pseudo components				Replicate response CBR (%)				Real components (mix ratios)		
X_1	X_2	X_3	Response symbol	1	2	Average response (%)	Predicted values (%)	Z_1	Z_2	Z_3
1	0	0	Y_1	14.0	15.4	14.7	14.7	1.00	2.00	0.120
0	1	0	Y_2	14.7	16.1	15.4	15.4	1.00	2.50	0.130
0	0	1	Y_3	15.4	17.5	16.5	16.5	1.00	3.00	0.140
1/2	1/2	0	Y_{12}	16.1	14.0	15.1	15.1	1.00	2.25	0.125
1/2	0	1/2	Y_{13}	16.1	17.5	16.8	16.8	1.00	2.50	0.130
0	1/2	1/2	Y_{23}	16.8	19.6	18.2	18.2	1.00	2.75	0.135
Control points (7-12)										
1/3	1/3	1/3	C_1	16.8	18.2	17.5	17.09	1.00	2.50	0.130
1/3	2/3	0	C_2	16.8	14.0	15.4	15.21	1.00	2.33	0.127
0	1/3	2/3	C_3	16.8	18.9	17.9	18.13	1.00	2.83	0.137
1/6	1/6	2/3	C_4	17.5	18.2	17.9	17.56	1.00	2.75	0.135
1/6	2/3	1/6	C_5	16.8	16.1	16.5	16.62	1.00	2.50	0.130
2/3	1/6	1/6	C_6	16.1	16.1	16.1	15.92	1.00	2.25	0.125

$$\beta_1 = Y_1 = 14.70$$
$$\beta_2 = Y_2 = 15.40$$
$$\beta_3 = Y_3 = 16.50$$
$$\beta_{12} = 4Y_{12}\text{-}2Y_1\text{-}2Y_2 = 0.20$$
$$\beta_{13} = 4Y_{13}\text{-}2Y_1\text{-}2Y3 = 4.80$$
$$\beta_{23} = 4Y_{23}\text{-}2Y_2\text{-}2Y_3 = 9.00$$

Substituting these coefficients in Eq. 9, the required optimisation model for the CBR of soil stabilized with quarry dust becomes:

$$\hat{Y} = (14.7)X_1+(15.4)X_2+(16.5)X_3+(0.2)X_1X_2+(4.8)X_1X_3+(9.0)X_2X_3 \quad (17)$$

The predicted values from Eq. 17 are presented in Table 2, Column 9.

Validation of the model (test for adequacy): The model equation was tested to check whether the model results agree with the actual experimental results. Let, the statistical null hypothesis be denoted by H_o and the alternative by H_I.

- H_o: there is no significant difference between the analytical (experimental) and the model predicted results
- H_I: there is significant difference between the analytical and the predicted results

The fisher statistical test was used to test the adequacy of the model. The model predicted values (Y_M) for the control points were obtained by substituting the corresponding pseudo-components (X_i) into the model equation, i.e., Eq. 17. These results were compared with the experimental results (Y_E) given in Table 3. Average experimental response $Y_{EA} = \sum Y_E/N = 101.30/6 = 16.883$; Average model-predicted response $Y_{EA} = \sum Y_E/N = 100.53/6 = 16.755$ where N is the number of responses. Computing the variance for both experimental and model results:

Table 3: F-statistics for the control points

Response	Y_E	Y_M	Y_E-Y_{EA}	Y_M- Y_{MA}	$(Y_E\text{-}Y_{EA})^2$	$(Y_M\text{-}Y_{MA})^2$
C_1	17.5	17.09	0.617	0.335	0.3807	0.1122
C_2	15.4	15.21	-1.483	-1.245	2.1993	1.5500
C_3	17.9	18.13	1.017	1.375	1.0343	1.8906
C_4	17.9	17.56	1.017	0.805	1.0343	1.6480
C_5	16.5	16.62	-0.383	-0.135	0.1467	0.0182
C_6	16.1	15.92	-0.783	-0.835	0.6131	0.6972
SUM	101.30	100.53	-	-	5.4084	4.9162

Y_E = Experimental CBR, Y_M = Model CBR, Y_{EA} = Average experimental CBR; Y_{MA} = Average model CBR, N = Number of points of observation

$$S_E^2 = \frac{\sum(Y_E\text{-}Y_{EA})^2}{(N\text{-}1)} = \frac{5.4084}{5} = 1.08168$$

$$S_M^2 = \frac{\sum(Y_M\text{-}Y_{MA})^2}{(N\text{-}1)} = \frac{4.9162}{5} = 0.98324$$

The fisher test statistic factor is given by $F = S_E^2/S_E^2$, since, S_E^2 is higher, i.e, F = 1.08168/0.98324 = 1.100. From the standard F-distribution Table for (N-1) degree of freedom the F-statistic, $F_{0.95}(5, 5) = 5.05$. Since, this is higher than the calculated value of 1.100, we accept the null hypothesis. Therefore, the model equation is adequate.

CONCLUSION

A comparison of the predicted results with the analytical results shows that the average percentage difference is 1.43% which is acceptable. Also, the fisher test used in the statistical hypothesis showed that the developed model is adequate and reliable at 5% significance level for predicting the CBR value of loose lateritic soil stabilized with quarry dust. The optimal mix proportion obtained by the model is 1:2.75:0.135

(lateritic soil: quarry dust: water) with a CBR value of 18.2%. This represents an increase of about 56.9% in the CBR value of the original lateritic soil. The CBR value of the experimental mix is suitable for use in the improvement of the subgrade. To achieve a higher CBR value for use as sub-base it will be necessary to use a lateritic soil of higher quality or add some quantity of cement or lime to the mix.

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