Journal of Engineering and Applied Sciences 13 (12): 4250-4259, 2018
ISSN: 1816-949X

Optimal Systematical Nodes Numbering Technique in the Three Dimensional Analysis of Structural Elements

Hayder M.K. Al-Mutairee
Department of Civil Engineering, College of Engineering, University of Babylon, Hillah, Iraq

Abstract: This scientific study involves a comparative study of eighteen systematical technique that can be used in numbering the nodes of finite elements meshes in the three dimensional analysis of structural elements which have a rectangular section. The BandWidth (BW) equations of these techniques were derived by using 8-nodes linear brick element and the optimal technique, called as (SML, Small Medium Large) is selected according to the minimum BW. Besides, the general equations of computing the BW for 8, 20 and 32-nodes brick element are derived depending on the SML technique. The research produces a simple optimal systematical nodes numbering technique that can be understood and used by engineers and researchers easily. The optimal systematical technique SML avoids the computational effort required for reordering the coefficients of matrix or renumbering the nodes which are usually used in traditional techniques. The study proves that the numbering technique SML is efficient, applicable and easy to be implemented. The technique also reduces the execution time to about 99% in some cases. For practical benefit, nodes mesh generation program for straight beam or wall is prepared according to the numbering technique SML.

Key words: Nodes numbering, bandwidth, finite elements, three dimensional analysis, SML, beam

INTRODUCTION

Recently, many structural three dimensional analysis studies have been done utilizing the Finite Element Method (FEM) which represents a powerful tool to analyze the structural elements (slabs, beams, columns, walls, foundations, ..., etc). These analyses always produce a symmetric and well diagonal blocked “banded” stiffness matrix which means that the non-zero terms have been situated around the major diagonal. Generally, the FEM produces a large linear system of:

$$[K]\{d\} = \{F\} \quad (1)$$

The [K] is a positive square symmetrical known as N×N, stiffness matrix and it is extremely sparse. It has a large number of zero coefficients in a typical picture due to the absence of some nodes numbers locate between the largest and smallest node number within the element. {d} is a vector of unknowns displacements of length N while the vector {F} represents the forces which are the known vector of length N, where N represents the total number of equations. Mathematically, it is preferable to make reduction in the size of [K] to avoid the unnecessary operations with zero values during the subsequent numerical calculations. This is achieved by limiting the number of zero in order to minimize the memory space and finally reduce the execution time. The execution time is approximately proportional either to the cubic of N, if the bandedness is not considered or to the square of the BW when the bandedness is accounted (Quoc and O’Leary, 1984; Papadopoulos, 2005). The need to minimize the BW of [K] of the finite element meshes becomes very important in the nonlinear analysis where it is often necessary to solve algebraic equations many times. Almost commercial programs have a built-in nodes numbering technique. This technique is usually optional where it requires more computational effort and time to perform the minimization which performed either by internally renumbering the nodes using one of algorithms of direct schemes (in these schemes for output purposes, the nodes numbering scheme defined in the input data used but internally the program uses the optimal nodes numbering scheme) or by changing the profile of [K] in some manners to reduce the BW by using one of algorithms of iterative schemes.

Many research studies have been done previously concerning minimizing the execution time of solving the global matrix which can be classified into two types: iterative and direct schemes. All these studies are efficient in reducing the BW of the global matrix and/or the profile. The iterative algorithms (Always and Martin, 1965; Rodrigues, 1975; Veldhorst, 1982) such as Jacobi, gauss-seidel, conjugate gradient and steepest descent are basically dependent on reordering the coefficients of the matrix to minimize its BW by finding the non-zero coefficients that cause the largest BW and then interchanging columns and rows in some manner that leads to reduce the BW. This category requires substantial high speed storage and it is more efficient for large systems. While the direct schemes (Gibbs *et al.*, 1976; Sloan and Randoph, 1983; Boutora *et al.*, 2007; Wang and Shi, 2009) such as gauss elimination, choleski and frontal are generally based on some consideration of theoretical graph concepts including renumbering the nodes to reduce the BW of matrix. The direct schemes are used for relatively small systems ($N \leq 10^6$) (Papadopoulos, 2005). Many bibliographies of renumbering algorithms are also presented in reference (Everstine, 1979). Many researchers illustrated the effects of nodes numbering scheme on the BW of matrix, their examples focused on structures of one and/or two direction (s) (Sloan and Randolph, 1983; Wang and Shi, 2009; Hearn, 1997; Kaveh, 1995; Kress, 2014). Throughout the search for available previous researches, the subject of nodes numbering and its effect on the size and profile of [K] of structural element in three dimensional analyses is rarely addressed by the researchers, thus, this research is devoted for this purpose.

MATERIALS AND METHODS

Systematical nodes numbering techniques: The numbering scheme of the nodes plays an important role in the size and shape of the profile of stiffness matrix [K]. Eighteen possible manners of systematical (regular) nodes numbering in three dimensional analysis of any traditional structural element have a rectangular section presented in this study.

Techniques codes: To illustrate the manner of coding the techniques as well as the effect of nodes numbering technique on the BW of structural stiffness matrix, consider the beam shown in Fig. 1. The 8-nodes brick element with three degree of freedom (dof) per node is adopted. The finite element mesh is selected here to consider beam consisted of 24 elements, (NX×NY×NZ) where NX, NY and NZ represent the number of elements in direction X, Y and Z, respectively. In the first technique (LMS) as shown in Fig. 2a and in Table 1, the numbering starts along the dimension that contains the largest number of elements (it coincides here with the direction +Z), then moves gradually along the dimension that has the medium number of elements (it coincides here with the direction +Y) and finally the numbering moves step by step along the dimension +X which has the smallest number of elements (Note: after moving, the numbering still in the same original direction of numbering, i.e., +Z-direction). Thus, this system is called "LMS". In other words, the first term always refers to the general direction of numbering while the second and third terms always refer to the first and second direction of moving or transition, respectively but the name of each term follows the number of elements in the corresponding direction. The other seventeen techniques shown in Fig. 2 had been coded according to the same manner.

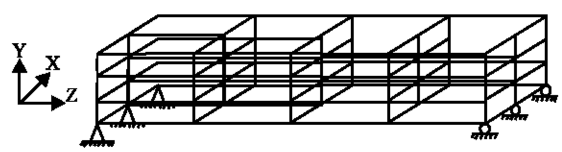

Fig. 1: A simple beam consists of 24 element

Table 1: The details of eighteen techniques by using 8-nodes brick elements

Technique number	Technique code	Main direction of numbering	Directions of moving or transmission	The general equation of (BW)	BW as calculated
1	LMS	+Z, Fig. 2a	+Y then+X	(NZ×NY+2NZ+NY+4)×dof	27×3 = 81
2	LSM	+Z, Fig. 2b	+X then+Y	(NZ×NX+2NZ+NX+4)×dof	22×3 = 66
3	MLS	+Y, Fig. 2c	+Z then+X	(NY×NZ+2NY+NZ+4)×dof	26×3 = 78
4	MSL	+Y, Fig. 2d	+X then+Z	(NY×NX+2NY+NX+4)×dof	18×3 = 54
5	SLM	+X, Fig. 2e	+Z then+Y	(NX×NZ+2NX+NZ+4)×dof	20×3 = 60
6	SML	+X, Fig. 2f	+Y then+Z	(NXNY+2NX+NY+4)×dof	17×3 = 51
7	L±MS	±Z, Fig. 2g	+Y then+X	(NZ×NY+3NZ+NY+3)×dof	30×3 = 90
8	L±SM	±Z, Fig. 2h	+X then+Y	(NZ×NX+3NZ+NX+3)×dof	25×3 = 75
9	M±LS	±Y, Fig. 2i	+Z then+X	(NY×NZ+3NY+NZ+3)×dof	28×3 = 84
10	M±SL	±Y, Fig. 2j	+X then+Z	(NY×NX+3NY+NX+3)×dof	20×3 = 60
11	S±LM	±X, Fig. 2k	+Z then+Y	(NX×NZ+3NX+NZ+3)×dof	21×3 = 63
12	S±ML	±X, Fig. 2l	+Y then+Z	(NX×NY+3NX+NY+3)×dof	18×3 = 54
13	L±M±S	±Z, Fig. 2m	±Y then+X	[(NZ+1)(NY+1)×2]× dof	40×3 = 120
14	L±S±M	±Z, Fig. 2n	±X then+Y	[(NZ+1)(NX+1)×2]× dof	30×3 = 90
15	M±L±S	±Y, Fig. 2o	±Z then+X	[(NY+1)(NZ+1)×2]× dof	40×3 = 120
16	M±S±L	±Y, Fig. 2p	±X then+Z	[(NY+1)(NX+1)×2]× dof	24×3 = 72
17	S±L±M	±X, Fig. 2q	±Z then+Y	[(NX+1)(NZ+1)×2]× dof	30×3 = 90
18	S±M±L	±X, Fig. 2r	±Y then+Z	[(NX+1)(NY+1)×2]× dof	24×3 = 72

The sign (±) in Table 1 and Fig. 2 refers to possibility of alternative direction of numbering or transmission according to the position of sign

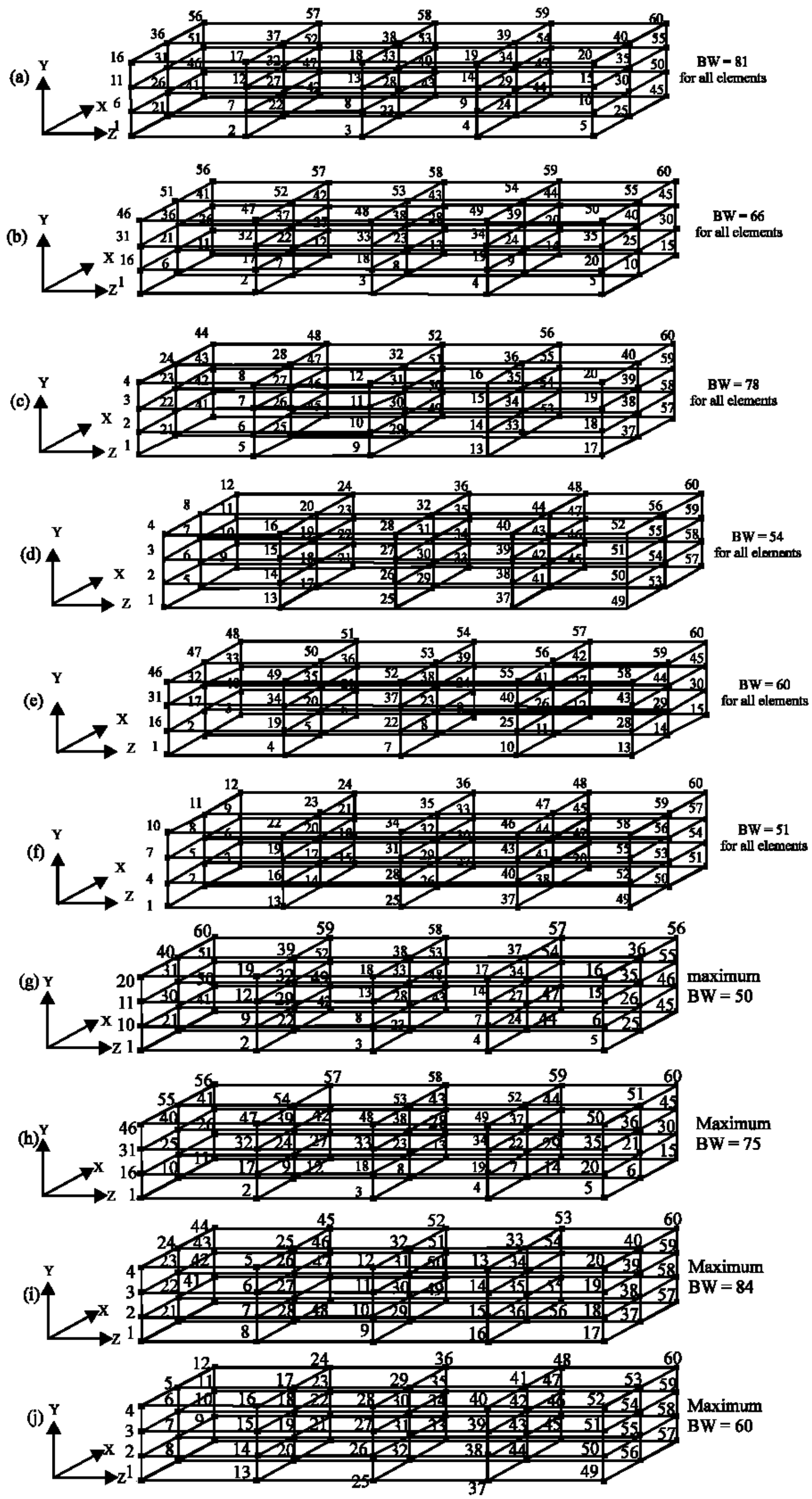

Fig. 2: Continue

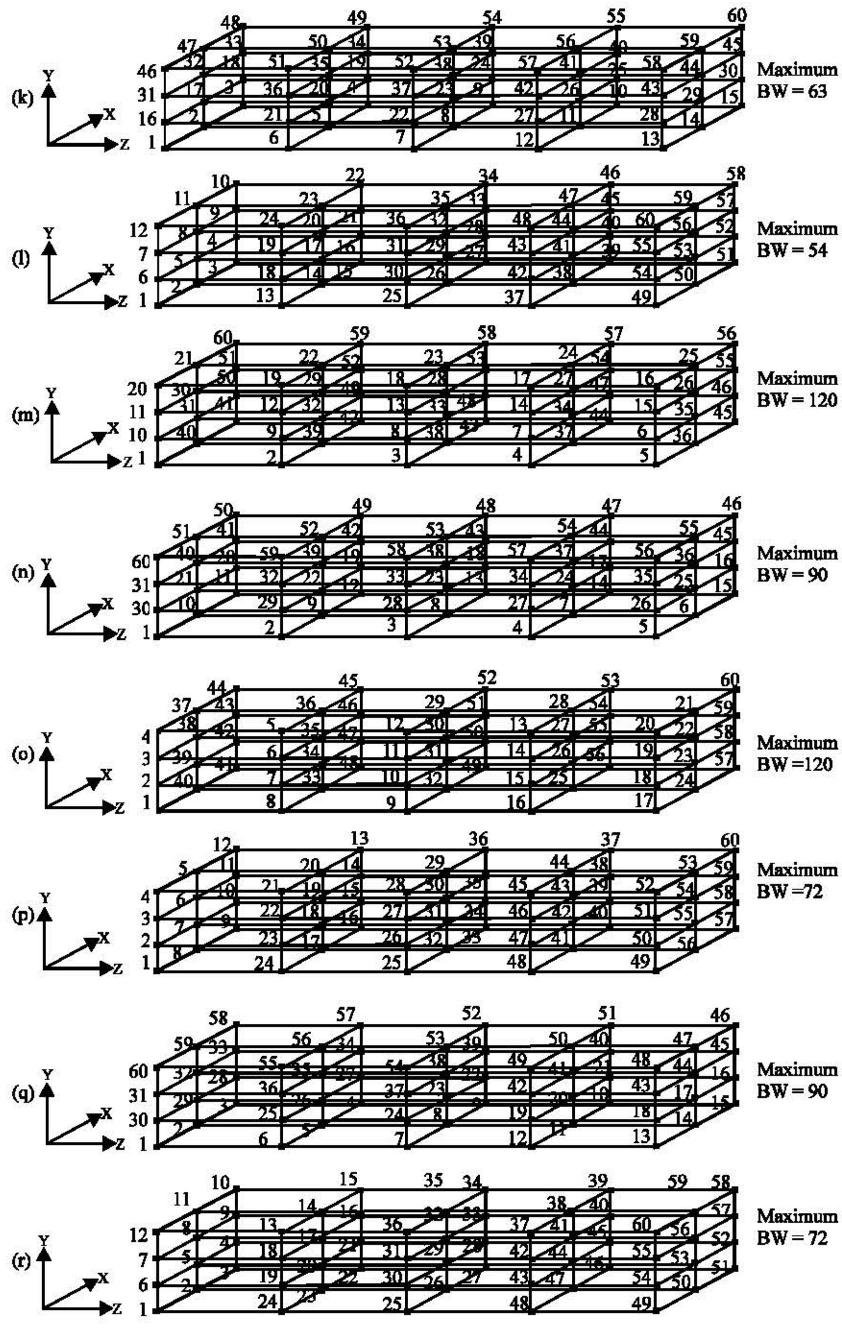

Fig. 2: Details of eighteen systematical nodes numbering techniques presented in this study

The BW of each technique: The BW of [K] depends directly on nodes numbering technique. The BW is determined according to the maximum difference among the nodes numbers connected by any one of the finite elements multiplied by the number of dof per node. The equations of BW for all techniques are derived and written in Table 1 according to the following concept (Segerlind, 1984; Fernando, 2015):

$$BW = \left(\max\left[BW^e\right]+1\right)\times dof \tag{2}$$

where, BW^e represents the difference in nodes numbers which is determined by calculating the difference in nodes numbers for each element of a finite element model while dof represents the degree of freedom per each node as it is mentioned previously.

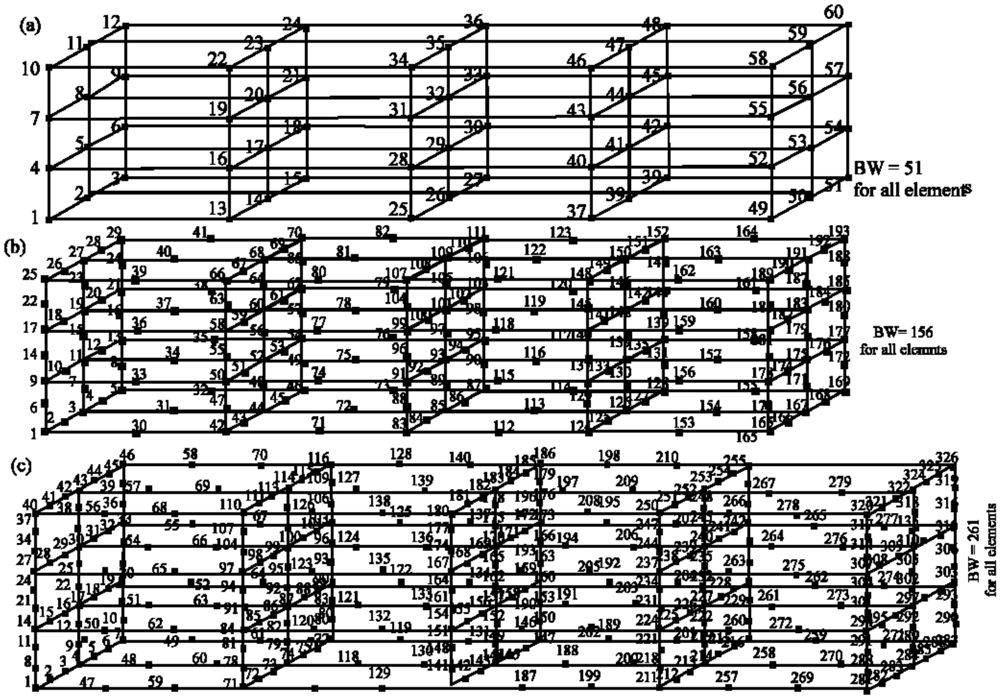

Fig. 3: a-c) SML technique with 8, 20 and 32 nodes brick elements for straight beam of 24 element

It is important to note that the first six techniques (LMS, LSM, MLS, MSL, SLM, SML) represent the most regular techniques because the numbering and translations always have one direction that coincides with the positive direction of coordinates of axes as it is shown in Fig. 2a-f, respectively. Moreover, these techniques produce a uniform profile of [K] where all elements have the same BW (Fig. 3 and 4).

Optimal systematical technique: Obviously, the last equation shows that the optimal technique is one which keeps the nodes numbers of each element as close as possible. The results of all techniques are tabulated in the last column of Table 1 for eight nodes brick elements as it is shown in this table and in Fig. 2f. The most suitable (optimal) systematical technique which produces the minimum BW is the SML technique. In this technique, the numbering has been done section by section. In each section the numbering must be along the direction that coincides with the dimension which contains the minimum numbers of elements (i.e., smallest), then the numbering moves step by step within the section along the other dimension that contains the medium number of elements and finally moves to numbering another section along the direction that coincides with the dimension which contains the maximum numbers of elements (i.e., largest).

The general equation of BW according to the SML technique: To avoid the direction of axis, one can transform the general equation of BW from terms of NX and NY to N1 and N2. Thus, for 8-nodes linear brick elements, the general equation of BW which is previously derived can be expressed as:

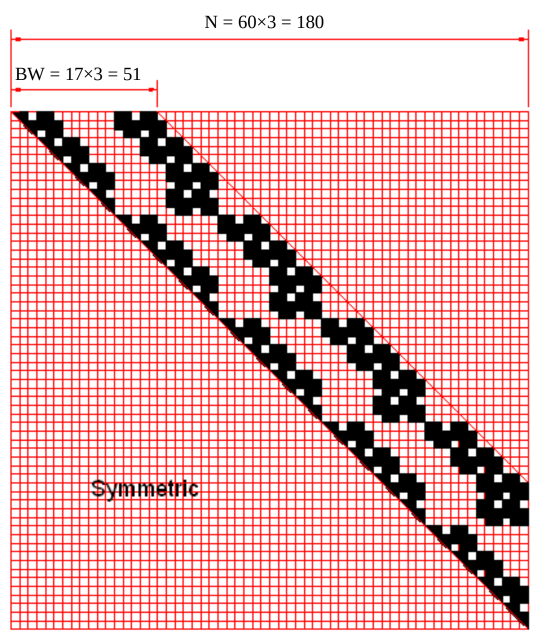

Fig. 4: Structural stiffness matrix according to SML technique for straight beam of 24 element, the BW for each element is equal to 51

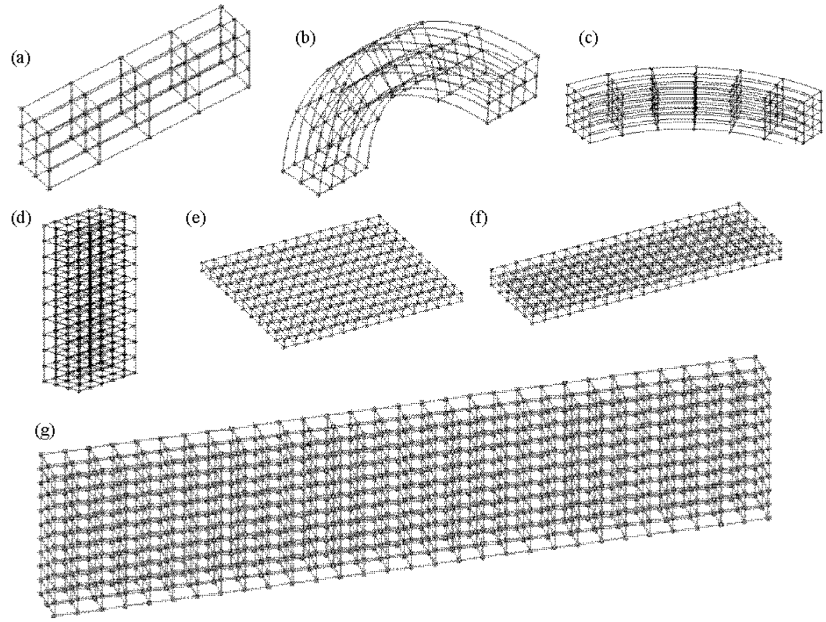

Fig. 5: Applications of SML technique in the structural members which have a rectangular section: a) Case 1: Straight beam; b) Case 2: Arch; c) Case 3: Horrizontally curved beam; d) Case 4: Coloum; e) Case 5: Slab; f) Case 6: Footing and g) Case 7: Wall

$$BW= \left(N_1N_2+2N_1+N_2+4\right).dof \quad (3)$$

In the present example, 24 elements are shown in Fig. 3a and 4. The BW is equal to 51 for all elements as it is calculated by Eq. 3. In Fig. 4 for simplicity, the nodes contributions are denoted as shaded squares while the empty squares are denoted by zeros and each square in this [K] diagram represents nine terms. The nodes numbers might be used instead of the degree of freedom, because this will not alter the conclusion in any way. Some structural applications require using 20-nodes quadratic brick elements or even 32-nodes cubic brick elements. However, Fig. 3-b illustrates the present example simulated by 20-nodes quadratic brick elements. The general equation of BW is derived and expressed in Eq. 4. The BW of [K] and each element is equal to 156. When using 32-nodes cubic brick elements for present example, as it is shown in Fig. 3-c, the BW of [K] and each element will be equal to 261. The general equation of BW is expressed in Eq. 5.

$$BW= \left(4N_1N_2+6N_1+3N_2+7\right).dof \quad (4)$$

$$BW= \left(7N_1N_2+10N_1+5N_2+10\right).dof \quad (5)$$

Where:

N_1 = Number of elements in the direction of numbering which must contain the smallest number of elements according to SML technique

N_2 = Nnumber of elements in the first direction of transmission that must contain the medium number of elements according to SML technique

As the last three equations show, the BW of equations according to the SML technique depends only on numbers of elements in two directions, the direction of numbering and the first direction of transmission. The BW of equation does not depend on the number of elements in the second direction of transmission N_3.

RESULTS AND DISCUSSION

Applications of SML technique in the structural elements: The SML technique can be used for any three dimensional member has rectangular section. In this article seven structural members are presented. The number of elements for each member is selected arbitrarily (the convergence study must be done by the researcher when he analyzes any member by increasing the number of elements of the member until the results of the last two successive meshes do not exceed 1% to achieve accurate results (Al-Mutairee, 2008; Thevendran *et al*., 1999) but the arrangement of the cases in Table 2 and in Fig. 5 was according to the ratio of Number of Elements in the Smallest Section (NESS) to the Total Number of Elements in the member (TNE). The reduction in the execution time due to use of BW is computed according to Eq. 6 and it is tabulated in the last column of Table 2. The results of the seven investigated cases have proved

Table 2: The details of the investigated cases using 8-nodes brick elements

Case	N_1	N_2	N_3	NESS	TNE	Total nodes	BW Eq. 3	N^*	(NESS/TNE) (%)	Reduction in execution time (%)
1	2	3	4	6	24	60	51	180	25.00	75.92
2	2	4	5	8	40	90	60	270	20.00	85.19
3	3	4	6	12	72	140	78	420	16.67	89.65
4	3	5	10	15	150	264	90	792	10.00	96.13
5	1	10	15	10	150	352	78	1056	06.67	98.36
6	2	5	20	10	200	378	69	1134	05.00	98.89
7	2	10	30	20	600	1023	114	3069	03.33	99.59

*The effect of restrained nodes is ignored in the calculation of N because it is different in each case according to the application and type of support

that SML technique has a very efficient effect on the application of traditional structural elements, especially when the ratio of (NESS/TNE) decreases, for example in the last case, the reduction in time reach to 99.59% when the user solved $[K]_{3069}\times_{114}$ instead of $[K]_{3069}\times_{3069}$.

$$\text{Reduction in exe. time \%} = 1\text{-}3\left(\frac{BW}{N}\right)^2 \quad (6)$$

Mesh Generation Program of Straight Beam and Wall (MGPSBW): In this study, a computer program introduced according to SML technique by using the FORTRAN language to provide a simple way to users. In this program, a seven input data which describe the geometry of structure required to analysis must be defined, these are:

- W = Width of beam or wall (mm) in X-direction
- D = Depth of beam or wall (mm) in Y-direction
- L = Length of beam or wall (mm) in Z-direction
- NEIW = Number of elements in width
- NEID = Number of elements in depth
- NEIL = Number of elements in length
- K = Type of brick element selected as 1, 2, 3 for 8, 20, 32 nodes brick element, respectively

All these input data within line 11 (or eleven line) of the program which contains the order PARAMETER. After the user inputs the seven required data, he must specify the filename which will contain the output data as it is shown in line 12 of the program within the order OPEN. The mesh generation of straight beam can be used for wall due to compatibility between them as it is shown in Fig. 5. The directions of axes shown in Fig. 1 are adopted in this program. For input data shown in line 11, the output will be as it is tabulated in Table 3 which contains four columns, the number of nodes and their coordinates. The order FORMAT in the tail of the program can be modified by the user according to the compatible form required by some computer programs such as ANSYS and ABAQUS.

Table 3: Output results of program MGPSBW, for straight beam consists 90 element as specified in the input data (line 11 in program)

Node number	X	Y	Z
1	0	0	0
2	150	0	0
3	300	0	0
4	0	166.67	0
5	150	166.67	0
6	300	166.67	0
7	0	333.33	0
8	150	333.33	0
9	300	333.33	0
10	0	500	0
11	150	500	0
12	300	500	0
13	0	0	333.33
14	150	0	333.33
15	300	0	333.33
16	0	166.67	333.33
17	150	166.67	333.33
18	300	166.67	333.33
19	0	333.33	333.33
20	150	333.33	333.33
21	300	333.33	333.33
22	0	500	333.33
23	150	500	333.33
24	300	500	333.33
25	0	0	666.67
26	150	0	666.67
27	300	0	666.67
28	0	166.67	666.67
29	150	166.67	666.67
30	300	166.67	666.67
31	0	333.33	666.67
32	150	333.33	666.67
33	300	333.33	666.67
34	0	500	666.67
35	150	500	666.67
36	300	500	666.67
37	0	0	1000
38	150	0	1000
39	300	0	1000
40	0	166.67	1000
41	150	166.67	1000
42	300	166.67	1000
43	0	333.33	1000
44	150	333.33	1000
45	300	333.33	1000
46	0	500	1000
47	150	500	1000
48	300	500	1000
49	0	0	1333.33
50	150	0	1333.33
51	300	0	1333.33
52	0	166.67	1333.33
53	150	166.67	1333.33
54	300	166.67	1333.33
55	0	333.33	1333.33
56	150	333.33	1333.33
57	300	333.33	1333.33
58	0	500	1333.33
59	150	500	1333.33
60	300	500	1333.33
61	0	0	1666.67
62	150	0	1666.67
63	300	0	1666.67
64	0	166.67	1666.67
65	150	166.67	1666.67
66	300	166.67	1666.67

Table 3: Continue

Node of number	X	Y	Z
67	0	333.33	1666.67
68	150	333.33	1666.67
69	300	333.33	1666.67
70	0	500	1666.67
71	150	500	1666.67
72	300	500	1666.67
73	0	0	2000
74	150	0	2000
75	300	0	2000
76	0	166.67	2000
77	150	166.67	2000
78	300	166.67	2000
79	0	333.33	2000
80	150	333.33	2000
81	300	333.33	2000
82	0	500	2000
83	150	500	2000
84	300	500	2000
85	0	0	2333.33
86	150	0	2333.33
87	300	0	2333.33
88	0	166.67	2333.33
89	150	166.67	2333.33
90	300	166.67	2333.33
91	0	333.33	2333.33
92	150	333.33	2333.33
93	300	333.33	2333.33
94	0	500	2333.33
95	150	500	2333.33
96	300	500	2333.33
97	0	0	2666.67
98	150	0	2666.67
99	300	0	2666.67
100	0	166.67	2666.67
101	150	166.67	2666.67
102	300	166.67	2666.67
103	0	333.33	2666.67
104	150	333.33	2666.67
105	300	333.33	2666.67
106	0	500	2666.67
107	150	500	2666.67
108	300	500	2666.67
109	0	0	3000
110	150	0	3000
111	300	0	3000
112	0	166.67	3000
113	150	166.67	3000
114	300	166.67	3000
115	0	333.33	3000
116	150	333.33	3000
117	300	333.33	3000
118	0	500	3000
119	150	500	3000
120	300	500	3000
121	0	0	3333.33
122	150	0	3333.33
123	300	0	3333.33
124	0	166.67	3333.33
125	150	166.67	3333.33
126	300	166.67	3333.33
127	0	333.33	3333.33
128	150	333.33	3333.33
129	300	333.33	3333.33
130	0	500	3333.33
131	150	500	3333.33
132	300	500	3333.33
133	0	0	3666.67

Table 3: Continue

Node number	X	Y	Z
134	150	0	3666.67
135	300	0	3666.67
136	0	166.67	3666.67
137	150	166.67	3666.67
138	300	166.67	3666.67
139	0	333.33	3666.67
140	150	333.33	3666.67
141	300	333.33	3666.67
142	0	500	3666.67
143	150	500	3666.67
144	300	500	3666.67
145	0	0	4000
146	150	0	4000
147	300	0	4000
148	0	166.67	4000
149	150	166.67	4000
150	300	166.67	4000
151	0	333.33	4000
152	150	333.33	4000
153	300	333.33	4000
154	0	500	4000
155	150	500	4000
156	300	500	4000
157	0	0	4333.33
158	150	0	4333.33
159	300	0	4333.33
160	0	166.67	4333.33
161	150	166.67	4333.33
162	300	166.67	4333.33
163	0	333.33	4333.33
164	150	333.33	4333.33
165	300	333.33	4333.33
166	0	500	4333.33
167	150	500	4333.33
168	300	500	4333.33
169	0	0	4666.67
170	150	0	4666.67
171	300	0	4666.67
172	0	166.67	4666.67
173	150	166.67	4666.67
174	300	166.67	4666.67
175	0	333.33	4666.67
176	150	333.33	4666.67
177	300	333.33	4666.67
178	0	500	4666.67
179	150	500	4666.67
180	300	500	4666.67
181	0	0	5000
182	150	0	5000
183	300	0	5000
184	0	166.67	5000
185	150	166.67	5000
186	300	166.67	5000
187	0	333.33	5000
188	150	333.33	5000
189	300	333.33	5000
190	0	500	5000
191	150	500	5000
192	300	500	5000

Algorithm; Mesh Generation Program of Straight Beam and Wall (MGPSBW):

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCC CCC MESH GENERATION PROGRAM FOR
STRAIGHT BEAM AND CCC
```

```
CCC         WALL OF RECTANGULAR SECTION CONSTRUCTED
THE      CCC
CCC      NODES AND THEIR COORDINATES ACCORDING TO THE
   CCC
CCC         SML TECHNIQUE BY 8, 20 and 32 BRICK ELEMENT
CCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCC
PROGRAM MGPSBW
IMPLICIT REAL*8 (A-H, O-Z)
INTEGER TE, TNON
REAL L
PARAMETER (W = 300, D = 500, L = 5000, NEIW = 2, NEID = 3,
NEIL = 15, K = 1)
DIMENSION COORN (9000, 4)
OPEN (1, FILE = 'StBeam-90E-8N.SOUT')
NES = NEIW*NEID
TE = NES*NEIL
NONS = (2*K-1)*(NEIW*NEID)+(K)*(NEIW+NEID)+1
NONMS = (K-1)*(NEIW+1)*(NEID+1)
TNON = NEIL*(NONS+NONMS)+NONS
CCCCC                COORDINATE OF NODES          CCCCC
      II = 0
      IZ = 0
      DZ = 0.0
      DO 10 NZ = 1, (NEIL*K+1)
      StepZ = L/(NEIL*K*1.0)
      IF (NZ.EQ.1) StepZ = 0.0
      DZ = DZ+StepZ
      KK = K
      IF (K.EQ.2) THEN
        IF ((INT(NZ/2.0)*2).EQ.NZ) KK = 1
      ENDIF
      IF (K.EQ.3) THEN
      LZ = NZ-IZ*3
      IF ((INT(LZ/2.0)*2).EQ.LZ) KK = 1
      IF ((INT(LZ/3.0)*3).EQ.LZ) KK = 1
      ENDIF
       IY = 0
      DY = 0.0
      DO 20 NY = 1, (NEID*KK+1)
      StepY = D/(NEID*KK*1.0)
      IF (NY.EQ.1) StepY = 0.0
      DY = DY+StepY
      KKK=K
                      IF (K.EQ.2) THEN
        IF ((INT(NY/2.0)*2).EQ.NY) KKK = 1
        IF ((INT(NZ/2.0)*2).EQ.NZ) KKK = 1
      ENDIF
      IF (K.EQ.3) THEN
        IF ((INT(LZ/2.0)*2).EQ.LZ) KKK=1
        IF ((INT(LZ/3.0)*3).EQ.LZ) KKK=1
        LY = NY-IY*3
        IF ((INT(LY/2.0)*2).EQ.LY) KKK = 1
        IF ((INT(LY/3.0)*3).EQ.LY) KKK = 1
      ENDIF
      DX = 0.0
      DO 30 NX = 1, (NEIW*KKK+1)
      StepX = W/(NEIW*KKK*1.0)
      IF (NX.EQ.1) StepX=0.0
      DX = DX+StepX
      II = II+1
      COORN(II,1) = II
      COORN(II,2) = DX
      COORN(II,3) = DY
      COORN(II,4) = DZ
30    CONTINUE
      IF ((INT(NY/3.0)*3).EQ.NY) IY = IY+1
20    CONTINUE
      IF ((INT(NZ/3.0)*3).EQ.NZ) IZ = IZ+1
10    CONTINUE
      DO 40 I = 1, TNON
40    WRITE (1,50) I, COORN(I,2), COORN(I,3), COORN(I,4)
50    FORMAT (I5,3F12.2)
      END
```

CONCLUSION

According to the scope of investigation, the following points can be concluded: Among eighteen systematical nodes numbering techniques, the SML technique represents the judiciously and optimal one which always gives the minimum BW. Whenever the SML technique is adopted, the general equation of BW (for 8, 20 and 32 node) depends directly on the number of elements in the smallest section rather than on the number of elements in the largest direction (i.e., the number of elements in the smallest section control the value of BW). According to the SML technique, the researcher can increase the number of elements in the case study without changing the size of BW by increasing the number of elements in the largest direction (N3) but the aspect ratio must be taken into account. The reduction in the execution time increases with the decreasing ratio of (NESS/TNE) and it reaches to about 99%. The SML technique enables the users to know the size of all matrices before the construction of the global stiffness matrix. This produces compacted storage of its components and in turns reduces the requirements of associated memory. The mesh generation program is easy to be constructed according to the SML technique. The SML technique saves the time which required in traditional techniques for renumbering the nodes or rearranging the coefficients of the stiffness matrix. The SML technique produces a uniform profile of stiffness matrix where all elements have the same BW. The presented program (MGPSBW) is very efficient; it provides the numbers and coordinates of nodes for straight beam or wall. It facilitates the job of researchers and saves their time. The required time to input data is about fourteen seconds while the running time is less than one second.

REFERENCES

Al-Mutairee, H.M.K., 2008. Nonlinear static and dynamic analysis of horizontally curved beams. Ph.D Thesis, University of Babylon, Hillah, Iraq.

Alway, G.G. and D.W. Martin, 1965. An algorithm for reducing the bandwidth of a matrix of symmetrical configuration. Comput. J., 8: 264-272.

Boutora, Y., N. Takorabet, R. Ibtiouen and S. Mezani, 2007. A new method for minimizing the bandwidth and profile of square matrices for triangular finite elements mesh. IEEE. Trans. Magn., 43: 1513-1516.

Everstine, G.C., 1979. A comparasion of three resequencing algorithms for the reduction of matrix profile and wavefront. Intl. J. Numer. Methods Eng., 14: 837-853.

Gibbs, N.E., W.G. Poole Jr. and P.K. Stockmeyer, 1976. An algorithm for reducing the bandwidth and profile of a sparse matrix. SIAM. J. Numer. Anal., 13: 236-250.

Hearn, E.J., 1997. Mechanics of Materials 1: An Introduction to the Mechanics of Elastic and Plastic Deformation of Solids and Structural Materials. 3rd Edn., Butterworth-Heinemann, Oxford, England, UK.,.

Kaveh, A., 1995. Structural Mechanics: Graph and Matrix Methods. 2nd Edn., Research Studies Press Publisher, UK., ISBN:9780863801860, Pages: 440.

Kress, G., 2014. Structural Analysis with FEM. ETH Zurich, Zurich, Switzerland,.

Papadopoulos, P., 2005. Introduction to the Finite Element Method. University of California Berkeley, Berkeley, California,.

Quoc, L.V. and J.R. O'Leary, 1984. Automatic node resequencing with constraints. Comput. Struct., 18: 55-69.

Rodrigues, J.S., 1975. Node numbering optimization in structural analysis. J. Struct. Divis., 101: 361-376.

Segerlind, L.J., 1984. Applied Finite Element Analysis. 2nd Edn., John Wiley & Sons, Hoboken, New Jersey, USA., ISBN:9780471806622, Pages: 427.

Sloan, S.W. and M.F. Randolph, 1983. Automatic element reordering for finite element analysis with frontal solution schemes. Intl. J. Numer. Methods Eng., 19: 1153-1181.

Thevendran, V., S. Chen, N.E. Shanmugam and J.R. Liew, 1999. Nonlinear analysis of steel-concrete composite beams curved in plan. Finite Ele. Anal. Des., 32: 125-139.

Veldhorst, M., 1982. An Analysis of Sparse Matrix Storage Schemes. Mathematical Centre Tracts Publisher, Amsterdam, The Netherlands, ISBN: 978-90-6196-242-7,.

Wang, Q. and X.W. Shi, 2009. An improved algorithm for matrix bandwidth and profile reduction in finite element analysis. Prog. Electromagnet. Res., 9: 29-38.