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Cellular Automata: From Theoretical Concepts to Urban Modeling a New Methodology and Perspective

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Abstract: Modeling and developing cellular automata applications differ from one field to another. In fact, before designing and developing any cellular automata models, specialists (researchers, designers, developers) should make a number of preliminary design choices. We propose in this study an easy to use methodology that can help to identify design choices of the oncoming CA Models regardless of field of application. Based on this methodology, we propose also a detailed discussion about the appropriate and the preferred design choices of CA Models in case of urban dynamics.

Key words: Cellular automata, urban modelling, complex dynamic systems, methodology, application, design

INTRODUCTION

Modelling and simulating urban systems is gaining great interests among researchers. Urban dynamics are effects of interactions between natural and human processes. The great challenge of urban modelling is interactions between natural and social processes that are results of complex temporal-spatial behaviours. Urban growth, population dynamics, soil erosion and wild fire spread are examples of phenomena that are manifesting complex behaviours over time and space.

Such phenomena are interesting and seem to be of great challenge for sustainable cities because of both their unexpected negative effects and new society needs they create. We inherited generalist techniques for urban planning that have significant weakness such as poor handling of space-time dimensions, cross sectional representation of data and top down approach which ultimately fails to reproduce realistic simulations of urban systems. Hence, it is necessary more than ever design new computer models for better understanding of urban systems as complex temporal-spatial dynamic systems.

Cellular automata are dynamic systems that date back to works of Van Neumann on self-reproducing automata. Van Neumann's machine had not been run under a real simulation on any computer (Langton, 1984). With the full development of computer graphics, geographical information systems and complex systems theories, CA had been applied gradually and perfectly in many dynamics systems from diverse research fields such as Biology, Chemistry, Physics and Computer vision.

In this study, we first present concepts of cellular automata, we expose diversity of use of CA throughout some interesting applications from fields of Physics, Chemistry, Biology and Computer vision. After that, we point out an easy to use methodology to help the specialists (researchers, designers, developers) to identify the preliminary design choices regardless of field of application. Finally, based on the mentioned methodology, we will discuss and select the preferred design choices in case of urban thematic.

MATERIALS AND METHODS

Basics of cellular automata: Cellular automata are discrete dynamic systems in which space and time are respectively, divided into regular spatial cells and discrete time steps. States are finite. Each cell behaves like a finite-state-automaton that change state over time by following a set of transition rules. The overall behaviour of the system is the combination of effects of individual behaviours of all cells. A cellular automaton consists of 5 elementary components.

Components

The cell: Cell is the elementary spatial unit in a cellular space. Cells in a cellular automaton are arranged in a spatial tessellation. A 2-dimensional grid of cells is the most known form of a cellular automaton. However, other arrangements such as a one and three dimensional cellular automaton have also been developed. The honeycomb arrangement is another arrangement that had been used in literature.

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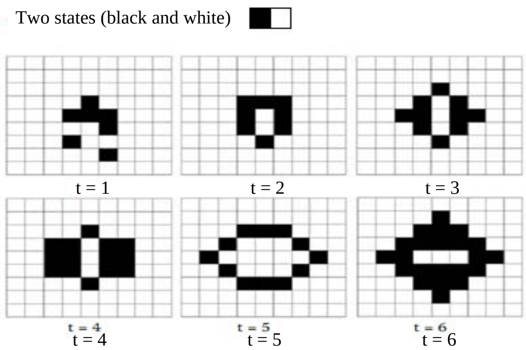

Fig. 1: A simple CA simulation based on Conway's "Game of Life"

The state: A state is a cell's attribute that represents the group to which the cell in question belongs. The set of states is often finite and it represents all cell's groups that exist in the system. Each cell has a state at particular time and it can move to another one in other time. The overall cells and theirs associated states at a specific time define the system state at that precise time. The state is frequently represented by a simple variable. For example, in the game of life as in Fig. 1, there are two groups of cells: the dead cells and the living cells. Thus, state could be represented by a variable that could have two values {0, 1}, zero to represent dead cells and one to represent alive cells.

The neighbourhood: Which contains a set of cells that form with the cell in question a neighbourhood relationship? Depending on cells arrangements, cell neighbourhood change. For example, in a 2D space, there are two basic types of neighbourhoods: the moore neighbourhood and the Von Neumann neighbourhood as in Fig. 2. Moore neighbourhood contains 8 cells that are disposed in the North-West, North-East, South-East and South-West directions the Von Neumann neighbourhood contains four cells which are arranged as the North, South, East and West neighbours of the cell in question.

The transition rule: Defines to which state the cell in question will transit depending on its current state and states of its neighbours. It is the essential component of cellular automata, it represents particularly algorithms that are responsible of driving changes of cells from one state to another over time. For a basic cellular automaton, the transition rules are uniform which means that are applied to all cells without exceptions and are synchronous which means that are applied simultaneously to all cells within

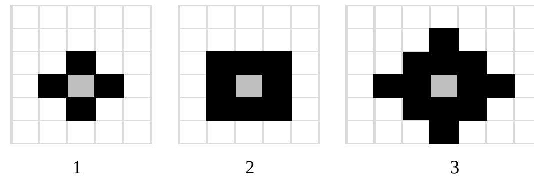

Fig. 2: Neighborhood types (1) Von Neumann, (2) Moore, (3) mixed Moore and Von Neumann

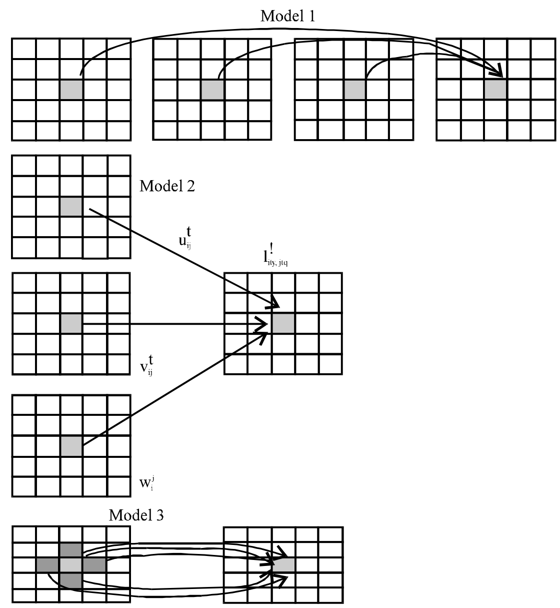

Fig. 3: Models of transition rules (Tobler, 1979)

the system. Tobler (1979) had presented 5 simple and abstract models of transition rules. We present the three most realistic ones which are the historical, multi variates and the geographical models. Figure 3 illustrates their principles. The historical model uses a series of past data of the concerned cell to predict its next state. The multivariate model uses other variables of the concerned cell at time t to predict its next state. The geographic model includes neighbours of the concerned cell to predict its next state. We could say that a model that combines the three could be best one.

The time: Time define the number of simulation's iterations. It specify the duration in which a cellular automaton performs. Based on the definition of cellular automata, each of all cells could update their states synchronously at each iteration over the time. The duration parameter is conventionally fixed in strict cellular automata models. However, some models released this restriction and operate different duration for different areas of the cellular space as was done by Uljee *et al.*

(1996) where they used for their simulated model two temporal scales: a monthly scale for the low-lying areas and a yearly scale for the upland areas.

Mathematical representation: Let, ${S_{Cij}}^t$ be the state of a cell C_{ij} at the position i, j at time t. ${S_{Cij}}^t$ is state from a finite set of states $\{S_0, \ldots, S_n\}$. Each cell of the cellular space has a state at a specific time. To compute ${S_{Cij}}^{t+1}$ the state of a cell at time t+1. Then:

$$S_{Cij}^{t+1} = g\left(S_{Cij}^{t}, S_{N_{Cij}}^{t}\right) \quad (1)$$

Where:

Nc_{ij} = Neighbors cells of cell
$C_{ij}, S_{t_{C_{ij}}}^{n}$ = A set of states of cells
N_{cij} = At time t
g = The transition rules function

If, we considered the cell itself as a member of its neighborhood, then Eq. 1 can be rewritten as:

$$S_{Cij}^{t+1} = g\left(S_{N_{cij}}^{t}\right) \quad (2)$$

Equation 2 is the engine of the cellular automata. Using it, we could create generation of CA at each next time. We usually construct transition rule function by using IF-THEN rules. IF-THEN rules could be expressed in a verbal and comprehensive syntax namely.

IF changements in neighbourhood of a cell, THEN change the cell at the next time step. A basic example of IF-THEN rules are the three rules of the Conways's game of life:

- IF A live cell has two or three live cells in its moore neighbourhood
- THEN the cell stays alive in the t+1
- IF A live cell has less than two or more than three live cells in its moore neighbourhood
- THEN the live cell dies in the t+1
- IF A dead cell has exactly three live cells in the moore neighbourhood
- THEN the dead cell becomes alive in t+1

Owing to this generic mathematical representation of cellular automata, cellular automata could be used as a framework for designing and simulating complex dynamic systems departing from clear and simple rules that will at end emerge significant global results.

RESULTS AND DISCUSSION

Fields of application of CA: CA is a flexible tool that can and had been used in a wide variety of fields. In this study, we outline some CA models to show its diversity of use in both function and fields of applications.

CA is a potential tool of physics field. It has shown a high degree of accuracy when used to simulate and predict N-Body interactions (Gardiner *et al*., 1998; Hayes, 2003; Ilachinski, 2001). One of works that obtained convening results which were nearly identical to lab experiments is the model (Lejeune *et al*., 1999). Researchers had chosen for the model conception, hexagonal cells rather than square cells. They have chosen also a circular neighbourhood with a radius equal to 2 spatial units.

In Chemistry, CA can be used to simulate, for example, hydrodynamics or "fluid" dynamics (Chen *et al*., 2015; Lupiano *et al*., 2015; Mareschal and Holian, 2013). A particular research subject is the gas cellular automata (Medvedev, 2016). The model comes first with square cells but later hexagonal cells have been used and showed more accuracy and realism. What makes it better is the neighbourhood type that is circular and in fact similar to the logic of gas propagation. This model need a specialised transition function that contains two parts: the propagation part that is responsible of the way particles move from one cell to another and the collision part that prevent how to handle with particles collision.

Myxobacteria are another example of CA applications from Biology. Myxobacteria are social bacteria that live, feed and develop cooperatively. When food lack then bacteria forms together a group called a body fruiting for energy saving. Researchers being interested in simulating such system have noted the usefulness of the cellular automata (Alber *et al*., 2004; Stevens, 2000; Holmes *et al*., 2010). Because of its similarity to the gas propagation system (Alber *et al*., 2004) had chosen to reuse "Gas Cellular Automata" and adopt it to bacteria propagation system. Therefore, they used rule based algorithms that mimic the manner in which bacteria move and group together.

Image processing was one of the first uses of CA (Preston and Duff, 2013; Rosin, 2010; Kauffmann and Piche, 2010). The use of cellular automata techniques is evident, since, digital images are divided into a cellular form composed of small regular cells that are pixels. Thus, each pixel of all ones has a state that is its colour and has neighbours depending on neighbourhood type. Pixels are tessellated in rectangular grid as square cells in fact circular neighbourhood are not preferred for digital images. Neighbourhood types that could be used are rectangular, Moore and Van-Neumann neighbourhoods.

Advantages of CA for urban modelling: Cellular automata are increasingly gaining favour in urban modelling. We describe in this section features that will made of CA a good urban modelling tool.

CA are spatial which means that are convenient to represent spatial problems like geographic problems as was confirmed by White and Engelen (1993). Spatial problems (e.g., fire spread, population or urban dynamics) acts in space usually of two dimensions, Hence, 2D lattice is appropriate for such problems. We could use 3D or honeycomb CA for representing other spatial problems, e.g., galaxy formation, 3D cities. However, researchers prefer when it is possible, 2D CA for its simplicity in model construction.

CA are self-organising systems that increase their internal structure without external guidance or source and as consequence emergent global patterns arise from local simple rules applied repetitively to all of CA cells over a calendar of time. This feature allows CA to be suitable to urban modelling. The process of urban spatial dynamics is local. Hence, we could apply local transition rules to each of spatial units of the urban space and in fact new global spatial patterns will emerge.

CA are dynamic systems (in contrast to static systems or cross sectional systems which are not time dependant) are time dependant and their final result is the calculation of adjustment processes over a calendar of time (Waddell and Ulfarsson, 2004). CA being dynamic, it are very much capable of modelling complex dynamic urban phenomena for instance urban sprawling, population and growth dynamics that could be achieved iteratively in short or long duration of time (e.g., months, years, semesters, etc).

CA are computational systems that consist of a large number of discrete cells which are locally connected. Each of all cells could represent an autonomous and interactive entity (e.g., automaton, machine, program, agent, etc.) that evolves simultaneously to other ones. Thus, CA could be seen a set of entities that are capable of supporting parallel computation, collective and cooperative processes.

Cellular automata modeling

CA modelling methodology: Regardless of field of application and before developing and designing any cellular automata model, designers should make a number of preliminary choices which will define specification of the oncoming CA Model. The analysis of CA Models of a variety of fields had led us to extract seven CA properties that are the most interesting features that need to be considered when attempting to use CA in any research field. Figure 4 point out the cycle of the seven CA properties.

By focusing on this cycle of CA features, we offer a methodology that the interested people (researchers, designers, developers) could follow in order to easily build the specification of its own CA Model which answers to the envisaged goals. For doing that, the specialist has to make a choice for each of CA properties. A choice that depends directly on the field of application and functionalities of the regarded CA Model. We collected in Table 1, a list of choices for each of the seven properties.

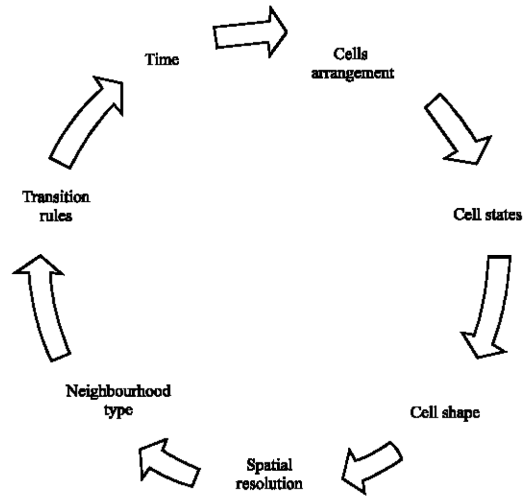

Fig. 4: Cycle of the seven proprieties of cellular automata

Table 1: List of choices of CA design

CA propriety	Choices
Cell's arrangement	One, two, three dimensions arrangement or honeycomb arrangement
Cell's shape	Square, triangular, circular or hexagonal form
Cell's states	The studied problem defines the set of land use types
Neighborhood type	Van-Neumann, Moore, circular, rectangular or mixed neighborhood
Transition rules	IF-THEN rules or probabilistic rules
Time	The studied problem defines the calendar of time
Spatial resolution	Available geographic data define the resolution, i.e., 10 m

CA for urban modelling: In this study, we will follow the mentioned methodology in order to discuss and select the preferred design choices in context of urban thematic.

Which arrangement to choose? Cell is the elementary spatial unit in a cellular space and it will represent the elementary spatial unit in the studied area. Spatial units of land areas are universally located by x and y coordinates in geographic data, i.e., satellite imagery, GIS imagery, etc. This is one of factors that simplified and in fact privileged the use of 2-dimensional cellular automata approach. In fact, to represent land use in the urban modelling, we prefer 2-dimensional grid. In 3-dimensional grid, the third dimension could be used to represent height of buildings in the urban models. However, this later arrangement is still not frequently used in urban modelling practice because of difficulties in model design and development. In addition to x and y coordinates which locate accurately all spatial units of the studied area in the cellular automata grid, we could add other attributes to cells such as state.

How to identify CA states?: State is one of others attributes we could associate to spatial units of the studied area. It is a particular attribute that represent the group to which the spatial unit belongs. In urban modeling, states of cells represent often the types of land use. Thus it could represent types of soil occupancy such as green area, habitat area, industrial area, etc. or any specific types of soil occupancy or it may be used to represents others features of the studied area such as categories of households (single household, family with dependent children, etc.) as was proposed by authors of (Haase *et al*., 2010). We note that the number of CA states in case of urban modeling is not a consensus between scholars. Moghadam and Helbich (2013) used five states which are built-up areas, water bodies, wetlands, forest and green space, open land and cropland. While others used more or less as done by Dubos-Paillard *et al*. (2003) that used fifteen states to represent types of soil occupancy.

What cell's shape to use: Shape of cells is an important specification of CA models. In urban modeling, cells of square shape are more suitable for representing spatial units. In fact, scholars use it more than hexagonal cells. Furthermore, coding and programming of square cellular automata is simple and easy comparing to hexagonal cellular automata. In spite of, some scholars have chosen to use hexagonal cellular automata as was done by Dubos-Paillard *et al*. (2003) that developed the SpaCelle application which integrates both square and hexagonal cells for simulating urban growth in the Rouen City and (Encinas *et al*., 2007) that developed hexagonal cellular automata for modeling and predicting forest fire spread. Triangular cellular automata are not used in urban modeling.

What determine spatial resolution? Spatial resolution of cellular automata in context of urban modeling is an important feature that is determined mainly by the available land use/cover data which comes essentially from aerial imagery, satellite imagery or GIS data. If available land use/cover data have a high resolution then CA could have a better accuracy. The more spatial resolution is high the better is the accuracy of analyses. For instance, a cellular automata with a spatial resolution of 10 m means that each of all cells could represent an object or part of object that have equal or larger size than 10 m square. However, cells could not represent objects that have inferior sizes than 10 m^2. Briefly, spatial resolution allows understanding how much a CA cell could represent spatially in ground. It are goals of urban model that defines which spatial resolution to use. For example (Agbossou *et al*., 2008) used a cells resolution of 10 m^2 for modeling and simulating short residential mobility in the Saone municipality. Objectives of this model were to identify and represent housing, i.e., owned or rented apartment, owned or rented single house in the CA grid. Thus, such resolution is appropriate for such objective, if we don't tell that it could be more accurate if we used a higher resolution.

Which neighbourhood type to select? Moore and Van-Neumann neighborhood have been widely used as CA neighborhood types. They are the most mentioned neighborhood types in case of urban modeling. These neighborhood types define neighborhood as cells immediately adjacent to the cell in question. They are founded under rule that say every change in cell state must be local.

Researchers extended this rule in way that neighbors could be located in a certain distance from the concerned cell. It is referred to as "action-at-a-distance". Depending on distance, we could explore an area that covers a district, a commune, a town, etc. For instance, White and Engelen (1993) defined a quite distant neighborhood comprising of 113 cells within a circle of a six-cell radius. For modeling and simulating a shrinking city (Haase *et al*., 2010) used a neighborhood with a radius of 500 m. Wu (1996) used a rectangular neighborhood when modeling and simulating land use development of a fast growing region in China. They defined a rectangle of a 5×5 which included 120 cells surrounding the central cell.

How to determine time? Time define the number of simulation's steps. It could be neglected in case of cross models that iterate at most for one step of simulations. However, dynamic models iterate for more than one step of simulation. So, we need to specify time of simulation that represents number of time steps. Number of time steps differs from one application to another. For instance, in application (Haase *et al*., 2010), time was short it was equal to 10 which represent teen years of simulation of the urban mobility of a shrinking city in Germany. In application, Jordan *et al*. (2012), time was long it was equal to 50 years of simulation of the residential mobility and housing choice of a district of leeds in the UK.

Which transition rules to use?: Transition rules are the core of Cellular automata. It represents how the process being modeled is working. Particularly, it represents algorithms that are responsible of driving changes of cells from one state to another over time. Different approaches have been employed for conceiving transition rules of urban cellular automata models. These approaches could be classified mainly in two classes if-then approaches and probabilistic approaches.

IF-THEN approach use simple rules that are implemented with if-conditions. For instance, Batty *et al.* (1989) used a very simple rule in their model of urban dynamic growth in which a vacant cell would convert to an occupied one if is surrounded in its neighborhood by an occupied cell. This model is too simple. In spite of, by combining great number of "IF" simple rules, we could build very complex cellular automata that could potentially simulate urban system' behavior. This was the idea behind many works such as SpaCelle application (Dubos-Paillard *et al.*, 2003) in which researchers defined dozen of transition rules for modeling urban growth of the Rouen City in France.

Probabilistic approach is based on probability distributions. It has become a standard method in developing models that attempts to predict individual choices among a finite set of alternatives (McFadden, 1973). Thus, each cell of all cells will transit from one state to another state with a transition probability. More specifically, we compute for each cell, probabilities to transit to each of all states and then the cell transit to the state that has the greatest transition probability. If at least two states have equal transition probabilities, then, we could expect that the cell could choose randomly a state or may be use another algorithm to choose the best state. An example of this approach include is the research by El-Alaouy *et al.* (2016). We usually use this approach by merging CA and Markov chain in order to benefit from complementary advantages of the two approaches. Du *et al.* (2010) reported that a combined MC-CA model that benefits from potentials of the two methods is a privileged one. CA represent the spatial distribution of the land use map and upon it, MC predict the total rates of land use changes induced from past changes.

CONCLUSION

This study introduced cellular automata, their diversity of use in many research fields and their potentials particularly for urban modelling. In the study, we propose a discussion about what need to be considered when transiting from theoretical concepts of CA to its application in urban dynamics.

Departing from the notice that CA had different implementations in different research fields. Let us interrogate about the usefulness of proposing a research work about the most CA features that need to be specified in any CA applications. By investigating and answering to this question, we were able to propose firstly an easy to use methodology that helps interested people (researchers, designers, developers) to identify the preliminary design choices of their oncoming CA Models. We accompanied this methodology that contains the list of choices for each of all CA features. Secondly, by following this methodology and applying it to urban modelling context, we were able to highlight through a discussion the preferred design choices that should be token in consideration when attempting to design CA models in context of urban thematic.

Finally, we could say that the methodology and the discussion, we proposed could give significant insights and supports to the specialists (researchers, designers, developers) in order to boosting design and development of cellular automata applications in urban thematic. We are working and hopefully we envisage in a coming future to extend this research in way to build a common Meta-model for boosting and developing urban cellular automata applications.

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