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Application New Class of P-Supremum Bounded Variation Double Sequences

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Abstract: The classical theorem on the uniform convergence of sine series with monotone decreasing coefficients have been proved by Chaundy and Jollife. Recently, the monotone decreasing coefficiets has been generalized by classes supremum bounded variation sequence and p-supremum bounded variation sequences. In two variables, class of supremum bounded variation double sequences and p-supremum bounded variation double sequences were studied under the uniform convergence of double sine series. This class has generalized to new class of p-supremum bounded variation double sequences. Futhermore, we discuss an application of class of p-supremum bounded variation double sequences

Key words: Application new class, bounded variation, double sequence, decreasing, variation, series

INTRODUCTION

The classical theorem on the uniform convergence of sine series with monotone decreasing coefficients have been proved by Chaundy and Jollife (1917) and Zygmund (1959) as stated in theorem 1.

Theorem 1: Suppose that $\{a_k\}\subset[0, \infty]$ is decreasing tending to zero. The necessary and sufficient conditions for the uniform convergence of the series:

$$\sum_{K=1}^{\infty} a_k \sin kx \quad (1)$$

where is $\lim_{k\to\infty} ka_k = 0$. As the single sine series Eq. 1, we discuss double sine series of two variables. Let $a = \{a_{jk}\}\subseteq \mathbb{C}$ and consider the double sine series of the form:

$$\sum_{j=1}^{m}\sum_{k=1}^{n} a_{jk} \sin jx \sin ky \quad (2)$$

As an idea in one variable, to investigate the uniform convergence of double sine series, the coefficients of the series by Korus (2011) are supposed to be member of class of general monotone double sequences. In two variables, Supremum Bounded Variation Double Sequences of first type (SBVSDS1) and supremum bounded variation double sequences of second type (SBVSDS2) have been introduced by Korus (2011).

Furthermore, Imron and Indrati (2013a, b) defined class of p-supremum bounded variation single sequences single and double sequences by Imron and Indrati (2013a, b). The new class single and double variable by Imron and Indrati (2013a, b) are defined as follow.

Definition 1: Let $a = \{a_k\}$ and $\beta = \{\beta_k\}$ be two sequences of complex and positive numbers, respectively. A couple (a, β) is said to be class of p-supremum bounded variation sequences second type of λ written $(a, \beta)\in SBVSI_p^{\lambda}$, if there exists C>0 and integer $\lambda\geq 1$ and $\{b(l)\}$ converges to infinity depend only $\{a_k\}$ such that:

$$\left(\sum_{k=n}^{2n-1}\left|\Delta a_k\right|^p\right)^{\frac{1}{p}} \leq \frac{C}{n}\sup_{m\geq b(n)}\sum_{k=m}^{2m}\beta_k,\ n\geq\lambda$$

for $1\leq p<\infty$

Definition 2: Let $a = \{a_{jk}\}$ and $\beta = \{\beta_{jk}\}$ be two double sequences of complex and positive numbers, respectively. A couple (a, β) is said to be class of λ p-supremum bounded variation double sequences second type of λ written $(a, \beta)\in SBVSI_p^{\lambda}$ if there exists C>0 and integer $\lambda\geq 1$ and $\{b(l)\}$, converging to infinity, depending only $\{a_{jk}\}$ such that:

$$\left(\sum_{j=m}^{2m-1}\left|\Delta_{10}a_{jn}\right|^p\right)^{1/p} \leq \frac{C}{m}\sup_{M\geq b(m)} \sum_{j=M}^{2M}\beta_{jn},\ m\geq\lambda,\ n\geq\lambda \quad (3)$$

$$\left(\sum_{k=n}^{2n-1}\left|\Delta_{01}a_{mk}\right|^p\right)^{1/p} \leq \frac{C}{n}\sup_{N\geq b(n)} \sum_{k=N}^{2N}\beta_{mk},\ m\geq\lambda,\ n\geq\lambda \quad (4)$$

$$\left(\sum_{j=m}^{2m-1}\sum_{k=n}^{2n-1}\left|\Delta_{11}a_{jk}\right|^p\right)^{1/p} \leq \frac{C}{mn}\sup_{M+N\geq b(m+n)} \sum_{j=M}^{2M}\sum_{k=N}^{2N}\beta_{jk},\ m, n\geq\lambda,\ \text{for } 1\leq p<\infty \quad (5)$$

In the present study, we will discuss an application of new classes of p-Supremum Bounded Variation Double Sequences of second type ($SBVDS2_p$).

MATERIALS AND METHODS

This research was done by study of literature, books and the supporting scientific journals to get a well understanding, then develop results related to the research that has been published in the journal. The results of this research are communicated in a seminar. In summary the method of the research is applying the new class of p-supremum bounded variation double sequences of second type.

RESULTS AND DISCUSSION

In this study, we study the uniform convergence of double sine Eq. 2 with coefficients of the class of p-supremum bunded variation double sequences second type of λ.

Theorem 3: Let $(a, \beta)\epsilon SBVDS2_p^{\lambda}$, $1 \leq p<\infty$, If sequence:

$$\left\{(mn)^{1-\frac{1}{p}} \sup_{M+N\geq b_{m+n}} \sum_{j=M}^{2M} \sum_{k=N}^{2N} \beta_{jk}\right\}$$

Converges to 0, for m+n→∞ then the series Eq. 2 uniformly convergence at $D = \{(x, y): 0\leq x, y\leq \pi\}$

Proof: To prove the uniformly convergent of double sine series Eq. 2, first by letting the single sine series with coefficients of double sequence $\{a_{jk}\}$:

$$\sum_{j=1}^{\infty} a_{jn} \sin jx,\ n = 1, 2, 3, ..., \quad (6)$$

$$\sum_{k=1}^{\infty} a_{mk} \sin ky,\ m = 1, 2, 3, ..., \quad (7)$$

Furthermore, by the condition:

$$\left\{(mn)^{1-\frac{1}{p}} \sup_{M+N\geq b_{m+n}} \sum_{j=M}^{2M} \sum_{k=N}^{2N} \beta_{jk}\right\}$$

Converges to 0 for m+n→∞, it can be proved that for any $m\geq\lambda(\{a_{mk}\}_{k=1}^{\infty}\beta)\epsilon SBVS2_p^{\lambda}$ and for any $n\geq\lambda(\{a_{jn}\}_{n=1}^{\infty} \beta)\epsilon SBVDS2_p^{\lambda}$. Hence, by condition of this theorem and according to Theorem 3 shown by Imron *et al*. (2013) implies the uniform convergence of the series Eq. 3 and 4. The second, we show that the double Eq. 2 is convergence at (x, y) for all $0\leq x,\ y\leq\pi$. Following can idea from Moricz (1991) for $(x, y) \neq (0, 0)$ we represent the rectangular partial sums: $S_{mn}(x, y) = \sum_{j=1}^{m}\sum_{k=1}^{n} a_{jk} \sin jx \sin kx,\ m, n\geq\lambda$ of Eq. 2 to perform a double summation by part yields:

$$S_{mn}(x, y) = \sum_{\lambda=1}^{m}\sum_{k=1}^{n} D_j^*(x)D_k^*(y)\Delta_{11}a_{jk} + \sum_{j=1}^{m} D_j^*(x)D_n^*(y)\Delta_{10}a_{j,\,n+1}) + \sum_{k=1}^{n} D_j^*(x) D_k^*(y)\Delta_{01}a_{m+1,\,k} + a_{m+1,\,n+1}D_m^*(x)D_n^*(y) \quad (8)$$

where, D_n^* conjugate Dirichlet kernel defined as:

$$D_n^*(x) = \sum_{k=1}^{n} \sin kx = \frac{\cos\frac{1}{2}x - \cos\left(n+\frac{1}{2}\right)x}{2\sin\frac{1}{2}x},\ n\geq\lambda$$

And $|D_n^*(x)|\leq\pi/x$ for $x\epsilon(0, \pi)$. Given condition in this theorem by Lemma 2.13 by Imron and Indrati (2013a, b) we obtained:

$$\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left|\Delta_{11}a_{jk}\right| < \infty$$

Thus:

$$\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left|D_j^*(x)D_k^*(y)\Delta_{11}a_{jk}\right| < \infty \quad (9)$$

Given condition of this theorem, by Lemmma 2.12 by Imron and Indrati (2013a, b):

$$\left|a_{jk}\right| \to 0,\ \text{for } j+k\to\infty \quad (10)$$

And from:

$$\Delta_{10}a_{mn} = \sum_{k=n}^{\infty}\Delta_{11}a_{mk},\ \Delta_{01}a_{mn} = \sum_{j=m}^{\infty}\Delta_{11}a_{jn}$$

We have:

$$\Delta_{10}a_{j,\,n+1} = \sum_{k=n+1}^{\infty}\Delta_{11}a_{jk}$$

Further:

$$\sum_{j=1}^{\infty}\left|\Delta_{10}a_{j,\,n+1}\right| \leq \sum_{j=1}^{\infty}\sum_{k=n+1}^{\infty}\left|\Delta_{11}a_{jk}\right| \to 0 \text{ as } n\to\infty$$

This implies that for all $0<x,\ y\leq\pi$:

$$\sum_{j=1}^{m} D_j^*(x)D_n^*(y)\Delta_{10}a_{j,\,n+1} \to 0 \text{ as } n\to\infty \quad (11)$$

uniformly in m. Similarly, for all:

$$0<x,\ y\leq\pi\sum_{k=1}^{n} D_m^*(x)D_k^*(y)\Delta_{01}a_{m+1,\,k} \to 0 \text{ as } m\to\infty \quad (12)$$

uniformly in n. By Eq. 7:

$$a_{m+1,\,n+1}D_m^*(x)D_n^*(y) \to 0,\ \text{as } m+n\to\infty \quad (13)$$

Consequently by Eq. 5-10. For:

$$\left(x, y\right) = \left(0, 0\right), \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} a_{jk} \sin jx \sin ky = 0$$

From the first and second part of this proof it can be concluded that Eq. 2 convergence is uniformly at (x, y) for all $0 \leq x, y \leq \pi$. The proof is complete.

Example 4: Set $m_r = 2^{(2^{2r})}$ for r = 1, 2, 3, ... and

$$a_{j1} = \begin{cases} 0 & \text{if} \quad 1 \leq j < m_1 \\ j^{-2} & \text{if} \quad j = m_r \\ 0 & \text{if} \quad n_j < j < m_r^2 \\ j^{-2} & \text{if} \quad m_r^2 \leq j < 2m_r^2 \\ 0 & \text{if} \quad 2m_r^2 \leq j < m_{r+1} \end{cases}$$

We define the sequence $\{\beta_{j1}\}$ where $\beta_{j1} = a_{j1}$ for every j. By Theorem 2. In Korus (2010) $(\{a_{j1}\}\ \{\beta_{j1}\}) \in SBVDS_p$ with constant C = 4 and b(j) = [j/2]. The second example:

$$a_{j2} = \begin{cases} 0 & \text{if} \quad 1 \leq j < m_1 \\ m_r^{-2} & \text{if} \quad j = m_r \\ 0 & \text{if} \quad m_j < j < m_r^2 \\ m_{r+1}^{-3/2} & \text{if} \quad m_j^2 \leq j \leq 2m_j^2 \\ 0 & \text{if} \quad 2m_r^2 < j < m_{r+1} \end{cases}$$

We define the sequence $\{\beta_{j2}\}$ where $\beta_{j2} = a_{j2}$ for every j. By Theorem 2. In Imron and Indatri (2013a, b) $(\{a_{j2}\}\ \{\beta_{j2}\}) \in SBVDS2_p$ with constant C = 2 and $b(j) = j^{1/2}$. Finally, we define $(\{a_{jk}\}\ \{\beta_{jk}\}) \in SBVDS_p$. Thus, $a_{jk} = 0$ for $j \geq 1$, $k \geq 3$.

Futhermore, the second and third condition of definition 1 are satiesfied for C = 4 and $b(\iota) = 1^{1/2}$ this means $(\{a_{jn}\}\ \{\beta_{jn}\}) \in SBVDS2_p$. Thus, by definition 2 $(a, \beta) \in SBVDS2_p^{\lambda}$ by $\lambda \geq 1$. It can be proved that the Eq. 2 uniform convergence.

Proof: It will be proved that sequence:

$$\left\{\left(mn\right)^{1-\frac{1}{p}} \sup_{M+N \geq b_{m+n}} \sum_{j=M}^{2M}\sum_{k=N}^{2N} \beta_{jk}\right\}$$

Converges to 0, for $m+n \to \infty$. Example for p = 1. For k = 1:

$$\sum_{j=M}^{2M} \beta_{jk} = \sum_{j=M}^{2M} j^{-2}$$

It can be proved that $\sum_{j=M}^{2M} j^{-2}$ converge to 0 and the same way for k = 2. By Theorem 3, the Eq. 2 converges uniformly at (x, y) for all $\leq x, y \leq \pi$.

CONCLUSION

In this study, we have introduced the application of class $SBVDS2_p^{\lambda}$, we have investigated that If sequence:

$$\left\{\left(mn\right)^{1-\frac{1}{p}} \sup_{M+N \geq b_{m+n}} \sum_{j=M}^{2M}\sum_{k=N}^{2N} \beta_{jk}\right\}$$

Converges to 0, for $m+n \to \infty$, then the Eq. 2 uniformly convergence at $D = \{(x, y): 0 \leq x, y \leq \pi\}$. For example of class of class of $SBVDS2_p^{\lambda}$ in example 4.

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