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On the Number of Ultra L-topologies in the Lattice of L-topologies

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Abstract: In this study, we investigate ultra L-topologies (dual atoms) in the lattice FX of all L-topologies on a given non empty set X when membership lattice L is a complete atomic Boolean lattice. All the ultra L-topologies and their number in the lattice FX are determined.

Key words: L-topology, lattice, complete atomic Boolean lattice, ultra L-topology, membership, Boolean lattice

INTRODUCTION

Lattice theory and topology are two related branches of mathematics, each in uencing the other. Many researchers have already undertaken the study of the lattice structure of the set of all topologies on a given set (Birkhoff, 1936; Frohlich, 1964; Gaifman, 1966; Hartmanis, 1958; Steiner, 1966; Vaidyanathaswamy, 1960; Rooij, 1968). It has been proved that this lattice is complete, atomic, dually atomic and complemented but neither modular nor distributive in general. Frohlich (1964) has determined dual atoms of this lattice and proved that it is also dually atomic and if $|X| = n$, then there are $n(n-1)$ dual atoms in the lattice of topologies on the set X. Analogously, the lattice structure of the set of L-topologies on a given set has been investigated by many researchers (Babusundar, 1989; Johnson, 1992, 2004). Johnson (1992, 2004) has investigated lattice structure of the set of L-topologies on a given set X and proved that this is complete, atomic but not modular, not complemented and not dually atomic in general.

In this study, we investigate the lattice structure of the lattice F_X of all L-topologies on a given non-empty set X when membership lattice L is a complete atomic boolean lattice. It is easy to see that F_X is complete, atomic, not modular and not distributive.

However, in this study we prove that if Y is the set of all atoms of a complete atomic Boolean lattice L, then for any non-empty set X, the number of ultra L-topologies in the lattice F_X is $|X|\ |Y|\ (|Y|-1)+|X||Y|^2\ (|X|-1)$. All the ultra L-topologies are also identified.

MATERIALS AND METHODS

Preliminaries: Throughout this study, X stands for a non-empty set, L for a complete atomic Boolean lattice with the least element 0 and the greatest element 1 and F_X stands for the lattice of all L-topologies on X. The constant function in L^X, taking value α is denoted by $\underline{\alpha}$ and x_γ where $\gamma(\neq 0)\in L$, denotes the L-fuzzy point defined by:

$$x(y) = \begin{cases} \gamma & \text{if} \quad y = x \\ 0 & \text{otherwise} \end{cases}$$

Any $f\in L^X$ is called as an L-subset of X. The following are some important definition reported by Balasubramanian (1992) and Davey and Priestley (2002):

Definition 2.1: An element of L is called an atom if it is a minimal element of $L\backslash\{0\}$.

Definition 2.2: An element of L is called a dual atom if it is a maximal element of $L\backslash\{1\}$.

Definition 2.3: Let (X, F) be an L-topological space and suppose that $g\in L^X$ and $g\notin F$. Then, the collection $F(g) = \{g_1\vee(g_2\wedge g): g_1, g_2\in F\}$ is called the simple extension of F determined by g.

Every complete atomic Boolean lattice is isomorphic to power set of some set, namely the set of all of its atoms. We assume that L is isomorphic to $(P(Y), \subseteq)$ where $Y = \{\alpha_i: I\in\Pi\}$ is the set of all atoms of L. Let $\beta_i = Y\backslash\{\alpha_i\}$, $i\in\Pi$. Then $\{\beta_i: i\in\Pi\}$ is the set of all dual atoms of L.

Consider the atom α_k and the set $A_{\alpha k} = \{\delta_p = \{\alpha_k, \alpha_p\}: p\in\Pi \text{ and } p\neq k\}$ of those $|Y|-1$ elements in L that immediately succeed α_k. Let $\delta_q = \{\alpha_k, \alpha_q\}$ be an arbitrary element of $A_{\alpha k}$ and L_q^k denotes the sublattice of L generated by the set $\{\alpha_k, \delta_p{:}\delta_p\in A_{\alpha k}$ and $p\in\Pi$ such that $p\neq q\}$. Then, L_q^k is a complete sublattice of L with the least element α_k and the greatest element β_q. $L\backslash L_q^k$ is also a complete sublattice of L generated by the set $\{\alpha_i, \delta_q: I\in\Pi$

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such that $i \neq k\}$ with the least element 0 and the greatest element 1. Clearly, (i) if $\alpha_k \leq \mu$ for some $\mu \in L \backslash L_q^k$, then $\delta_q \leq \mu$:

$$\delta_q \Lambda \gamma = \alpha_k, \forall \gamma \in L_q^k$$
$$L_q^k \cap (L \backslash L_q^k) = \phi$$

Throughout this study, we will use all these notations.

RESULTS AND DISCUSSION

Ultra L-topology: An L-topology F on X is called an ultra L-topology if the only L-topology on X strictly finer than F is the discrete L-topology.

Remark 3.1: Let U be an L-topology in F_X. In order to show that U is an ultra L-topology, it is sufficient to show that simple extension of U by any L-subset $g \in L^X$ such that $g \notin U$ is the discrete L-topology. Now certain properties of ultra L-topologies are derived. Let U be an arbitrary ultra L-topology in F_X.

Lemma 3.2: Atleast one L-fuzzy point does not belong to U.

Proof: Suppose all L-fuzzy points belong to U. Since, for each $f \in L^X$, $f = \vee x_\lambda$ such that $x_\lambda \leq f \Rightarrow f \in U$, $\forall f \in L^X \Rightarrow U = L^X$ which is a contradiction.

Lemma 3.3: If two L-fuzzy points a_λ and b_η do not belong to U, then a = b.

Proof: Suppose the lemma is not true, then the simple extension $U(a_\lambda)$ is an L-topology such that $b_\eta \notin U(a_\lambda) \Rightarrow U(a_\lambda) \neq L^X$. But $U \subset U\ (a_\lambda)$ which is a contradiction.

Lemma 3.4: There is exactly one element $a \in X$ and one atom $\alpha_i \in L$ for some $i \in \Pi$ such that $a_{\alpha i} \notin U$.

Proof: By lemmas 3.2 and 3.3, there exists exactly one element say $a \in X$ such that $a_\lambda \notin U$ for some $\lambda(\neq 0) \in L \Rightarrow a_{\alpha i} \notin U$ for some $i \in \Pi$ since L is atomic.

If possible, let $a_{\alpha i}, a_{\alpha j} \notin U$ for some $i, j \in \Pi$ such that $i \neq j$. Then, the simple extension $U(a_{\alpha i})$ is an L-topology such that $a_{\alpha j} \notin U(a_{\alpha i})) \Rightarrow U(a_{\alpha i}) \neq L^X$. But $U \subset U\ (a_{\alpha i})$ which is a contradiction.

Theorem 3.5: Let $a \in X$ be an arbitrary element. Then, $U_q^k(a) = \{f \in L^X$: $f(a) \neq \lambda$ for any $\lambda \in L_q^k\}$ is an ultra L-topology not containing the L-fuzzy point $a_{\alpha k}$.

Proof: Clearly, $\underline{0}, \underline{1} \in U_q^k(a)$. Let $\{f_i\}_{i \in \Delta}$ be an arbitrary family of L-subsets in $U_q^k(a)$. Then, $f_i\ (a) \in L \backslash L_q^k$, $\forall_i \in \Delta$ and since $L \backslash L_q^k$ is a complete sublattice of $L \Rightarrow \bigvee_{i \in \Delta} f_i\ (a)$, $\bigwedge_{i \in \Delta} f_i\ (a) \in L \backslash L_q^k \Rightarrow \bigvee_{i \in \Delta} f_i\ (a)$, $\bigwedge I_{\in \Delta} f_i \in U_q^k\ (a)$. Thus, $U_q^k(a)$ is an L-topology. Clearly, (i) $x_\eta \in U_q^k(a)$, $\forall x(\neq a) \in X$ and $\forall \eta(\neq 0) \in L$:

$$a_\gamma \in U_q^k(a), \forall \gamma(\neq 0) \in L \backslash L_q^k \Rightarrow a_{\alpha i} \in U_q^k\ (a),$$
$$\forall i \in \Pi \text{ such that } i \neq k$$

$$\alpha_k \in L_q^k \Rightarrow a_{\alpha k} \notin U_q^k(a)$$

Let g be an arbitrary L-subset not belonging to $U_q^k(a)$. Then, $g\ (a) = \xi$ for some $\xi \in L_q^k$. Let S = simple extension of $U_q^k(a)$ determined by g. Then, $g \in S$ and a_1, $a_{\delta q} \in U_q^k(a) \subset S \Rightarrow a_{\alpha k} \in S) \Rightarrow S = L^X$.

Thus, simple extension of $U_q^k(a)$ by any of the L-subset not belonging to it, makes $a_{\alpha k}$ an L-open set. Hence, $U_q^k(a)$ is an ultra L-topology.

Remark 3.6: Since, a can be replaced by any other element of X in the theorem 3.5, it follows that $U_q^k(a)$ is an ultra L-topology, $\forall x \in X$.

δ_q was an arbitrary element of $A_{\alpha k}$, any element δ_p can be choosen from $A_{\alpha k}$ and corresponding to the element δ_p the sublattice L_p^k and ultra L-topology $U_p^k\ (a)$ can be formed in the same way as formed for δ_q.

Therefore, there are |X| choices for a and |Y|-1 choices for δ_q, by multiplication principle, total number of such ultra L-topologies containing the L-fuzzy points a_1 but not containing the L-fuzzy points $a_{\alpha k}$, $\forall a \in X$ is |X| (|Y|-1).

The same process can be done for any atom. Hence, total number of ultra L-topologies $U_q^k(x)$ where $x \in X$ and k, $q \in \Pi$ such that $k \neq q$ is |X| |Y| (|Y|-1).

Theorem 3.7: Let U be any ultra L-topology in F_X such that $a_1 \in U$ and $a_\lambda \notin U$ for some $a \in X$ and $\lambda(\neq 0, 1) \in L$. Then, $U = U_q^k(a)$ for some k, $q \in \Pi$ such that $k \neq q$.

Proof; Case 1: λ is an atom. Then, $\lambda = \alpha_k$ for some $k \in \Pi$ and $a_{\alpha k} \notin U$. By lemma 3.3, $x_\eta \in U$, $\forall x\ (\neq a) \in X$ and $\forall \eta\ (\neq 0) \in L$ and by lemma 3.4, $a_{\alpha i} \in U$, $\forall i \in \Pi$ such that $i \neq k$. Clearly, no pair of the distinct L-fuzzy points from the set $\{a_{\delta p}$: $\delta_p \in A_{\alpha k}\}$ belongs to U and if $a_{\delta p} \notin U$, $\forall \delta_p \in A_{\alpha k}$, then $U \subset U_q^k(a)$, $\forall q \in \Pi$ such that $q \neq k$ which is a contradiction. Without loss of generality assume that $a_{\delta q} \in U$ for some $\delta_q \in A_{\alpha k}$. Then $a_{\delta q} \vee (\bigvee_{j \in \Delta} a_{\alpha k}) = a_1 \in U$, where $\Delta = \Pi \backslash \{k\}$.

If there exist an L-subset $f \in L^X$ in U such that $f\ (a) = \gamma$ for any $\gamma \in L_q^k$, then a_1, $a_{\delta q}$, $f \in U \Rightarrow a_{\alpha k} \in U$ which is a contradiction. Therefore, $f \in L^X$ such that $f\ (a) = \gamma$ for any $\gamma \in L_q^k$ are the only L-subsets not belonging to $U \Rightarrow U = \{f \in L^X$: $f\ (a) \neq \gamma$ for any $\gamma \in L_q^k\} = L_q^k(a)$ which is an ultra L-topology by theorem 3.5.

Case 2: λ is not an atom. Since, L is atomic, $a_\lambda \neq U \Rightarrow a_{\alpha i} \notin U$ for some $i \in \Pi$ and then by case 1, $U = U^i_t(a)$ for some $t \in \Pi$ such that $t \neq i$.

Remark 3.8: From theorem 3.7, it follows that $U^k_q(x)$ where $x \in X$ and k, $q \in \Pi$ such that $k \neq q$ are the only ultra L-topologies containing the L-fuzzy points x_1 but not containing the L-fuzzy points x_λ, $\forall x \in X$ and $\forall \lambda(\neq 0, 1) \in L$. Then by remark 3.6, total number of these ultra L-topologies are |X| |Y| (|Y|-1).

Remark 3.9: It is easy to see that if a and b are any two elements of X such that $a \neq b$, then $б_{a, b\alpha i} = \{f \in L^X: f(a) \neq 0 \Rightarrow b_{\alpha i} \leq\}$ is an L-topology.

Theorem 3.10: Simple extension $б_{a, b\alpha i}(a_{\beta k}) = \{f \in L^X: f(a) \vee \beta_k = 1 \Rightarrow b_{\alpha i} \leq f\}$ of the L-topology $б_{a, b\alpha i}$ by $a_{\beta k}$ for some $k \in \Pi$ is an ultra L-topology.

Proof: Let $f_\lambda = a_\lambda \vee b_{\alpha i}$, $\forall \lambda(\neq 0) \in L$. Clearly, $f_\lambda \in б_{a, b\alpha i} \subset б_{a, b\alpha i}(a_{\beta k})$, $\forall \lambda(\neq 0) \in L$. Then, $a_{\beta k} \wedge f_\lambda = a_\gamma \in б_{a, b\alpha i}(a_{\beta k})$, $\forall \gamma(\neq 0) \in L$ such that $\gamma \leq \beta_k$. Also, $x_\gamma \in б_{a, bi}$, $\forall x(\neq a) \in X$ and $\forall \gamma(\neq 0) \in L$. Therefore a_η, where $\eta(\neq 0) \in L$ such that $\eta \vee \beta_k k = 1$ are the only L-fuzzy points not belonging to $б_{a, bi}(a_{\beta k})$.

Hence, $б_{a, b\alpha i}(a_{\beta k}) = \{f \in L^X: f(a) \vee \beta_k = 1)\ b_{\alpha i} \leq f\}$ and $g \in L^X$ such that $g(a) = \ell$ and $g(b) = \mu$ where ℓ, $\mu(\neq 0) \in L$ such that $\ell \vee \beta_k = 1$ and $\mu \wedge \alpha_i = 0$ are the only L-subsets not belonging to $б_{a, b\alpha i}(a_{\beta k})$. Simple extension of $б_{a, b\alpha i}(a_{\beta\beta k})$ by any of the L-subsets not belonging to it, makes each a_η, where $\eta(\neq 0) \in L$ such that $\eta \vee \beta_k = 1$ an L-open set. Hence, $б_{a, b\alpha i}(a_{\beta k})$ is an ultra L-topology.

Remark 3.11: From the theorem 3.10, it follows that for any x, $y \in X$ such that $x \neq y$, the simple extension $б_{x, y\alpha i}(x_{\beta k})$ for any $k \in \Pi$ of the L-topology $б_{x, y\alpha i}$ for any $i \in \Pi$ is an ultra L-topology.

There are |X| choices for x and |X|-1 choices for y. L is a complete atomic Boolean lattice isomorphic to power set algebra P(Y), so, there are |Y| choices for α_i and |Y| choices for β_k. Therefore by multiplication principle, total number of such ultra L-topologies is $|X|\ |Y|^2\ (|X|-1)$.

Theorem 3.12: If U is an ultra L-topology in F_X such that $a_\gamma \notin U$ and $\{f_i\}_{i\in\Phi}$ is the collection of all those L-subsets in U which assumes value γ at a, then $\bigwedge_{i\in\Phi} f_i \in U$, $\bigwedge_{i\in\Phi} f_i \neq a_\gamma$ and $\bigwedge_{i\in\Phi} f_i = a_\gamma \vee b_{\alpha i}$ for some $b(\neq a) \in X$ and $\alpha_i \in Y$.

Proof: If X and L are finite, then it is clear that $\bigwedge_{i\in\Phi} f_i \in U$ and $\bigwedge_{i\in\Phi} f_i \neq a_\gamma$. Let $g = \bigwedge_{i\in\Phi} f_i$. Then, either $g = a_\gamma$ or $g \in L^X$ such that $g(a) = \gamma$ and $g(x) \neq 0$ for atleast one $x(\neq a) \in X$.

Case A: $g \in L^X$ such that $g(a) = \gamma$ and $g(x) \neq 0$ for atleast one $x(\neq a) \in X$. If $g \notin U$, then the simple extension U (g) is an L-topology such that $a_\gamma \notin U(g) \Rightarrow U(g) \neq L^X$. But $U \subset U(g)$ which is a contradiction.

Case B: $g = a_\gamma$. This case is possible only when atleast one of X or L is infinite.

Case 1: X is finite and L is infinite. Then, there exists some $x(\neq a) \in X$ such that $f_i(x) \neq 0$ for all $i \in \Phi$, $\bigwedge_{i\in\Phi} f_i(x) = 0$ and $\bigwedge_{i\in\Omega} f_i(x) \neq 0$ for any finite subfamily $\Omega \subset \Phi$. Since, L is infinite power set algebra, there exists some $\eta(\neq 0) \in L$ such that:

$$f_i(x) \neq \eta \text{ and } f_i(x) \wedge \eta \neq 0, \forall i \in \Phi$$

Let $h \in L^X$ such that $h(a) = \gamma$, $h(x) = \eta$ and $h(y) = 0$, $\forall y(\neq x, a) \in X$. Then, $h \notin U$ and the simple extension U (h) is an L-topology such that $a_\gamma \notin U(h)$. But $U \subset U(h)$ which is a contradiction.

Case 2: X is infinite and L is infinite. (i) the family $\{f_i\}_{i\in\Phi}$ contains L-subsets which can be written as the join of finitely many L-fuzzy points.

Let f_k be an arbitrary L-subset of the family $\{f_i\}_{i\in\Phi}$ which can be written as the join of least number of L-fuzzy points . Clearly, f_k is the join of exactly two L-fuzzy points and let $f_k(a) = \gamma$, $f_k(b) \neq 0$ for some $b(\neq a) \in X$, $f_k(x) = 0$ for all $x(\neq a, b) \in X$. $a_\gamma \notin U \Rightarrow f_i(b) \neq 0$ for all $i \in \Phi$, $\bigwedge_{i\in\Phi} f_k(b) = 0$ and $\bigwedge_{i\in\Omega} f_k(b) \neq 0$ for any finite subfamily $\Omega \subset \Phi$. Since, L is infinite power set algebra, there exists some $\eta(\neq 0) \in L$ such that:

$$f_i(b) \neq \eta \text{ and } f_i(b) \wedge \eta \neq 0, \forall i \in \Phi$$

Let $h \in L^X$ such that $h(a) = \gamma$, $h(b) = \eta$ and $h(y) = 0$, $\forall y(\neq x, a) \in X$. Then, $h \notin U$ and the simple extension U (h) is an L-topology such that $a\gamma \notin U(h)$. But $U \subset U(h)$, a contradiction. (ii) no member of the family $\{f_i\}_{i\in\Phi}$ can be written as the join of finitely many L-fuzzy points.

Corresponding to every member f_i of the family $\{f_i\}_{i\in\Phi}$, choose a pair of distinct elements x^i, $y^i(\neq a)$ of X such that $f_i(x^i)$, $f_i(y^i) \neq 0$ and corresponding to two different members of the family $\{f_i\}_{i\in\Phi}$, choose disjoint pairs of elements of X, i.e., if $f_j \in \{f_i\}_{i\in\Phi}$ such that $j \neq i$, then choose x^j, $y^j \in X$ such that (i) $x^j \neq x^i$, y^i (ii) $y^j \neq x^i$, y^i (iii) $f_j(x^j)$, $f_j(y^j) \neq 0$.

Let $f \in L^X$ such that (i) $f(a) = \gamma$ (ii) $f(x^i) = f_i(x^i)$, $f(y^i) = 0$, $\forall i \in \Phi$ (iii) $f(z) = 1$, $\forall z \in X$ such that $z \neq a$, x^i, y^i, $\forall i \in \Phi$. Then, $f \notin U$ and the simple extension U (f) is an L-topology such that $a_\gamma \notin U(f) \Rightarrow U(f) \neq L^X$. But $U \subset U(f)$, a contradiction.

Case 3: X is infinite and L is finite. Clearly, in this case, no member of the family $\{f_i\}_{i\in\Phi}$ can be written as the join of finitely many L-fuzzy points. Now same as Case 2(ii). $\Rightarrow \bigwedge_{i\in\Phi} f_i \neq a_\gamma$. Since, U is an ultra L-topology, $g = a_\gamma \vee b_{\alpha i}$ for some $b(\neq a) \in X$ and $\alpha_i \in Y$.

Theorem 3.13: Let $a \in X$ be an arbitrary element. If U is an ultra L-topology such that $a_1 \notin U$, then $U = б_{a, b\alpha i}(a_{\beta k})$ for some $b(\neq a) \in X$ and i, $k \in \Pi$.

Proof; By lemma 3.3: $x_\lambda \in U$, $\forall x\ (\neq a) \in X$ and $\forall \lambda(\neq 0) \in L$. $a_1 \notin U \Rightarrow a_{\alpha k} \notin U$ for some $k \in \Pi$ and by lemma 3.4, $a_{\alpha k} \in U$, $\forall i \in \Pi$ such that $i \neq k \Rightarrow a_{\beta k}$, $a_\lambda \in U$, $\forall \lambda(\neq 0) \in \Pi$ L such that $\lambda \leq \beta_k$.

Clearly, β_k is the only dual atom in L such that $a_{\beta k} \in U$. Let $B = \{\gamma \in L: \alpha_k \leq \gamma\}$. For any $\gamma \in B$, $\gamma \vee \beta_k = 1 \Rightarrow a_\gamma \notin U \Rightarrow a_\gamma$, where $\gamma \in B$ are the only L-fuzzy points not belonging to U.

Let $\gamma\ (\neq 1) \in B$ be an arbitrary element. If possible, let there exists no L-subset in U which assumes value γ at a and let $f \in L^X$ such that $f(a) = \gamma$ and $f(x) = 1$, $\forall x\ (\neq a) \in X$. Then, $f \notin U$ and the simple extension U (f) is an L-topology such that $a_\gamma \notin U(f) \Rightarrow U(f) \neq L^X$. But $U \subset U(f)$, a contradiction. Similar is the case when $\gamma = 1$ in this case consider $f \in L^X$ defined as $f(x) = 0$ for some $x\ (\neq a) \in X$ and $f(y) = 1$, $\forall y\ (\neq x) \in X$.

Let $\{f_i\}_{i \in \Phi}$ be the collection of all those L-subsets in U which assumes value γ at a and $g = \bigwedge_{i \in \Phi} f_i$. By theorem 3.12, $g \in U$, $g \neq a_\gamma$ and $g = a_\gamma \vee b_{\alpha i}$ for some $b\ (\neq a) \in X$ and $\alpha_i \in Y$.

Since, $\gamma \in B$ was an arbitrary element, the same process can be done for any element of B. Let δ_1, $\delta_2 \in B$ be two arbitrary elements. Then, $\alpha_k \leq \delta_1$ and $\alpha_k \leq \delta_2 \Rightarrow \alpha_k \leq \delta_1 \wedge \delta_2 = \delta_3$(say). Let $\{h_i\}_{i \in \Theta}$ and $\{g_j\}_{j \in \Psi}$ be the collections of all those L-subsets in U which assume value δ_1 and δ_2, respectively at a. Let $G = \bigwedge_{i \in \Theta} h_i$ and $H = \bigwedge_{i \in \Psi} g_j$. Then, G; $H \in U$ and $G = a_{\delta 1} \vee b_{\alpha i}$, $H = a_{\delta 2} \vee c_{\alpha j}$, where b, c $(\neq a) \in X$ and α_i, $\alpha_j \in Y$. If $b \neq c$ or $I \neq j$, then $G \wedge H = a_{\delta 3} \in U$, a contradiction since, $\delta_3 \in B \Rightarrow b = c$ and $\alpha_i = \alpha_j \Rightarrow$ there exists a unique element $b\ (\neq a) \in X$ and a unique atom α_i such that $b_{\alpha i} \leq h$, $\forall h \in U$ such that $h(a) = \gamma$ for any $\gamma \in B$. Clearly, there is no L-subset g_1 in U such that $g_1(a) = \gamma$ and $g_1(b) = \rho$, where $\gamma \in B$ and $\rho \in L$ such that $\rho \wedge \alpha_i = 0$. Hence, $U = \{f \in L^X: f(a) \vee \beta_k = 1 \Rightarrow b_{\alpha i} \leq f\} = б_{a,\ b\alpha i}(a_{\beta k})$ which is an ultra L-topology by theorem 3.10.

Remark 3.14: By remark 3.11 and theorem 3.13, it follows that there are $|X|\ |Y|^2\ (|X|-1)$ ultra L-topologies not containing the L-fuzzy points x_1, $\forall\ x \in X$.

Theorem 3.15: Let X be a non-empty set and Y be the set of all atoms of a complete atomic Boolean lattice L, then there are $|X|\ |Y|\ (|Y|-1)+|X|\ |Y|^2\ (|X|-1)$ ultra L-topologies in the lattice F_X.

Proof: Let U be an arbitrary ultra L-topology in F_X. By lemma 3.2, $a_\lambda \notin U$ for some $a \in X$ and $\lambda(\neq 0) \in L$.

Case 1: $\lambda(\neq 0) \in L$ such that $0<\lambda<1$. Then by theorem 3.7, $U = U_q^k(a)$ for some $a \in X$ and k, $q \in \Pi$ such that $q \neq k$.

Case 2: $\lambda = 1$. Then by theorem 3.13, $U = б_{a,\ b\alpha i}(a_{\beta k})$ for some a, $b \in X$ such that $a \neq b$ and i, $k \in \Pi$. $\Rightarrow U_q^k(a)$ where $a \in X$ and k, $q \in \Pi$ such that $q \neq k$ and $б_{a,\ b\alpha i}(a_{\beta k})$ where a, $b \in X$ such that $a \neq b$ and i, $k \in \Pi$ are the only ultra L-topologies in F_X. Therefore, by remarks 3.8 and 3.11, total number of ultra L-topologies in F_X is $|X|\ |Y|\ (|Y|-1)+|X|\ |Y|^2\ (|X|-1)$.

Remark 3.16: By using the formula given by theorem 3.15, number of ultra L-topologies in F_X can be found for any non-empty set X and any complete atomic Boolean lattice L.

CONCLUSION

In this study, we have identified ultra L-topologies in the lattice of all L-topologies on a non-empty set X when membership lattice L is a complete atomic Boolean lattice. Topological properties of these ultra L-topologies will be identified in future studies.

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