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Precise Long-Term Prediction of Behavior in a 3-D Chaotic Map

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Abstract: In this study, the prediction accuracy on long-term of future behavior in a 3-D chaotic map is possible. This study introduces and justifies by numerical simulation a new phenomenon shown by the 3-D map which is the behavior in the 3-D map is always regular and periodic and bounded on large time, i.e. that the behavior repeats itself regularly after cycles on large time intervals. The new 3-D map produces new several chaotic attractors obtained via quasi-periodic route to chaos.

Key words: Precise prediction, 3-D map, future behavior, symmetric intervals, regular behavior, time intervals

INTRODUCTION

Since, the discovery of the first famous three dimensional Lorenz chaotic attractor (Lorenz, 1963), chaos as a most fascinating phenomenon in non-linear dynamical systems, in recent years has been a considerable increase in their study (Rossler, 1976; Chua *et al.*, 1968; Lu and Chen, 2002; Hu *et al.*, 2012; Mandal *et al.*, 2013, 2011; Mammeri, 2016; Gonchenko *et al.*, 2005, 2007; Gonchenko and Meiss, 2006). The correct prediction on long-term of the behavior of solutions in non-linear dynamical systems is interesting and important in understanding of evolution of behavior of chaotic systems. It is well known the chaotic behavior to be strongly dependent on initial conditions, small changes in initial conditions can possibly lead to immense changes in subsequent on large time intervals and is what so ever difficult to precisely predict the future in next few decades. For example, in life sciences an interesting still open problem is to predict human future behavior from the actions they took in the past in other word it is possible to use someone past actions to predict his future behavior? Many other researchers have investigate the prediction of future behavior see for instance the research done by Lehnertz and Elger (1998), Romanelli *et al.* (1988) Jeong (2002), Arrow *et al.* (2008), Boettiger and Hastings (2013) and Dauwels *et al.* (2009). In this study, the accurate prediction of future behavior in 3-D chaotic discrete system on large time intervals is possible. This short study reports and investigate the effect of the sine map in 3-D discrete systems, the modified 3-D discrete map (Eq. 1) obtained via direct modification of the 3-D discrete system proposed by Dullin and Meiss (2000). The map (Eq. 1) is defined with two sine nonlinearities topologically different from any other know 3-D systems. In this study, we have studied some impotents basic properties for a new 3-D map and introduces and justifies numerically a new physical phenomenon shown by the new 3-D map (Eq. 1) which is the behavior of the map (Eq. 1) is always regular and periodic, i.e. that the behavior repeats itself regularly after cycles (periods) on large time symmetry intervals. Furthermore the behaviors observed for the map 1 are bounded and symmetric about the origin. Considered essentially the following modified 3-D map Eq. 1:

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} y_t \\ \sin z_t \\ a+bx_t+cy_t-\sin z_t \end{pmatrix} \quad (1)$$

where, (a-c) $\in \mathbb{R}^3$ are bifurcation parameters and $(x_t, y_t, z_t)\in\mathbb{R}^3$ are the state variables. The choice of sinusoidal map has an important role is to guarantee the boundedness of the orbits of the map (Eq. 1) for all values of a-c. Generally, the map (Eq. 1) is not symmetric and the associated map of the new 3-D map (Eq. 1) is continuous and differentiable on $\mathbb{R}^3$. Furthermore the Jacobian matrix of the map (Eq. 1) is not constant and equal to b cos z ($b\neq0$ and $\cos z\neq0$). The system (Eq. 1) can be transform into a third-order difference Eq. 2 as follows:

$$z_{t+1} = a+b\sin z_{t-2}+c\sin z_{t-1}-\sin z_t \quad (2)$$

MATERIALS AND METHODS

Qualitative properties of the map: In this study, we will show that the all orbits of the map (Eq. 1) are bounded and are lies inside in a box and we investigate domains for the bifurcation parameters $(a, b, c) \in \mathbb{R}^3$ in which the fixed points of the map (Eq. 1) are asymptotically stable.

Theorem 1: The all orbits of the map (Eq. 1) are bounded for every parameters (a-c) $\in \mathbb{R}^3$ and t>2 and for all finite initial conditions (x_0, y_0, z_0).

Proof: We use the following standard results, the real sequence $(z_n)_n$ is bounded if there is one positive real such that $|z_n|\leq k$ for every n$\in\mathbb{N}$. In our case the sequence $(z_t)_t$ given in satisfies the following inequality, $|z_t|\leq 1+|a|+|b|+|c|$ because $|\sin z|\leq 1$ for every z$\in\mathbb{R}$. Since, the real $1+|a|+|b|+|c|$ is positive, thus, the sequence $(z_t)_t$ is bounded for every (a, b, c)$\in\mathbb{R}^3$ and t>2. Thus, implies the all orbits of the map (Eq. 1) are bounded for every (a, b, c)$\in\mathbb{R}^3$, t>2 and for all finite initial conditions (x_0, y_0, z_0)$\in\mathbb{R}^3$.

Conclusion 1: The all orbits of the map (Eq. 1) are lies inside in the box:

$$\left\{(x,\ y,\ z)\in \mathbb{R}^3:\ |x|\leq 1,\ |y|\leq 1,\ |z|\leq 1+|a|+|b|+|c|\right\} \quad (3)$$

Proof: It's very easy to prove this conclusion, since, the map (Eq. 1) is equivalent to:

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \\ z_{t+1} \end{pmatrix} = \begin{pmatrix} \sin\ z_{t-1} \\ \sin\ z_t \\ a+b\ \sin z_{t-2}+c\ \sin\ z_{t-1}-\sin z_t \end{pmatrix} \quad (4)$$

Theorem: The fixed point A(x, y, z) of the map (Eq. 1) is asymptotically stable for all a$\in\mathbb{R}$ and if and only if (b, c)$\in\cup_{i=1}^{i=2}$ where:

$$\Omega_1: \begin{cases} -1<b<1 \\ \dfrac{b\ (b+1)\cos\ z^2-1}{\cos\ z}<c<\dfrac{1-(1-b)\cos\ z}{\cos\ z} \end{cases} \quad (5)$$

$$\Omega_2: \begin{cases} -1<b<1 \\ \dfrac{1+(1-b)\cos\ z}{\cos\ z}<c<\dfrac{b\ (b+1)\cos\ z^2-1}{\cos\ z} \end{cases} \quad (6)$$

Proof: The characteristic polynomial of the Jacobian matrix of the map (Eq. 1) calculated at the fixed point A (x, y, z) which takes the form, $P_A(\lambda) = \lambda^3$, according to the result available in Ogata (1995), we conclude that the fixed point A of the map (Eq. 1) is asymptotically stable if the following conditions hold |b cos z|<1, 1+cos z-c cos z-b cos z>0, 1-cos z-c cos z+b cos z-c cos z>0 and 1-b^2 $\cos^2$ z>b $\cos^2$ z-c cos z-c cos z. From Eq. 1, we have Eq. 5 |b|<1 and from Eq. 2, Eq. 3 and Eq. 5, we have Eq. 6 c cos z<1-(1-b) |cos z|, from Eq. 4 we have Eq. 7 c cos z>b (b+1) $\cos^2$ z-1 and from Eq. 6 and 7, we get Eq. 8 b (b+1) $\cos^2$ z-1<c cos z<1-(1-b) |cos z|. Finally, the conditions Eq. 1 and 8 give the all conditions Eq. 5 and 4 of asymptotic stability for the fixed point A. For example, the fixed points of the map (Eq. 1) are the real solutions of the system:

$$x = y,\ y = \sin z,\ z = a+bx+cy-\sin z$$

Hence, one may easily obtain Eq. 7:

$$z-(b+c)\ \sin z+\sin z-a = 0$$

Can't be compute the fixed points of the map (Eq. 1) analytically, we remark if a = 0, the point (0, 0, 0) it is fixed point of the map (Eq. 1) for all values of the bifurcation parameters (b, c)$\in\mathbb{R}^2$. Then we have the following theorem:

Theorem 3: If a = 0, the fixed point (0, 0, 0) of the map (Eq. 1) is asymptotically stable if and only if the following conditions holds:

$$\begin{cases} -1<b<1 \\ b\ (b+1)-1<c<b \end{cases} \quad (7)$$

If we choose a = 0, b = 0.8 and c = 0.1. Then with this values the fixed point (0, 0, 0) is asymptotically stable and we have the following three eigenvalues λ_1 = -0.8566-0.6229i λ_2 = -0.8566+0.6229i and λ_3 = -0.3380, thus $|\lambda_{1,2,3}|<1$.

Bifurcation analysis of the map: In this study, we will illustrate some observed chaotic behaviors, the dynamical behaviors of the map (Eq. 1) are investigated numerically. Figure 1-6 show the bifurcation diagram and the diagram of the variation of the largest Lyapunov exponent of the map (Eq. 1) that are obtained at different values of parameter a, a$\in$ [-4, 4]. However, we deduce from the bifurcation diagram Fig. 1a that the proposed map (Eq. 1) exhibit a quasi-periodic bifurcation scenario route to chaos for the selected values of the bifurcation parameter a.

First, we fix the initial condition $x_0 = y_0 = z_0 =$ 0.01 and b = 0.8, c = 0.9 and let the parameter a vary in the interval [-4, 4], the map (Eq. 1) exhibits the following dynamical behaviors as shown in Fig. 4a and b). For the quasi-periodic with periodic windows at the point a = -2.72 the dynamical behavior the map (Eq. 1) is in the

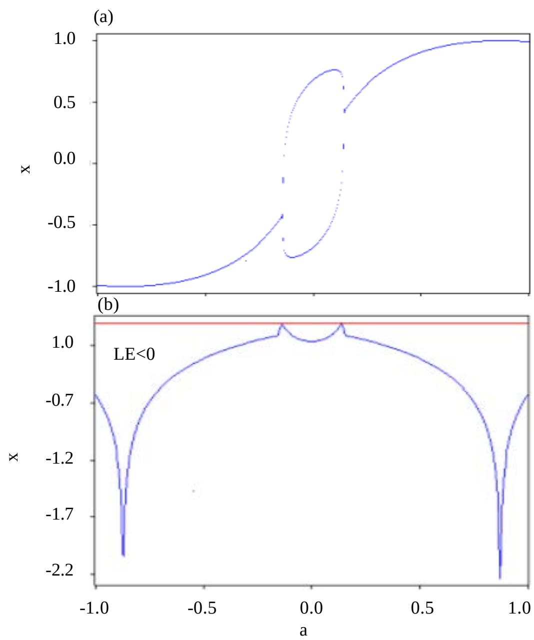

Fig. 1: a) The symmetric bifurcation diagram for the map (Eq. 1) obtained for b = 0.8, c = 0.9 and $-1 \leq a \leq 1$ and b) Variation symmetry of Lyapunov, exponent of the map (Eq. 1) for b = 0.8, c = 0.9 and $-1 \leq a \leq 1$

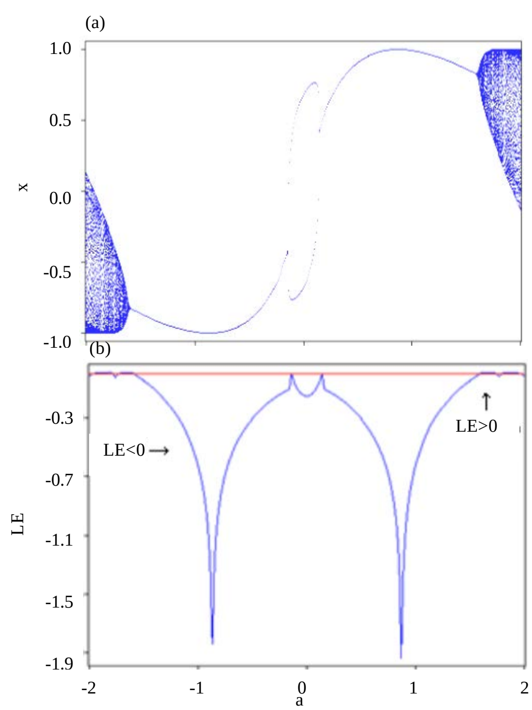

Fig. 2: a) The symmetric bifurcation diagram for the map (Eq. 1) obtained for b = 0.8 c = 0.9 and $-2 \leq a \leq 2$; and b: Variation symmetry of Lyapunov exponent of the map (Eq. 1) for b = 0.8, c = 0.9 and $-2 \leq a \leq 2$

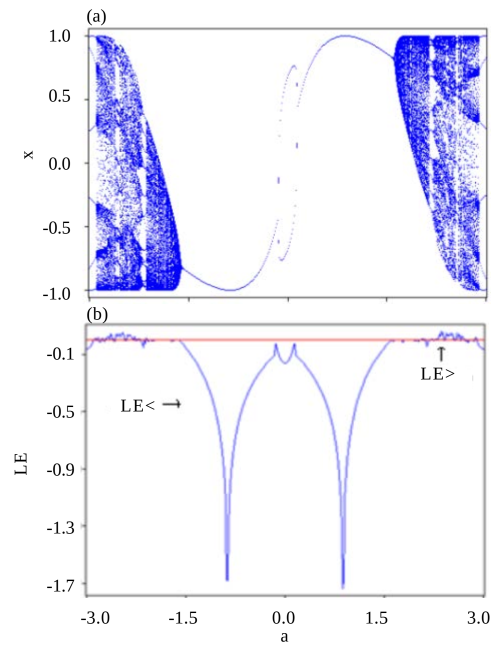

Fig. 3: a) The symmetric bifurcation diagram for the map (Eq. 1) obtained for b = 0.8 c = 0.9 and $-3 \leq a \leq 3$ and b) Variation symmetry of Lyapunov exponent of the map (Eq. 1) for b = 0.8, c = 0.9 and $-3 \leq a \leq 3$

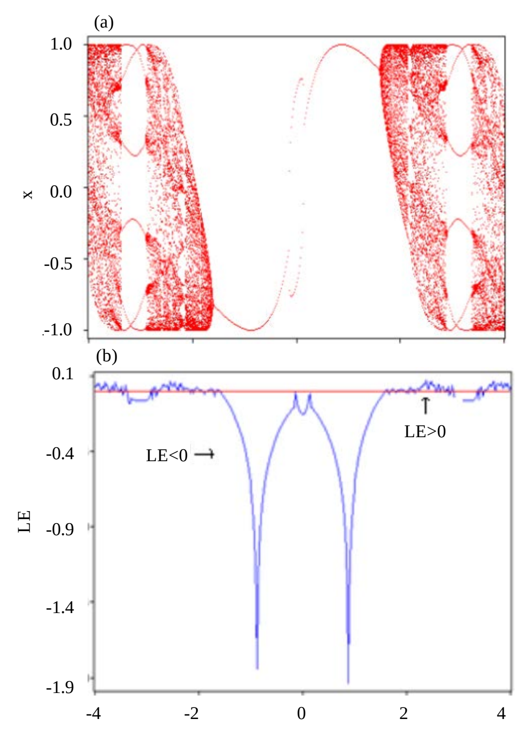

Fig. 4: a) The symmetric bifurcation diagram for the map (Eq. 1) obtained for b = 0.8 c = 0.9 and $-4 \leq a \leq 4$ and b) Variation symmetry of Lyapunov exponent of the map (Eq. 1) for b = 0.8, c = 0.9 and $-4 \leq a \leq 4$

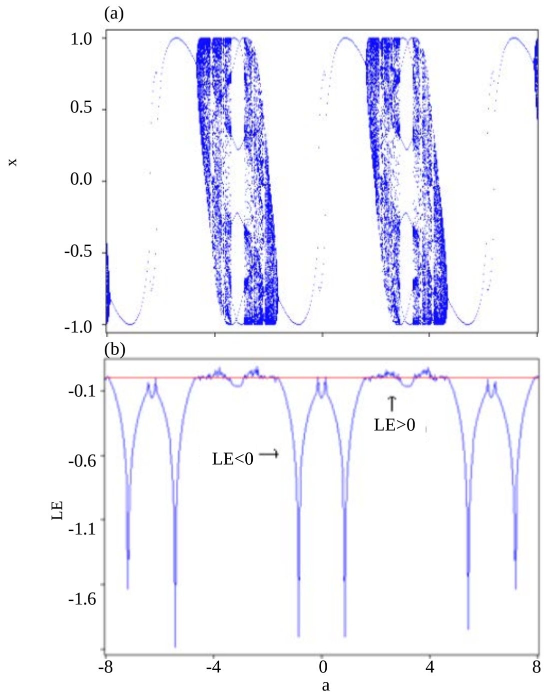

Fig. 5: a) The symmetric bifurcation diagram for the map (Eq. 1) obtained for b = 0.8 c = 0.9 and -8≤a≤8 and b) Variation symmetry of Lyapunov exponent of the map (Eq. 1) for b = 0.8, c = 0.9 and -8≤a≤8

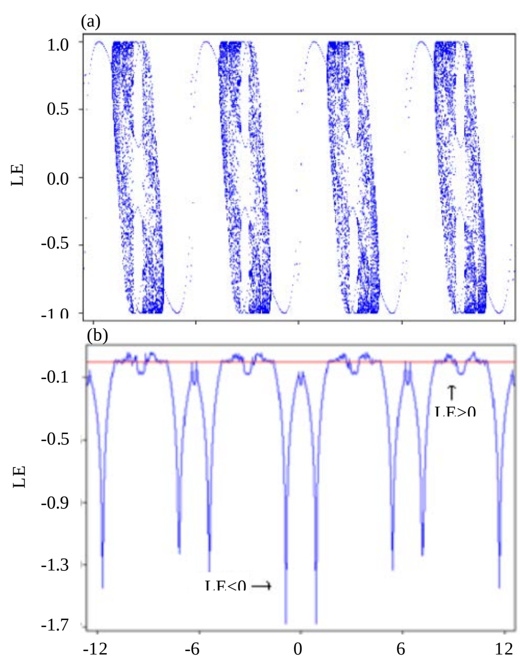

Fig. 6: a) The symmetric bifurcation diagram for the map (Eq. 1) obtained for b = 0.8 c = 0.9 and -12≤a≤12 and b) Variation symmetry of Lyapunov exponent of the map (Eq. 1) for b = 0.8, c = 0.9 and -12≤a≤12

quasi-periodic-40 as shown in Fig. 7d and at the point a = -3.52 the behavior is quasi-periodic as shown in Fig. 7c, for the range -4≤a≤-2.72 the behavior of the map (Eq. 1) is chaotic with periodic windows in the chaotic band which is verified by the corresponding largest Lyapunov exponent is positive, Fig. 7a and b shows, respectively the chaotic behaviors of the map (Eq. 1) when a = -3.76 and a = -3.6 (Fig. 1-3).

Secondly, for the range 0≤a<1.68 the dynamical behavior of the map (Eq. 1) is periodic which is verified by the corresponding largest Lyapunov exponent is negative as shown in Fig. 4b for the range 1.68≤a<2.32 the dynamical behavior of the map (Eq. 1) is quasi-periodic orbits with periodic windows at the point a = 2.24 the dynamical behavior of the map (Eq. 1) is in the quasi-periodic-19 as shown in Fig. 8e and f shows the quasi-periodic attractor of the map (Eq. 1) when a = 1.84 for the range 2.32≤a<2.72 the dynamical behavior of the map (Eq. 1) is chaotic with periodic windows in the chaotic band which is verified by the corresponding largest Lyapunov exponent is positive. For the range 2.72≤a<3.6 the dynamical behavior of the map (Eq. 1) is quasi-periodic with periodic windows at the point a = 3.52 the dynamical behavior of the map (Eq. 1) is in range-1.68<a≤0 the dynamical behavior of the map (Eq. 1) is periodic which is verified by the corresponding largest Lyapunov exponent is negative as shown in Fig. 4b for the range -2.32<a≤-1.68 the dynamical behavior the map (Eq. 1) is quasi-periodic with periodic windows at the point a = -2.24 the dynamical behavior the map (Eq. 1) is in the quasi-periodic-19 attractor as shown in Fig. 7e and f shows the quasi-periodic attractor of the map (Eq. 1) when a = -1.84 for the range -2.72<a≤-2.32 the dynamical behavior of the map (Eq. 1) is chaotic with periodic windows in the chaotic band which is verified by the corresponding largest Lyapunov exponent is positive. For the range -3.6<a≤-2.72 the dynamical behavior of the map (Eq. 1) is the quasi-periodic-40 as shown in Fig. 8d and at the point a = 3.52 the dynamical behavior is quasi-periodic as shown in Fig. 8c for the range 3.6≤a≤4 the dynamical behavior of the system (Eq. 1) is chaotic with periodic windows in the chaotic band which is verified by the largest Lyapunov exponent is positive, Fig. 8a and b, respectively show the dynamical behavior of the map (Eq. 1) when a = 3.76 and a = 3.6.

Precise long-term prediction of behavior: Dynamical systems theory is an area of mathematics used to describe

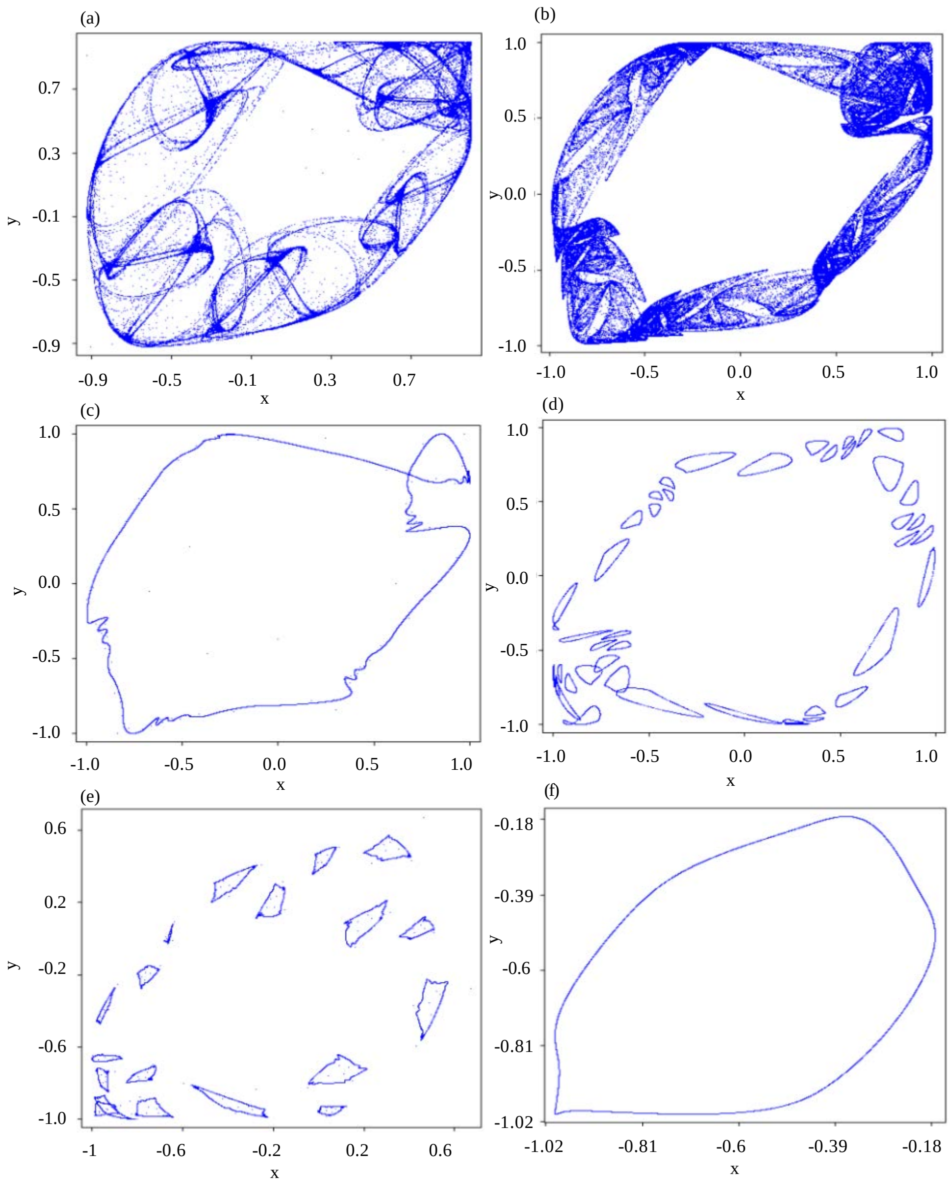

Fig. 7: a) a = -3.76; b) a = -3.76; c) a = -3.52; d) a = -2.72; e) a = -2.24 and f) a = -1.84

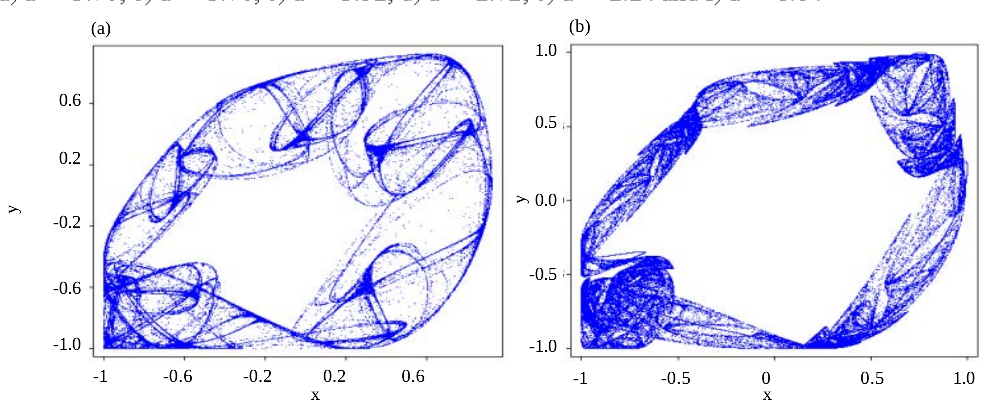

Fig. 8: Continue

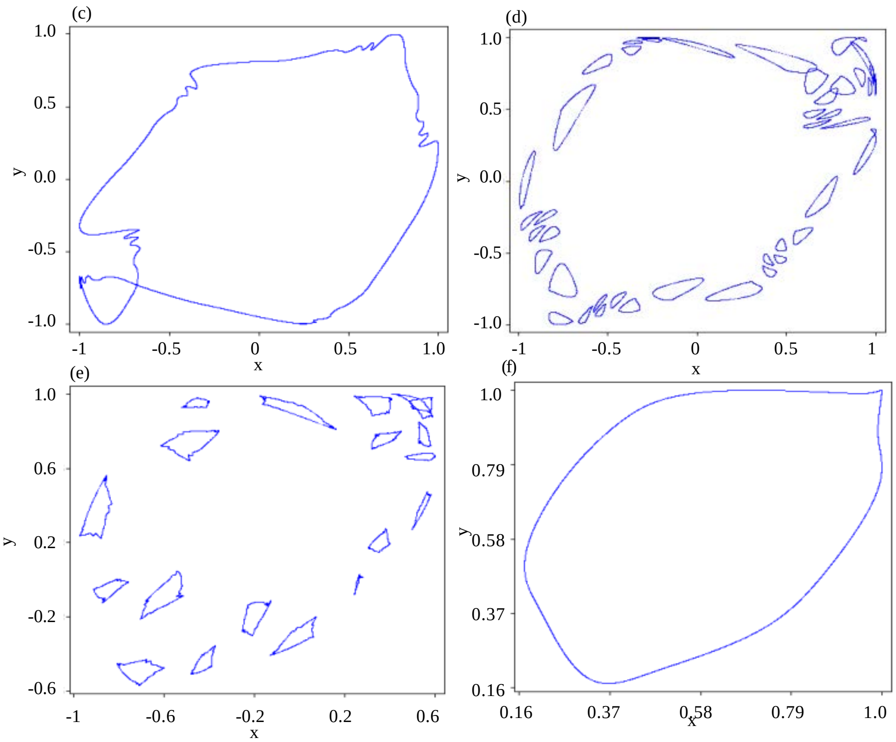

Fig. 8: a) a = 3.76; b) a = 3.76; c) a = 3.52; d) a = 2.72; e) a = 2.24 and f) a = 1.84

the behavior of dynamical systems. The correct prediction of the behavior of orbits of dynamical map (Eq. 1) is important in understanding of evolution process and reduces uncertainty. From the above analysis and the bifurcation diagrams in Fig. 1a, b and 6a, b we deduce that the dynamical behavior on long-term of the map (Eq. 1) is always regular periodic and bounded and has a horizontal stretch, i.e. that the dynamical behavior repeats itself on large time symmetry intervals ([-a, a], a∈ℝ) as it moves along the a-axis and the cycles of this regular repeating are called periods and the amplitude of the behavior bands will be 1 (confined between-1 and +1) as shown in Fig. 1a, b and 6a, b. Furthermore, the dynamical behavior of map (Eq. 1) given in Fig. 7 and 8 are respectively symmetric about the origin and inside the two finite symmetry intervals [-a, 0] and [0, a], a∈ℝ (Fig. 7a-f and 8a-f).

CONCLUSION

In this study, the prediction accuracy on long-term of future behavior in 3-D systems is possible this phenomenon is justified by numerical investigation. In other hand an analytic properties for the dynamics of this system is also presented in term of a singlebifurcation parameter. Furthermore, the new 3-D system produces new several chaotic attractors observed via. quasi-periodic route to chaos.

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