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# Portfolio Selection Optimization Which Involves Minimum Transaction Lot and Transaction Cost Using Rank Dependent Expected Utility

Pasrun Adam and La Gubu
Department of Mathematics, Kampus Bumi Tridharma Anduonohu, Universitas Halu Oleo, 93232 Kendari, Indonesia

**Abstract:** The purpose of this research was to develop a model of a portfolio selection optimization problem involving transaction lot and transaction cost in accordance with conditions in the Indonesia Stock Exchange. In the development of this model, the rank dependent expected utility theory was used. From the simulation result using the daily data of stock price, it was found that the behaviour of the objective function G(w) decreases, if parameter value a and b increase.

**Key words:** Portfolio selection**,** RDEU theory, quadratic utility function, transaction lot, transaction cost, decreases

## INTRODUCTION

A lot represents a standard quantity of financial instruments established by a financial institution or stock exchange. In Indonesia, the institution that sets the lot of transaction is set by the Indonesia Stock Exchange. In stock trading, the transaction lot specifies the minimum number of shares that can be traded. So, when investors buy stock instruments, the unit of transaction used is the lot.

On the other hand, transaction cost is these that investors must pay when they buy or sell equity. Institutions that charge this transaction fee are brokers or financial intermediaries appointed by the government. The transaction cost will weigh on the investors, so, it will reduce their profits.

Mansini *et al.* (2015) divided the transaction cost structure in several types or models, namely fixed transaction cost, proportional transaction cost, convex piecewise linear cost, concave piecewise linear cost and linear cost with minimum charge. In addition, other researchers used different types of transaction costs, for example concave transaction cost (Xue *et al*., 2006; Gao *et al*., 2009). In the calculation of transaction costs, the use of these types of transaction costs gives a different impact on the magnitude of the profits of investors in investing.

Nowadays, the development of the model of portfolio selection problem has been done by many researchers. However, they are still rarely involving transaction lot or/and transaction cost in their models. In general, a mean-variance model is widely used in the development of their models, for example (Xue *et al*., 2006; Lin *et al*., 2005).

This study aimed to build a model of portfolio selection optimization problem involving transaction lot and proportional transaction cost. The proportional transaction cost is involved in the model because it has been used on practical issues in Indonesia. The theory used in the development of this model is the theory of Rank-Dependent Expected Utility (RDEU) developed by Quiggin (1982, 1993). Thus, the main contribution of this study is the finding of a portfolio selection optimization model involving transaction lot and transaction cost that can be beneficial to investors in their portfolio diversification.

## MATERIALS AND METHODS

In this research, the development of model of portfolio selection problem using the RDEU theory proposed by Quiggin (1982, 1993). This model uses three types of functions namely, utility function, cumulative distribution function and probability weight function. There are many types of utility functions that have been used by previous researchers to analyze the portfolio selection including: linear utility function (Cenci and Fillipine, 2006; Gao *et al*., 2010), quadratic utility function (Rahadi *et al*., 2015; Adam and Gubu, 2017; Koo *et al*., 2016), isoelastic form utility function (Bahaji and Casta, 2016; Sims, 2015) and logarithmic utility function (Xuan and Yue, 2011; Barrachina *et al*., 2012).

**Corresponding Author**: Pasrun Adam, Department of Mathematics, Kampus Bumi Tridharma Anduonohu, Universitas Halu Oleo, 93232 Kendari, Indonesia

Suppose that financial markets have uncertain events in the probability space (Ω, f, p) where, Ω is the set of universal events, f is a field or set collection of events and P is a probability measure defined in subcollections of the set of events E of f such that $0 \leq p(E) \leq 1$. Assume that X is a random variable defined in the probability space (Ω, f, p). This random variable has a cumulative distribution function $F_X(x) = pr(x \leq x)$, $x \in (-\infty, +\infty)$. The function $F_X(x)$ is an increasing function such that $F_X(-\infty) = 0$ and $F_X(+\infty) = 1$.

Furthermore, the RDEU theory according to Quiggin (1982, 1993) and had been referenced by Adam and Gubu (2017) is as follows. Suppose that the random variable X has the values $x_i (i = 1, 2, ..., n)$ with $x_i \leq x_2 \leq, ..., x_n \leq$ and has a probability measure $p_1, p_2, ..., p_n$. The formulation of discrete form of the RDEU theory is:

$$RDEU(X) = \sum_{i=1}^{n} U(x_i)\left[g\left(\sum_{j=1}^{i} p_j\right) - g\left(\sum_{j=1}^{i-1} p_j\right)\right] \quad (1)$$

where, U: $(0, \infty) \rightarrow (-\infty, +\infty)$ is a utility function of investor preferences in the portfolio selection. The function U(x) is an increasing function with U(0) = 0. The function g: $[0, 1] \rightarrow [0, 1]$ is called the probability weight function. This function is an increasing function with g(0) = 0 and g(1) = 1. Equation 1 can be expressed in the form of Lebesque-Stieltjes integral:

$$RDEU(X) = \int_{\Omega} U(x) dg(F_x(x)) \quad (2)$$

If the cumulative distribution function $F_X(x)$ has a derivative $f(x) = F_X(x)$ and the functions U(x) and g(x) are continuous, then Eq. 2 can be expressed in the form of Riemann integral:

$$RDEU(X) = \int_{-\infty}^{+\infty} U(x) g'(F_x(x)) f(x) dx \quad (3)$$

The function f(x) is a density function of the random variable X. Next, let ≥ be a relation of investor preference to select an asset or a portfolio. Suppose that there are two random variables X and Y with their cumulative distribution functions are $F_X$ and $F_Y$. Then $X \geq Y$ if and only if RDEU (X) ≥ REDU(Y). In this case, the investor will choose X, if the RDEU(X) value is greater than the RDEU(Y) value or the investor can choose either X or Y if RDEU(X) = RDEU(Y).

Adam and Gubu (2017) developed the RDEU Model in which the utility function is a quadratic function:

$$U(x) = x - \alpha x^2, 0 \leq x < \frac{1}{2\alpha}, 0 < a < 1$$

And the probability weight function g(x) is:

$$g(x) = \begin{cases} bx, \text{ if } 0 \leq x \leq Pr(X \leq \nu) \\ cx+d, \text{ if } Pr(X \leq \nu) < x \leq 1 \end{cases} \quad b > c > 0$$

The function g(x) is a concave peacewise linear function and continuous on [0, 1]. Further, it is assumed that the random variable X is normally distributed with the probability function:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-m^2)}{2\sigma^2}\right), x \in (-\infty, +\infty)$$

where, m = E(x) is the mean and σ is standard deviation of X. With this assumption and the monotonic, concavity and continuity properties of the function g(x), it is obtained $0<c<1<b<2$. If the value of a is close to 0, Adam and Gubu (2017) had shown that the approximate value of RDEU(x) is:

$$RDEU(X) = -\alpha\sigma^2 + \frac{(2-2b)(1-2\alpha m)\sigma}{\sqrt{2\pi}} + (m - \alpha m^2) \quad (4)$$

The investor portfolio has a weight vector $w = (w_1, w_2, ..., w_n)$ with $\sum_{i=1}^{n} w_i = 1$. Furthermore, the returns of equity $R_i$ i, (i = 1, 2, ..., n) are mutually independent and normally distributed. Then, the random variable return of the portfolio $R_p = \sum_{i=1}^{n} w_i R_i$ is normally distributed which has the expected return $E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$ and the standard deviation:

$$\sigma_p = \left(\sum_{i=1}^{n} w_i w_j \sigma_{ij}^2\right)^{\frac{1}{2}18}$$

Cenci and Filippini (2006) provided a formula for risk based on the RDEU theory, namely φ (X) = U(E (X))-RDEU(X). By using this formula for $X = R_p$ and Eq. 4, then the portfolio standard risk approximation value (Adam and Gubu, 2017) of the investor is:

$$\varphi(R_p) = \alpha\sigma_p^2 + \frac{(2b-2)(1-2\alpha E(R_p))\sigma_p}{\sqrt{2\pi}} \quad (5)$$

where, $0<\alpha<1$ and $1< b<2$. In the results study, Eq. 5 is used to develop a model of objective function of the portfolio selection optimization problem involving transaction lot and transaction cost and its constraints. Furthermore, a simulation is performed to evaluate the behaviour of the objective function. The stock price data used in this simulation is daily stock price data from ten companies where the ten stocks of the companies are in the LQ45 stock group in the Indonesia Stock Exchange. The daily stock price data was selected from the closing price that spans from June, 20, 2016-June, 16, 2017.

## RESULTS AND DISCUSSION

Suppose that $M(M_0 \leq M \leq M_1)$ represents the amount of money invested in the securities of the stock and let $k_i$ is a number of shares of the securities $I(i = 1, 2, ..., n)$. Thus, the investors have a portfolio. Suppose that $n_i$ is a minimum lot of transaction purchased by the investor for securities i, then $K_i$ is a multiple of $n_i(i = 1, 2, ..., n)$. Next, let also $o_i$ is a number of shares per lot for securities $(i = 1, 2, ..., n)$. Thus, we obtain $k_i = n_i o_i$, $(i = 1, 2, ..., n)$. If $p_i$ is a price of each share, then the total purchase price of each securities i is:

$$M_i = k_i p_i = n_i o_i p_i, (i = 1, 2, ..., n) \quad (6)$$

Based on Eq. 6, the total purchase price of n type of securities (or amount of initial wealth) of the investor is:

$$M = \sum_{i=1}^{n} M_i = \sum_{i=1}^{n} k_i p_i = \sum_{i=1}^{n} n_i o_i p_i \quad (7)$$

where, $M_0 \leq M \leq M_1$, $M_0$ is a lower bound of investment and $M_1$ is an upper bound of investment. Further, let $\alpha$ $(0<\alpha<1)$ is the proportional transaction cost or fee to be paid by the investor to the broker for the purchase of the security. The amount of commission received by the broker is:

$$\alpha M = \sum_{i=1}^{n} \alpha k_i p_i = \sum_{i=1}^{n} \alpha n_i o_i p_i \quad (8)$$

The random variable of return of the securities i is $R_i$ with the expected return $(R_i)$, $(i = 1, 2, ..., n)$. If the weight of each securities in the portfolio W is $w_i = k_i p_i/M = n_i o_i p_i/M(i = 1, 2, ..., n)$ and $R_p^{wlc}$ is the return of portfolio W which involves the transaction lot and proportional transaction cost, then from Eq. 7 and 8 the expected return of portfolio W with weight vector $w = (w_1, w_2, ..., w_n)$ is:

$$E\left(R_p^{wlc}\right) = \sum_{i=1}^{n} w_i E(R_i) - \frac{\sum_i^n \alpha n_i o_i p_i}{M} = \sum_{i=1}^{n} \frac{n_i o_i p_i}{M} E(R_i) - \alpha \quad (9)$$

In accordance with investment theory and practical review, the investors usually want the return from their investment to be maximum, so, the value of $E\left(R_p^{wlc}\right)$ in Eq. 9 is maximum. The problem of portfolio selection optimization is the combination of Eq. 5 and 9 as follows:

$$\text{Min}\,\varphi(R_p) = \text{Min}\left(\alpha\sigma_p^2 + \frac{(2b-2)\left(1-2\alpha E(R_p)\right)\sigma_p}{\sqrt{2\pi}}\right) \quad (10)$$

$$\text{Maks}\, E\left(R_p^{wlc}\right) = \text{Maks}\left(\sum_{i=1}^{n} \frac{n_i o_i p_i}{M} E(R_i) - \alpha\right) \quad (11)$$

$$M_0 \leq M \leq M_1 \quad (12)$$

$$\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} \frac{n_i o_i p_i}{M} = 1 \quad (13)$$

$$w_i \leq 0, (i = 1, 2, ..., n) \quad (14)$$

By using the concept of a multiobjective function, then the portfolio selection objective function which involves transaction lot and transaction cost is:

$$G(w) = -\beta\left(\alpha\sigma_p^2 + \frac{(2b-2)\left(1-2aE(R_p)\right)\sigma_p}{\sqrt{2\pi}}\right) + (1-\beta)\left(\sum_{i=1}^{n} \frac{n_i o_i p_i}{M} E(R_i) - \alpha\right) \quad (15)$$

where, $0<\alpha<1$, $1<b<2$ and $0<\beta<1$. The value of the objective function G(w) at Eq. 11 must be maximum with constraints (Eq. 12-14). Variance $\sigma_p^2$ and expected return $E(R_p)$ at Eq. 11 are:

$$E(R_p) = \sum_{i=1}^{n} \frac{n_i o_i p_i}{M} E(R_i)$$

And:

$$\alpha_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{n_i o_i p_i}{M} \frac{n_i o_i p_i}{M} \sigma_{ij}^2$$

The objective function in Eq. 11 has the assumption that the investors only make one purchase. The behaviour of the objective function G(w) is evaluated by selecting the values of a and b for a vector weighted portfolio $w = (w_1, w_2, ..., w_n)$ through a simulation. In this simulation the data used is daily stock price data from ten companies where the ten stock companies are listed in the Indonesia Stock

Table 1: The company names, stock code, purchase price and expected return

| Company name | Stock code | Purchase price in rupiah currency ($p_i$) | Expected return $E(R_i)$ |
|---|---|---|---|
| Astra Agro lestari Tbk | AALI | 16000 | -0.001050 |
| Adhi Karya (Persero) Tbk | ADHI | 16000 | 0.000879 |
| Adaro Energy Tbk | ADRO | 1750 | -0.000729 |
| AKR Corporindo Tbk | AKRA | 6650 | -0.000382 |
| Aneka Tambang (Persero) Tbk | ANTM | 810 | -0.001016 |
| Astra International Tbk | ASII | 8050 | 0.001009 |
| Alam Sutera Realty Tbk | ASRI | 386 | -0.001942 |
| Bank Central Asia Tbk | BBCA | 15425 | 0.001685 |
| Bank Negara Indonesia (Persero) Tbk | BBNI | 5950 | 0.001098 |
| Bank Rakyat Indonesia (Persero) Tbk | BBRI | 11950 | 0.002381 |

Table 2: The values of portfolio selection objective function

| Values | Variables | G(w) |
|---|---|---|
| a = 0.1 | b = 1.1 | 0.005324 |
| a = 0.3 | b = 1.3 | 0.005319 |
| a = 0.5 | b = 1.5 | 0.005314 |
| a = 0.7 | b = 1.7 | 0.005309 |
| a = 0.9 | b = 1.9 | 0.005304 |

Exchange. These stocks are included in the LQ45 stock group. The stock code, the purchase price and the stock return and the names of the ten companies are shown in Table 1.

The Indonesia Stock Exchange has determined the number of shares per lot in a transaction, namely 100 shares per lot, so $n_1 = n_2 = \cdots = n_n$, =100. Transaction cost of purchase is 0.2%, so, $\alpha$ = 0.0002. Suppose that the initial investment is IDR 10,000,000, so, M = 10,000,000. The result of data processing is shown in Table 2.

The objective function values G(w) for some values a and b as shown in Table 2, it seems that the value of G(w) is decreased if the values of a and b are greater towards its the upper bound. This is due to the declining value of $-\varphi(R_p)$.

The decreasing value of $-\varphi(R_p)$ in Eq. 11 is due to the rise in the value of $-\varphi(R_p)$ in Eq. 5 if values a and b rise (Adam and Gubu, 2017). Furthermore, the results of this study differ from those of Lin *et al.* (2005) and Gao *et al.* (2009). These differences are due to the fact that in the model built by them, the lot is used as a unit of portfolio vector elements. Meanwhile, in this study, the units of portfolio vector elements used are the number of shares.

## CONCLUSION

This study aimed to develop a model of optimization problem of portfolio selection using the RDEU theory involving transaction lot and transaction cost. In the development of this model, it is assumed that the investor only makes one purchase. The transaction cost to be paid by the investor only at the time of purchase. The simulation result shows that the bigger the value of a and b, the smaller the expected return value of the investor.

## RECOMMENDATIONS

This study does not consider the cost of sale transaction in the model. In the next study, the model development can consider the cost of sale transaction. Because in fact, the investors can buy shares more than once in a certain time period, so in the next study, the payment of transaction cost more than once can also be considered in the development of the model.

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