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Effective Turbulence Model for High Speed Flow for General Engineering Applications

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Abstract: This study deals with the revised version of k-epsilon-V_n developed by the researchers earlier. Intentionally the model was developed to deal with free-stream gas jets. In present research the model was adapted for universal usage both in free-stream and boundary-layer flows. The model has the following distinctive features.The model is of the nonlinear eddy viscosity group. Eddy viscosity is proportional to turbulent time scale and scale of velocity fluctuation normal to streamlines with velocity fluctuation V_n' determined via. separate differential equation. Time scale is determined dissipation rate parameter 1/ω unlike other V_n Models utilizing ratio between turbulence kinetic energy and dissipation-k/epsilon.

Key words: Computational fluid dynamics, turbulance, compressibility, k-epsilon, eddy viscosity, nonlinear eddy

INTRODUCTION

Turbulent flow is often prevailing in modern vehicles and machinery. Turbulence is one of the underlying phenomena in fluid dynamics and heat transfer. The phenomenon of turbulence was discovered more than a century ago, yet it has not been fully understood. One of the biggest problems that remain in this area is investigation of turbulent gas flow at high speeds.

As it is known, one of the main features of any gas media is compressibility. In certain conditions (generally characterized by the magnitude of Mach number) compressibility is starting to affect the flow parameters greatly.

A typical supersonic effect is a drastic reduction in the growth rate of a plane free-shear layer flow. The reduction was confirmed in a plenty of experiments under various conditions (Papamoschou and Roshko, 1988; Goebel and Dutton, 1991).

This effect plays an important role in present-day problems of rocket and airspace engineering. For example, in a supersonic combustion ramjet reduced turbulence levels can be highly detrimental as they reduce the rate of fuel and oxidizer mixing. Compressibility changes the nature of boundary layer transition to turbulence in flows over hypersonic vehicles during re-entry.

Nevertheless, it must be noted that in some cases compressibility makes a very small impact on turbulence. For example, the experimental data obtained by the Goebel and Dutton (1991) showed that axial turbulence intensity actually does not depend on Mach number. Compressibility also has a very little impact on the intensity of turbulence in the near-wall region.

Computation of turbulent flows is performed using different approaches. The most wide-spread of them is utilization of Reynolds (Favre) averaged Navier-Stokes equations along with the turbulence model. By present day a plenty of turbulence models has been created. Nevertheless, all of them have certain limitations. Some of them are limited only to free stream conditions, others to the boundary layers.

Compressibility could be another limiting factor of a particular turbulence model implementation. The local density extensions of standard incompressible turbulence models were found to be inadequate in duplicating the experimentally observed reduction in growth rate of the mixing layer with increasing Mach number. Therefore, special treatment is needed in this case.

MODEL DESCRIPTION

This study deals with the revised version of k-ε-V_n developed by the researchers earlier. Initially the model was developed to deal with free-stream gas jets. In present research the model was adapted for universal usage both in free-stream and boundary-layer flows.

The model has the following distinctive features. The model is of the nonlinear eddy viscosity group. Eddy viscosity is proportional to turbulent time scale and scale

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of velocity fluctuation normal to streamlines with velocity fluctuation V_n determined via., separate differential equation. As noted by Durbin (1991) this approach is considered to be more mathematically correct than formulae used in standard k-ε and k-ω Models.

This is due to fact that as it is shown in classical fluid dynamics, eddy viscosity is proportional to velocity fluctuations normal to streamlines. At the same time, standard k-ε and k-ω Models utilize square root of turbulence kinetic energy $\sqrt{k}$ as a velocity scale which can exceed velocity fluctuation V'_n greatly.

Time scale is determined via. dissipation rate parameter $1/\omega$ unlike other V'_n Models utilizing ratio between turbulence kinetic energy and dissipation $\neg k/\varepsilon$. Classical formulation of epsilon-equation gives acceptable results in free-stream conditions but for certain reasons fails to determine turbulence parameters correctly in the near-wall region.

Approach utilizing omega-equation on the contrary performs greatly in the near-wall region but results given by such models in free-stream zone strictly depend on turbulence conditions in outer region of boundary layer. In this paper approach developed by Menter (1994) is utilized: omega-equation is rewritten in such form that inside boundary layer it performs as standard omega equation while switching to epsilon-like behavior in outer region.

Omega-equation has yet another disadvantage connected with determining wall boundary conditions as on the wall surface omega goes to infinity which creates certain handicaps for numerical solution. In the present work omega equation is replaced by equation for g-parameter determined as:

$$g^2 = \frac{1}{\beta^*\omega} = \frac{k}{\varepsilon}$$

As it was shown by Kalitzin *et al*. (1996) this grants natural boundary conditions on the wall-g = 0.5. Compressibility is treated using DNS-data based approach developed earlier by Molchanov and Bykov (2013). The equations of the model can be written as follows:

$$\frac{\partial}{\partial t}(\rho k)+\frac{\partial}{\partial x_k}(\rho u_k k) = \frac{\partial}{\partial x_k}\left[\left(\mu+\frac{\mu_T}{\sigma_K}\right)\frac{\partial k}{\partial x_k}\right]+P^* - \frac{\rho k}{g^2}$$

$$\frac{\partial}{\partial t}(\rho g)+\frac{\partial}{\partial x_k}(\rho u_k g) = \frac{\partial}{\partial x_k}\left[\left(\mu+\frac{\mu_T}{\sigma_g}\right)\frac{\partial g}{\partial x_k}\right] - $$

$$-\frac{\alpha}{2}\frac{g}{k}P^* + \frac{\beta\rho}{2\beta^* g} - \frac{3}{g}\left(\mu+\frac{\mu_T}{\sigma_g}\right)\frac{\partial g}{\partial x_k}\frac{\partial g}{\partial x_k} +$$

$$+\frac{(1-F_1)}{k}\cdot\left\{\frac{2\mu_T}{\sigma_g}\frac{\partial k}{\partial x_k}\frac{\partial g}{\partial x_k} + \frac{g}{2}\frac{\partial}{\partial x_k}\left[\mu_T\left(\frac{1}{\sigma_K}-\frac{1}{\sigma_\varepsilon}\right)\frac{\partial k}{\partial x_k}\right]\right\}$$

$$\frac{\partial}{\partial t}\left(\rho\overline{V_n'^2}\right)+\frac{\partial}{\partial x_k}\left(\rho u_k\overline{V_n'^2}\right) = \frac{\partial}{\partial x_k}\left[\left(\mu+\frac{\mu_T}{\sigma_K}\right)\frac{\partial \overline{V_n'^2}}{\partial x_k}\right]+$$

$$+\frac{2}{3}C_{\Pi 1}C_2 P - \left[C_1\frac{\overline{V_n'^2}}{k}+\frac{2}{3}(1-C_1)\right]\frac{\rho k}{g^2}$$

$$\mu_T = \min\left(C_D\rho\overline{V_n'^2}g^2;\ \frac{\rho a_1 k}{SF_2}\right)$$

where, is a Bradshaw constant and blending functions are defined similarly by Menter (1994):

$$F_1 = \tanh\left(\arg_1{}^4\right);$$

$$\arg_1 = \min\left[\max\left(\frac{g^2\sqrt{k}}{y}, \frac{500\mu\beta^* g^2}{\rho y^2}\right), \frac{1}{CD_{K\omega}}\frac{4\rho k}{\sigma_\varepsilon y^2}\right];$$

$$CD_{K\omega} = \max\left(-\frac{4\rho}{\sigma_\varepsilon g}\frac{\partial k}{\partial x_j}\frac{dg}{dx_j},\ 1.0\times 10^{-10}\right);$$

$$F_2 = \tanh\left(\arg_2{}^2\right);\ \arg_2 = \max\left(2\frac{g^2\sqrt{k}}{y}, \frac{500\mu\beta^* g^2}{\rho y^2}\right)$$

Model coefficients are defined as follows:

$$\beta^* = C_\mu = 0.09,\ \alpha_1 = 5/9,\ \beta_1 = 0.075,\ \sigma_{K1} = 1.176$$

$$C_D = \frac{(1-C_2)}{C_1},\ C_1 = 1.8,\ C_2 = 0.6,\ \sigma_{g1} = 2.0,\ \sigma_{K2} = 1.0,$$

The classical turbulence source term is modified as stated:

$$P^* = P\left[1-(1-F_1)C_{\Pi 2}\right]$$

Functions $C_{\Pi 1}$ and $C_{\Pi 2}$ depend on turbulent Mach number M_T:

$$C_{\Pi 1}(M_T) = \begin{cases} 1, & M_T \le \alpha_{\Pi 1} \\ 1-3\zeta_1^2+2\zeta_1^3, & \alpha_{\Pi 1} < M_T < \beta_{\Pi 1} \\ 0, & M_T \ge \beta_{\Pi 1} \end{cases}$$

$$C_{\Pi 2}(M_T) = \begin{cases} C_{\Pi 2,\max}\left(3\zeta_2^2+2\zeta_2^3\right), & \alpha_{\Pi 2} < M_T \le \beta_{\Pi 2} \\ C_{\Pi 2,\max}\left(1-3\zeta_3^2+2\zeta_3^3\right), & \beta_{\Pi 2} < M_T < \gamma_{\Pi 2} \\ 0, & M_T \le \alpha_{\Pi 2} \cup M_T \ge \gamma_{\Pi 2} \end{cases}$$

$$\zeta_1 = \frac{(M_T-\alpha_{\Pi 1})}{(\beta_{\Pi 1}-\alpha_{\Pi 1})},\ \zeta_2 = \frac{(M_T-\alpha_{\Pi 2})}{(\beta_{\Pi 2}-\alpha_{\Pi 2})},\ \zeta_3 = \frac{(M_T-\beta_{\Pi 2})}{(\gamma_{\Pi 2}-\beta_{\Pi 2})},$$

$$\alpha_{\Pi 1} = 0.1;\quad \beta_{\Pi 1} = 0.27;\quad \beta_{\Pi 2} = 0.315;$$

$$\gamma_{\Pi 2} = 10;\quad C_{\Pi 2,\max} = 0.65$$

By turbulent Mach number in this work is defined in the following way:

$$M_T = \frac{\min\left(0.2857 L_T S; \sqrt{2k}\right)}{a}$$

where, $S = \sqrt{2 S_{ij} S_{ij}}$ ¬ is invariant of the strain tensor $S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$, $L_T = \min\left(g^2 \sqrt{k}; y\right)$ ¬ length scale where y is a distance to the nearest wall.

MODEL VALIDATION

The model was validated in boundary layer and free-stream flow cases. Experimental flow data obtained by Goebel and Dutton (1991) and Fernholz and Finley (1977) was used. The calculation results using k-ε-V_n Model are also compared with the results obtained with Menter's SST Model (Menter, 1994) and Sarkar's Model (Sarkar *et al.*, 1993).

Free stream validation was performed using (Goebel and Dutton, 1991) experimental data. In these experiments mixing of two parallel gas flows was investigated. The flows had different speed and density. Tests were performed on all the regimes given in the research by Goebel and Dutton (1991). Results of modeling one of the regimes are shown in Fig. 1 and 2. This regime has the following parameters: U_1 = 830 msec, U_2/U_1 = 0.16, ρ_2/ρ_1 M_1 = 2.27, M_2 = 0.38, T_1 = 332 K, T_2 = 292 K, p = 32 kPa.

By examining these results, we can infer the following. As one can see increase in relative speed M^r yields to decrease of shear stress (Fig. 1) and considerable decrease of lateral velocity fluctuations (Fig. 2).

This means that compressibility first of all influences on velocity fluctuations normal to streamlines and through $\overline{v'n^2}$ -on shear stress.

At the same time the influence on streamwise fluctuations is negligible. The proposed turbulence model reflects these statements rather well while SST and Sarkar's model do not.

For validation of the model in boundary layer conditions, test data by Fernholz and Finley (1977) was used. In this research, near-wall compressible gas flows were investigated with Mach number equal to 4.544 (regimea), 2.244 (regime b) and 5.29 (regime c). Sarkar's Model was not tested here as it is a High-Reynolds Model.The results of calculations are presented in Fig. 3 and 4.

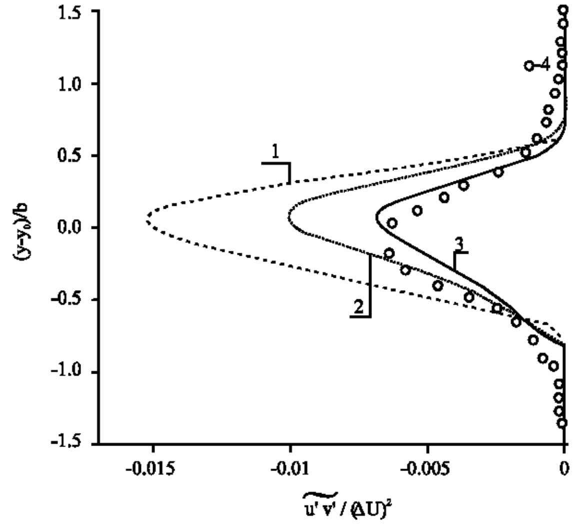

Fig. 1: Shear stress in 2D mixing layer; 1) Calculation results using SST; 2) Sarkar's Model; 3) Present Model and 4) Experimental data by Goebel and Dutton (1991)

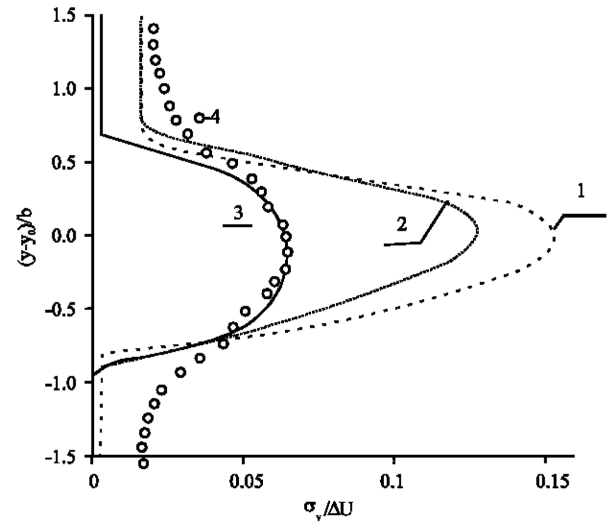

Fig. 2: Lateral stress in 2D mixing layer; 1) Calculation results using SST; 2) Sarkar's Model; 3) Present Model and 4) Experimental data by Goebel and Dutton (1991)

By considering the obtained results, it is possible to conclude that the present model permits to gain more accurate velocity profiles, closer to experimental data.

Results for incompressible flows are not presented in the study. However, one can easily deduce that in low-speed limit the present model will behave similarly to SST. Nevertheless, the issue may need further investigation

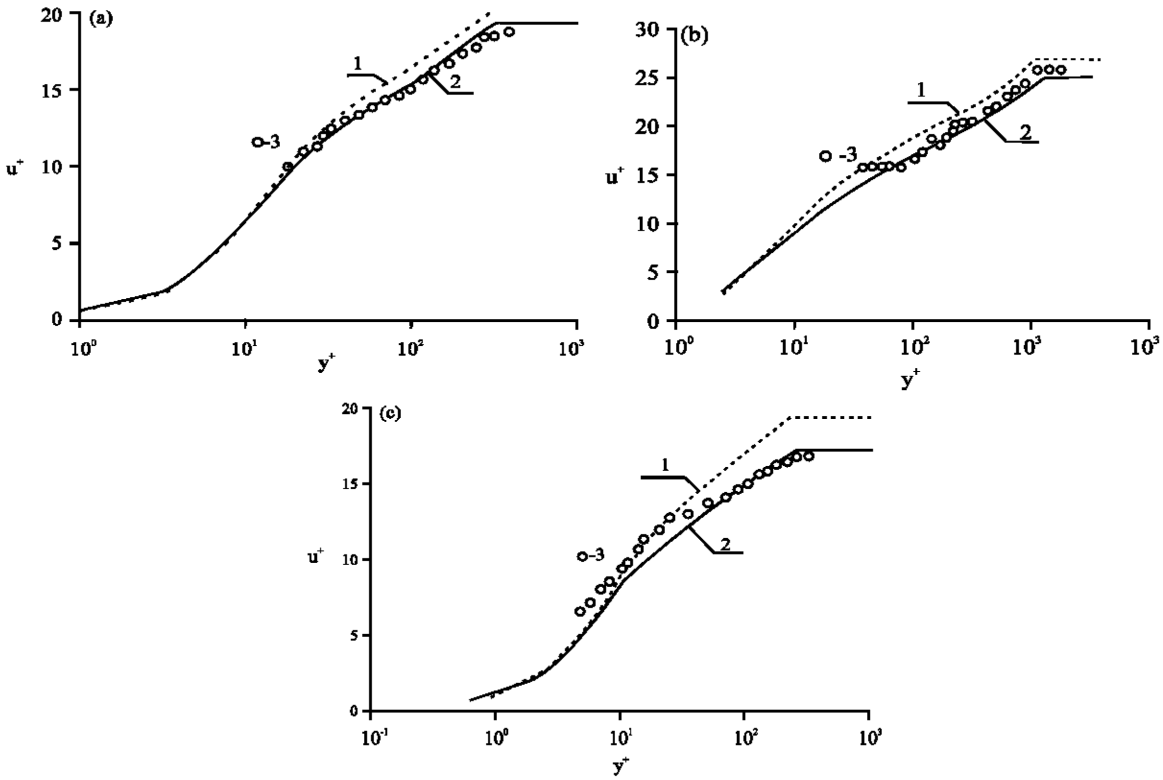

Fig. 3: Dimensionless velocity $U^{+}(y^{+})$ profile; 1) Results using SST; 2) Present model and 3) Experimental data by Fernholz and Finley (1977)

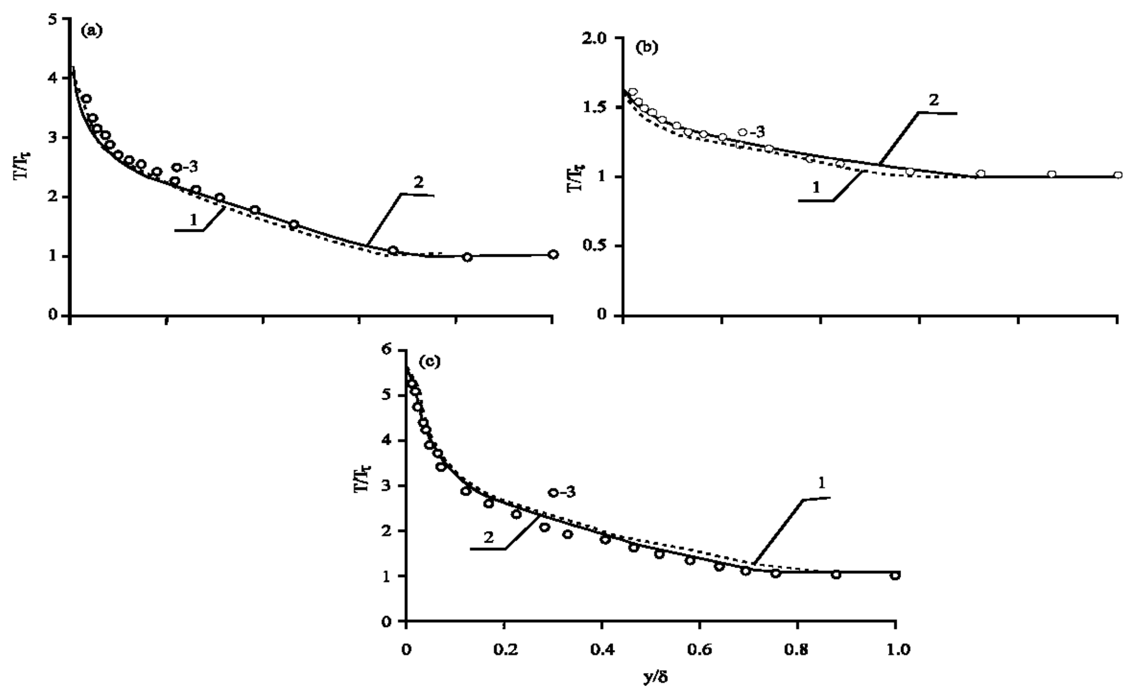

Fig. 4: Dimensionless temperature profile; 1) Results using SST; 2) Present model and 3) Experimental data by Fernholz and Finley (1977)

CONCLUSION

The developed model gives good agreement with the test data for free-stream and boundary layer flows in a wide range of flow parameters is rather numerically stable and easy to handle which makes it useful for general engineering applications. In this model the ideas used in k-ε-V_n, v^2f, SST, k-g are utilized. However, the present model lacks several disadvantages of the latter, like problems with isotropy and boundary condition definition.

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