

Synthesis Method of Sliding-Mode Observers for Identification of Values of Faults in Nonlinear Objects

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Abstract: The study presents a method for synthesizing a system for identification of faults that appear in various elements of complex mechatronic systems described by nonlinear differential equations. This method is based on the use of diagnostic observers and the logical-dynamic approach which makes it possible to apply linear methods in the diagnosis of faults in nonlinear systems as well as introducing special feedback by residual signal to the observers which provides an accurate estimation of the values of the emerging faults in real time. On the example of identification of faults arising in electric drives of a multi-link manipulator, the operability and high efficiency of the proposed method are shown.

Key words: Fault identification, diagnostic, nonlinear systems, residual, described, mechatronic

INTRODUCTION

Timely detection of faults that occur in elements and subsystems in the process of operation is one possibility to improve the efficiency of the functioning of technical systems. Operative detection, localization and identification of these faults are conditions for the implementing fault-tolerant control that allow the stabilization of the most important characteristics of the system in the presence of faults.

Methods of technical diagnostics which include the steps of forming the residual and decision-making are commonly used for the detection of faults. The residual is the result of a mismatch between the behavior of a real diagnosed system and the behavior of its mathematical model (diagnostic observer).

There are several methods for constructing diagnostic observers for nonlinear dynamical systems, including those based on the algebra of functions, the differential geometric approach (Shumsky and Zhirabok, 2006), logic-dynamic approach (Zhirabok and Usoltsev, 2001; Filaretov *et al.*, 2012, 2013) and others (Isermann, 2006; Frank, 1990; Staroswiecki *at al.*, 2006; Siqueira and Terra, 2004). The first two allow obtaining the optimal solution with a minimum dimension of observer, however, they offer a rather complicated process of finding a solution because of complex analytical calculations. The logic-dynamic approach does not guarantee a minimum dimension but the process of finding a solution on its

basis is quite simple with the use of linear methods. The problem of identifying the values of the faults using logic-dynamic approach has been solved by Filaretov *et al.* (2012, 2013) by introducing the feedback by the residual into the diagnostic observers. However, the existing approach is only valid when diagnostic observers are of the first order and the state vector of the diagnosing object is fully measurable.

In this study, a special feedback by residual signal is introduced in the diagnostic observers created with the logic-dynamic approach. It ensures the operation of observer in sliding mode and solves the problem of fault value identification. It provides an accurate definition of the values of faults occurring in multidimensional nonlinear systems, even in the case of the variability of fault values and partial measurability of the state vector of the object.

MATERIALS AND METHODS

Model of diagnosed object: In general, Diagnosed Object (DO) can be described by a system of non-linear differential equations which can be presented in the matrix form:

$$\dot{x}(t) = Fx(t) + B(x(t), u(t)) + Gu(t) + Ld(t), y(t) = Hx(t) \quad (1)$$

where, $x \in \mathbb{R}^n$ is the state vector, n is the dimensionality of DO, $y \in \mathbb{R}^m$ is the output vector, $u \in \mathbb{R}^1$ is the input

vector, p-dimensional vector $d(t)$ which describes the errors occurring in the system due to occurrence of faults in it (in the presence of a fault the corresponding element of the vector becomes an unknown function of time); $F \in \mathbb{R}^{n \times n}$ is the matrix of system dynamics; $G \in \mathbb{R}^{n \times l}$ is the matrix that determines the effect of the input to the system; $H \in \mathbb{R}^{m \times n}$ is the matrix that determines the relationship of output with the state vector; $H \in \mathbb{R}^{m \times n}$ is a vector which determines a non-linear part of the system. In this study, a case where each of elements of vector $d(t)$ included only in one of Eq. 1 is considered which means that there is only one nonzero element in each of the columns of matrix $L \in \mathbb{R}^{n \times p}$.

The problem of fault detection and identification of their values are invited to decide on the basis of the Logic-Dynamic Approach (LDA) which allows the construction of diagnostic observers using only linear methods. The proposed method provides a bank of nonlinear diagnostic observers, each of which is sensitive to one of the faults and insensitive to the others. Each of observers can be described by system of Eq. 2:

$$\begin{aligned} \dot{x}^*(t) &= F^*x^*(t) + B^*(x^*(t), u(t)) + G^*u(t) + Jy(t) \\ y^*(t) &= H^*x^*(t) \end{aligned} \quad (2)$$

where, $x^* \in \mathbb{R}^k$ is the state vector of observer, k is the dimensionality of observer; y^* is the output signal of observer; $F^* \in \mathbb{R}^{k \times k}$, $G^* \in \mathbb{R}^{k \times l}$, $H^* \in \mathbb{R}^{l \times k}$, $B^*(x^*(t), u(t))$ are the matrixes corresponding to the matrices of system Eq. 1, $J \in \mathbb{R}^{l \times m}$ is the matrix which defines the use of the DO output in observer. All these matrices must be determined during the synthesis of the observer. During synthesis of observer matrices F^* and H^* can be chosen as:

$$F^* = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, H^* = [1 \quad 0 \quad 0 \quad \dots \quad 0] \quad (3)$$

The state vectors of the DO and observer are associated by the matrix $\Phi \in \mathbb{R}^{k \times n}$ and the following equations are satisfied in the absence of a mismatch between the DO and the observer:

$$x^* = \Phi x, G^* = \Phi G, CH = H^* \Phi, \Phi F = F^* \Phi + JH \quad (4)$$

On the basis of the outputs of the DO and observer the following residual is generated:

$$r(t) = Cy(t) - y^*(t) \quad (5)$$

where, C is the m -dimensional vector, defining the use of the DO output in residual generation. In case of any fault, the residual Eq. 5 becomes different from zero and Eq. 4 is no longer fulfilled. To solve the problem of identification of faults, consider how the residual will change in the event of faults. After differentiating the expression Eq. 5 taking into account 1 and 2 it can be obtained:

$$\begin{aligned} \dot{r} &= CH(Fx + Gu + B(x, u) + Ld) - \\ &H^*(F^*x^* + G^*u + B^*(x^*, u) + Jy) \end{aligned}$$

$$\begin{aligned} \dot{r} &= CHFx + CHGu + CHB(x, u) + CHLd - H^*F^*x^* - \\ &H^*G^*u - H^*B^*(x^*, u) - H^*Jy \end{aligned}$$

In view of Eq. 4 it can be obtained:

$$\begin{aligned} \dot{r} &= H^*\Phi Fx - H^*F^*x^* - H^*JHx + H^*\Phi B(x, u) - \\ &H^*B^*(x^*, u) + H^*\Phi Gu - H^*G^*u + H^*\Phi Ld \end{aligned}$$

$$\dot{r} = H^*(F^*(\Phi x - x^*) + \Phi B(x, u) - B^*(x^*, u) + \Phi Ld)$$

Denote the value of $\Phi x - x^*$ as a vector $e \in \mathbb{R}^k$ of a mismatch of states of the DO and the observer:

$$e = \Phi x - x^* \quad (6)$$

Considering introduced vector, it can be obtained:

$$\dot{r} = H^*(F^*e + \Phi B(x, u) - B^*(x^*, u) + \Phi Ld) \quad (7)$$

Consider the relationship of the vector of mismatch e with the residual r . In view of Eq. 5 it can be obtained:

$$r = Cy - y^* = CHx - H^*x^* = H^*\Phi x - H^*x^* = H^*e \quad (8)$$

Thus, taking into account Eq. 3, the residual r is the first element of mismatch vector e :

$$r = e_1$$

Then, taking into account Eq. 6 and 7 it can be obtained:

$$\dot{e} = F^*e + \Phi B(x(t), u(t)) - B^*(x^*(t), u(t)) + \Phi Ld \quad (9)$$

As can be seen from Eq. 9 the derivative of the mismatch vector depends on the element $\Phi B(x(t), u(t)) - B^*(x^*(t), u(t))$ which characterizes the non-linearity of the DO and the observer. In the case of incomplete

measurability of the DO, the state vector x and the value of the element $\Phi B(x(t), u(t))$ is unknown. It makes it difficult to identify the value of faults. In LDA the observer is constructed in way to compensate the effect of non-linearity in the residual signal, so that, $\Phi B - B^* = 0$. However, this condition is fulfilled only when there is no mismatch between the state vectors of the DO and the observer and $e = 0$. To determine the value of the specified fault, d_i , feedback by residual signal which is formed in such way to eliminate mismatch even in the case of faults should be introduced.

RESULTS AND DISCUSSION

Introduction of feedback: After the introduction of feedback by residual signal in the observer this model assumes the form:

$$\begin{aligned} \dot{x}^*(t) &= F^*x^*(t) + B^*(x^*(t), u(t)) + G^*u(t) + \\ &w(r) + Jy(t) \quad y^*(t) = H^*x^*(t) \end{aligned} \quad (10)$$

where, $w(r) \in R^k$ is a vector specifying feedback by the residual signal. Taking into account the introduced feedback, change of the mismatch vector will be determined by the Eq. 11:

$$\dot{e} = F^*e + \Phi B(x, u) - B^*(x^*, u) + \Phi Ld - w(r) \quad (11)$$

If formed feedback ensures that the equalities 4 and $B^*(x(t), u(t)) = \Phi B(x^*(t), u(t))$ are performed, nonlinearity in the process of identification of faults will be compensated. To provide the sensitivity of the observer to the fault d_i and the invariance to the other faults, matrix Φ is constructed in such way that all the columns of matrix ΦL except i th were zero, so, the observer will be affected only by one element of fault vector d . Next, consider a common case, when the Eq. 11 can be represented as:

$$\begin{aligned} \dot{e}_1 &= e_2 - w_1(e_1) \\ \dot{e}_2 &= e_3 - w_2(e_1) \\ &\dots \\ \dot{e}_v &= e_{v+1} + f_v d_i + a_v - w_v(e_1) \\ &\dots \\ \dot{e}_k &= -w_k(e_1) \end{aligned} \quad (12)$$

where, $a = \Phi B(x(t), u(t)) - B^*(x^*(t), u(t))$, f_v is the element of the i th column of the matrix ΦL . In this case, the fault and uncompensated nonlinear component a_v are present in only one of Eq. 12. In the case of $w_{v+1}(e_1) = 0, \dots,$

$w_k(e_1) = 0$ and the consistency of the initial conditions of the DO and the observer, e_{v+1}, \dots, e_k are zero regardless of the occurrence of the fault. Then further, it is possible to consider only a part of the system of Eq. 12:

$$\begin{aligned} \dot{e}_1 &= e_2 - w_1(e_1) \\ \dot{e}_2 &= e_3 - w_2(e_1) \\ &\dots \\ \dot{e}_v &= f_v d_i + a_v - w_v(e_1) \end{aligned} \quad (13)$$

Let us to consider the next steps of synthesis of the fault identification system for the case of the second order observer. Equation 13 in this case will be:

$$\begin{aligned} \dot{e}_1 &= e_2 - w_1(e_1), \\ \dot{e}_2 &= f_2 d_i - w_2(e_1) \end{aligned} \quad (14)$$

Let the feedback elements be $w_1 = T_1 e_1$ and $w_2 = T_2 e_1 + z$ where, T_1 and T_2 are coefficients to ensure stability of the system and z is the control signal providing the observer operation in the sliding mode. Using the sliding mode observers provides the elimination of mismatch between the DO and the observer and the fault identification even in case of a variable value of fault. To achieve stability and the required performance in the synthesis of the second order observer it is advisable to select the feedback coefficients as follows:

$$T_1 = \frac{10}{T_c}, T_2 = \frac{25}{T_c^2}$$

where T_c is the time during which the output of the system enters the 5% area of the value of the desired fault. Accepting $T_c = 0.1$ sec, we get $T_1 = 100, T_2 = 2500$. Next, consider the form of control law z . After a change of variables in the system of Eq. 13, $x = e_1, y = \dot{e}_1$, it will be:

$$\dot{x} = y, \quad \dot{y} = -T_2 x - T_1 y + f_2 d_i + a_v - z \quad (15)$$

Phase portrait of this system is shown on Fig. 1. For given feedback coefficients this system is stable and $e_1 \rightarrow d_i + a_v - z$ when $t > T_c$. The phase trajectories of system Eq. 15 are parts of parabolas, the tangent to them is determined by the Eigenvector v of the matrix of the system dynamics \tilde{F} :

$$\tilde{F} = \begin{bmatrix} 0 & 1 \\ -T_2 & -T_1 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -50 \end{bmatrix}$$

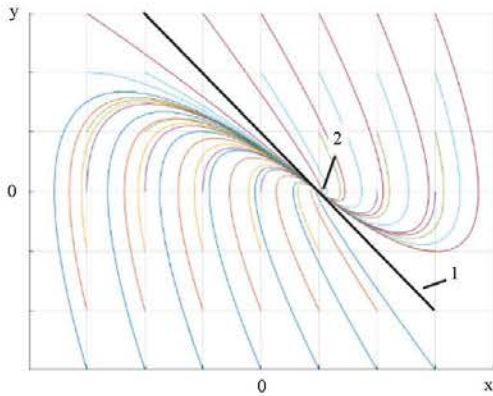


Fig. 1: Phase portrait of the system Eq. 15. 1 is tangent line, 2 is the point $x = d_1 + a_v - z$

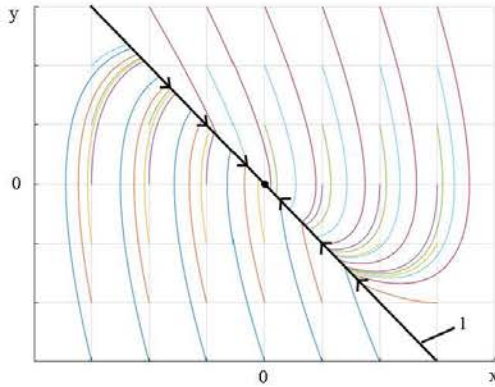


Fig. 2: Phase portrait of the system with sliding mode. 1 is the switching line

The line, described as $p = [x, y] v = 0$ can be adopted as the switching line, then control law z is:

$$z = -z_0, \text{ if } p < 0$$

$$z = z_0, \text{ if } p > 0$$

where, $z_0 > |d_1 + a|$. In this case, any phase trajectory of the system will reach the switching line in a finite time period and will continue to move along the switching line until achievement of point $e_1 = x = y = 0$ (Fig. 2). Thus, after the introduction of this kind of feedback, residual e_1 will tend to zero after the completion of the transition process, including in the event of faults. This will ensure the synchronization of the state vectors of the DO and the observer. From Eq. 14 it is follows that value of fault can be found from averaging of signal z : $d_1 = -z/f_2$. Block diagram of the synthesized system of the fault identification is shown in Fig. 3.

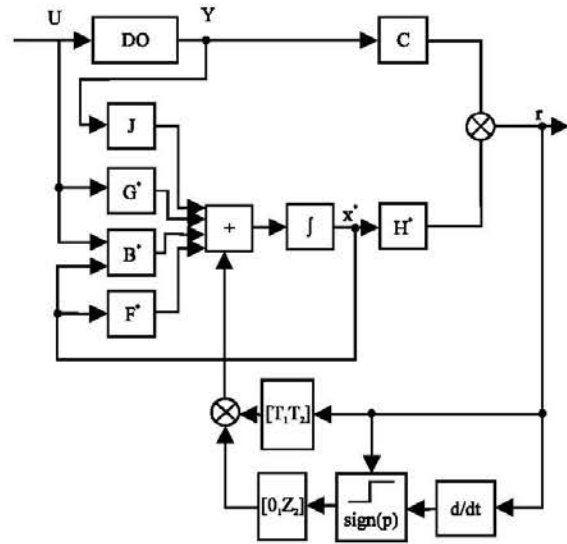


Fig. 3: Block diagram of the fault identification system

Example: Let us consider the Multilink Manipulator (MM) with n degrees of freedom which is driven by n actuators with the DC motors. Each actuator has sensors of the position and current. It is assumed that the following types of defects are possible in the actuators of MM: d_1 , fault caused by an increase of moment of dry friction M_{dfo} in the actuators; d_2 , fault caused by the change of active resistance of the rotor circuit of the DC motors (for example, when there is a significant change of temperature). The presence of these defects significantly reduces the quality of the actuators and accuracy of performance of specific technological operations. In the presence of these faults, each actuator of the MM without feedback can be described by the differential equation in form Eq. 1 where:

$$F = \begin{bmatrix} 0 & 1/i & 0 \\ 0 & -(k_v + h)/(J+H) & k_m/(J+H) \\ 0 & -k_p/L & -R/L \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -M_E/(J+H) - M_{dfo} \text{sign}(x_2)/(J+H) \\ 0 \end{bmatrix}$$

$$d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} G = \begin{bmatrix} 0 \\ 0 \\ k_a/L \end{bmatrix}$$

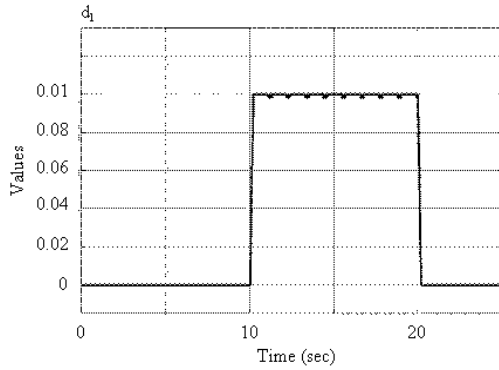


Fig. 4: Output of observer at constant fault value

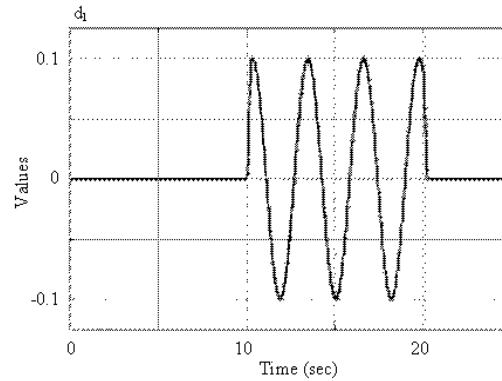


Fig. 5: Output of observer at variable fault value

Where:

- R, L, x_3 = Resistance, inductance and current of the DC motor rotor circuits, accordingly
- k_w = A counter-EMF coefficient
- k_a = An amplifier coefficient
- k_m = A moment coefficient
- J = A moment of inertia of the DC motor shaft and rotating parts of the reducers
- x_1 = A turn angle of an electric motor shaft
- x_2 = A rotor speed
- I = A reducing ratio of the reducer
- h, H, M_E = The variable components of torque influences on actuators including the interactions between all the degrees of freedom of the manipulator
- u = A input voltage of amplifiers

In this case, the dimension of the output vector of the system is less than dimension of state vector. Thus, only the value of x_1 and x_3 (the rotation angle of the output shaft and the current) are measured and x_2 (angular speed) is not. Next, consider the synthesis of the identification system for fault d_1 . The diagnostic observer of the second order which is sensitive to this fault and insensitive to fault d_2 will be described by system of Eq. 10 and the following matrix:

$$F^* = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, J = \begin{bmatrix} \frac{h+k_v}{H+J} & 0 \\ 0 & -\frac{k_m}{i(H+J)} \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 1 & 0 & 0 \\ \frac{h+k_v}{H+J} & \frac{1}{i} & 0 \end{bmatrix}, C = [1 \quad 0], G = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$B^* = \begin{bmatrix} 0 \\ \frac{M_E + M_{dfo} \text{sign}(x_2)}{i(H+J)} \end{bmatrix}$$

Since, the value of x_2 from the vector B^* is not measured by sensors, it must be defined from Eq. 4:

$$x_2 = i(x_2^* + \frac{h+k_v}{H+J} x_1)$$

To verify the functionality and effectiveness of the proposed method, the modeling of the synthesized faults accommodation system for the model of the actuator of the second degree of freedom (q_2) of PUMA-type manipulator was carried out. Faults were simulated by introducing error signals $d_1 = 0.1$ (Fig. 4) and $d_1 = 0.1 \sin(2t)$ (Fig. 5) from time $t_1=10$ to $t_3=20$ sec. As can be seen from the figures the proposed system accurately determines the presence or absence of a fault d_1 and its value.

CONCLUSION

In this study, we considered a new method of detection, localization and identification of values of faults of the nonlinear dynamic systems. This method consists of the application of a logic-dynamic approach for synthesis of diagnostic observers, guaranteeing the independence of the detection and localization of possible faults and the introduction of special feedback for diagnostic observers, providing observers operation in sliding mode and identifying values of faults. The advantage of this method is the simplicity of implementation and accuracy of the identification of faults, even in case of their variability. Efficiency of the proposed method of synthesis of diagnostic observers for identification of faults in nonlinear dynamic systems was confirmed by the results of mathematical modeling.

RECOMMENDATION

The subject of further research will be the application of the proposed method for synthesis of the fault accommodation and failsafe control systems for such objects as industrial manipulators and underwater vehicles.

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