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Application of Mathematical Models for Calculating the Stability of Complex Systems

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Abstract: Power plants of autonomous facilities usually generate power by means of multiple paralleled Synchronous Generators (SyG). When applying or cutting the load, reactive and active power is transferred between paralleled generators because the static and dynamic properties of generator units are not identical. This problem is usually solved by PID controllers. However, they have to be re-configured in operation due to alterations in the parameters of diesel engines, generators and ACS components that form the Generator Unit (GU). Such altered parameters of GU components are referred to as destabilizing factors as they have a negative impact on the quality of power and the stability of the system.

Key words: Autonomous system, generators, excitation system, analog circuit, electromechanical, stability

INTRODUCTION

To illustrate an autonomous system, we analyze a Ship Power Plant (SPP) that consists of the main power source and the Main Power Plant (MPP). The main power source consists of diesel generators DG1, DG2, ..., made up of synchronous generators SyG1, SyG2, ..., powered by diesel engines DE1, DE2, ..., (Fig. 1).

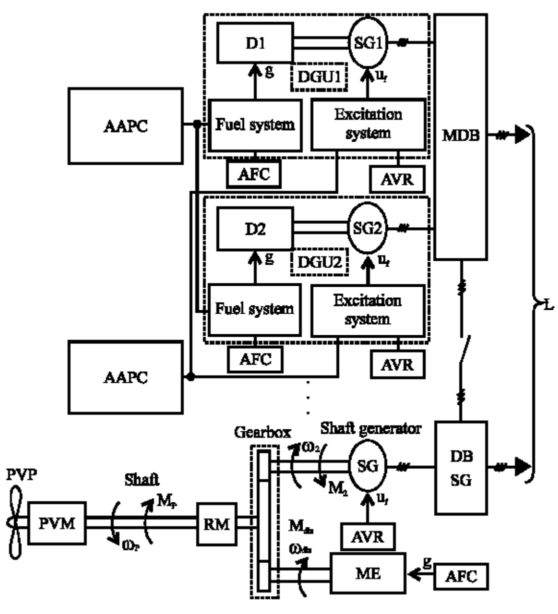

Fig. 1: Electromechanical layout with withdrawal of power from the MPP

RESEARCH OBJECT

The Main Power Plant (MPP) consists of the Main Engine (ME) that propels the vessel by means of a Controllable Pitch Propeller (CPP). The shaft line is coupled with the ME via. the Elastic Coupling (EC) and a Reduction Drive (RD).

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The motion speed of the vessel is adjusted by turning the CPP blades by means of the Pitch Control Mechanism (PCM). The Reduction Drive (RD) from which the shaft line exits is connected to the Shaft Generator (SG) that is also, a power withdrawal generator (Nyrkov *et al*., 2017).

All the generators can be parallel-connected to the common buses of the Main Switchboard (MS). Receivers (R) receive power from the MS and the SGS. The SPP is power-limited. Excitation voltage u_f is applied to all generators; All diesel engines get fuel with a consumption value g. The master diesel generator in the group of paralleled generators employs Automatic Voltage Control (AVC) and Automatic Frequency Control (AFC) systems. The slave diesel generators in the group of paralleled generators have their active and reactive power controlled by means of controllers (Automatic Active Power Controllers and Automatic Reactive Power Controllers or

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AAPC and ARPC) that control the diesel engine by means of the fueling system whereas the generator is controlled by means of the excitation system (Rutkowska *et al*., 2004).

When n generators operate in parallel, the power portion of each is described by differential equations of the 5П order due to the reciprocal effects of such generators. Such high order of SyG-specific differential equations prevents any synthesis of controllers and makes them excessively difficult to configure in operation (Malioutov *et al*., 2016).

This study is to mathematically describe an electromechanical systems while factoring in the destabilizing factors it is exposed to and the reciprocal effects its elements can have on each other.

MATHEMATICAL MODELING

Consider the analog circuit Fig. 2 of a generator (index 1) paralleled with other generators (index 2). In the Fig.2: E_{sd1} and E_{sq1} are the longitudinal- and transver-axis components of the generator G1 Electromotive Force (EMF); i_{s1d}, i_{s1q} are the components of the G1 current vector on the axes d and q of the generator; i_{Hd}, i_{Hq} are the components of the load current vector i_H on the axes d and q of the generator G1. Generators paralleled with G1 feed the load with the current i_{s2}. Project the current i_{s2} onto the axes d and q of the generator G1. We therefore, obtain the corresponding values of infeed currents i_{s2d}, i_{s2q} on the axes d and q of the generator G1.

To factor in the infeed currents introduce the coefficients K_d, K_q (Eq. 1) that factor in the ratio of the currents i_{s1}, i_H and the phase shift between them:

$$K_d = \frac{i_{HD}}{i_{s1d}} = \frac{\left|\vec{i}_H\right| \cdot \sin(\gamma)}{\left|\vec{i}_{s1}\right| \cdot \sin(\psi)}, K_q = \frac{i_{Hq}}{i_{s1q}} = \frac{\left|\vec{i}_H\right| \cdot \cos(\gamma)}{\left|\vec{i}_{s1}\right| \cdot \cos(\psi)} \quad (1)$$

Where:

- γ = The angle between the full load current vector and the axis q of the generator
- ψ = The angle between the full generator current vector and the axis q of the generator. For most operating conditions, those angles are within -90 to +90° (Chaudhuri and Stenger, 2005)

In case of such equivalence, the operation of each of the paralleled generators can be seen as autonomous operation with a load with variable resistance $X_H K_q$, $X_H K_d$, $R_H K_q$, $R_H K_d$, Fig. 3.

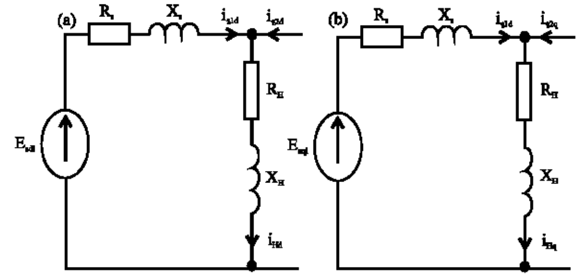

Fig. 2: a, b) Analog circuits

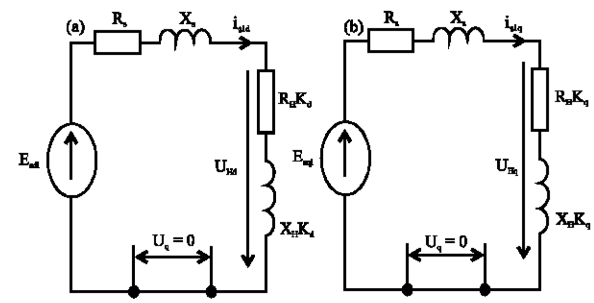

Fig. 3: a, b) Diagrams for calculating the load voltage

The active and the reactive components of drop of the load voltage X_H, R_H on the axes d, q with the formerly added coefficients Eq. 1 K_d, K_q factored in are equal:

$$\begin{aligned} X_H.i_{Hq} &= X_H.\left(K_q.i_{i1q}\right) = \left(X_H.K_q\right).i_{\Gamma i1q} \\ X_H.i_{Hd} &= X_H.\left(K_d.i_{i1d}\right) = \left(X_H.K_d\right).i_{\Gamma i1d} \\ X_H.i_{Hq} &= R_H.\left(K_q.i_{i1q}\right) = \left(R_H.K_q\right).i_{\Gamma i1q} \\ R_H.i_{Hd} &= R_H.\left(K_d.i_{i1d}\right) = \left(R_H.K_d\right).i_{\Gamma i1d} \end{aligned} \quad (2)$$

The ship power network is a limited-power network rendering inappropriate the classical use of complete Park-Gorev equations to calculate the voltage at SyG clamps:

$$\begin{cases} p\psi_d = (1+s)\psi_q - R_{Hd} i_d,\ p\psi_q = -(1+s)\psi_d - R_{Hq} i_q \\ p\psi_f = u_f - K_R R_f i_f,\ p\psi_{yd} = -K_R R_{yd} i_{yd}, \\ p\psi_{yq} = -K_R R_{yq} i_{yq} \end{cases} \quad (3)$$

$$X_{dH} = X_{ad} K_\mu K_\omega + X_S + X_H K_d \quad (4)$$

$$X_{qH} = X_{aq} K_\mu K_\omega + X_S + X_H K_q \quad (5)$$

$$X_f = X_{sf} + X_{ad} K_\mu K_\omega \quad (6)$$

$$X_{yd} = X_{syd} + X_{ad} K_\mu K_\omega \quad (7)$$

$$X_{yq} = X_{syq} + X_{aq} K_\mu K_\omega \quad (8)$$

$$R_{Hd} = R_s K_R + R_H K_d \quad (9)$$

$$R_{Hq} = R_s K_R + R_H K_q \quad (10)$$

$$R_{yq} = 0{,}75 R_{yd} \quad (11)$$

Linkages Ψ_d, Ψ_f, Ψ_{dy}, Ψ_q Ψ_{yq} can be determined using the expressions:

$$\begin{cases} \psi_d = (X_s + X_H \cdot K_d) i_d + K_\mu K_\omega X_{ad} (i_d + i_f + i_{yd}) \\ \psi_f = X_{sf} i_f + K_\mu K_\omega X_{ad} (i_d + i_f + i_{yd}),\ \psi_{yd} = X_{shd} i_{yd} + \\ K_\mu K_\omega X_{ad} (i_d + i_f + i_{yd}),\ \psi_q = (X_s + X_H \cdot K_q) i_q + \\ K_\mu K_\omega X_{aq} (i_q + i_{yq}),\ \psi_{yq} = X_{syq} i_q + K_\mu K_\omega X_{aq} (i_q + i_{yq}) \end{cases} \quad (12)$$

In the existing power plant automation systems, there are no tools to measure the infeed currents i_{s2d}, i_{s2q} (Fig. 1). This is why it is not technically possible to calculate K_d, K_q. Sliding-Mode Controllers (SMC) can be used to automatically control excitation. The specific feature of SMC is that one does not have to know the values K_d, K_q at any moment of time to synthesize such a controller. It is sufficient to know their limit values provided that on top of a rapidly changing SMC control signal, the coefficients K_d, K_q are quasi-steady.

By transforming Eq. 3-13, we obtain a transfer function between the excitation current and the excitation winding voltage:

$$\frac{i_f(p)}{u_f(p)} = W_f(p) = \frac{p^4 b_0 + p^3 b_1 + p^2 b_2 + b_4}{p^5 + p^4 a_1 + p^3 a_2 + p^2 a_3 + p a_4 + a_5} \quad (13)$$

Where:

$$b_0 = \frac{\frac{X_{yd} X_{dH}}{A} - A}{\frac{X_{yd} X_{dH}}{A} - A X_{dH} + 2 (K_\mu K_\omega)^2 X_{ad}^2 - A X_{yd} - A X_f} \quad (14)$$

$A = K_\mu K_\omega X_{ad}$. The values of the other coefficients in Eq. 13 can be derived similarly. Find the value b_0 at $K_d \to \infty$. This matches the idle-mode operations of a generator with $i_{s1d} \to 0$. Substitute Eq. 4 in Eq. 14:

$$b_0 = \frac{\frac{X_{yd}(A + X_s + X_H K_d)}{A} - A}{\frac{X_{yd}(A + X_s + X_H K_d) X_f}{A} - A(A + X_s + X_H K_d) + 2 (K_\mu K_\omega)^2 X_{ad}^2 - A X_{yd} - A X_f} \quad (15)$$

Divide the numerator and the denominator of the fraction Eq. 15 by K_d and find the limit of the function b_0 a $K_d \to \infty$:

$$\lim_{K_d \to \infty} b_0 = \frac{\frac{X_{yd}\left(\frac{A + X_s}{K_d} + X_H\right)}{K_\mu K_\omega X_{ad}} - \frac{A}{K_d}}{\frac{X_{yd}\left(\frac{A + X_s}{K_d} + X_H\right) X_f}{A} - A\left(\frac{A + X_s}{K_d} + X_H\right) + \frac{2 (K_\mu K_\omega)^2 X_{ad}^2 - A X_{yd} - A X_f}{K1}}$$

As a result:

$$\lim_{K_d \to \infty} b_0 = \frac{x_{yd}}{x_{yd} x_f - (K_\mu K_\omega x_{ad})^2} \quad (16)$$

Expressions Eq. 15 and 16 are to be used to synthesize a sliding-mode controller. When synthesizing a sliding mode controller and modeling the transient processes of generator rotation speed alterations, we shall use the derived values of partial derivatives: those of the generator torque with respect to the rotation speed $(\partial M_s/\partial \omega)_0$ the excitation current $(\partial M_s/\partial \omega i_f)_0$ the load resistance $(\partial M_s/\partial R_H)_0$, $(\partial M_s/\partial X_H)_0$.

To model the speed alteration process under disturbance signals, add the excitation current alteration signal Δ if and the electric load signal ΔX_H, ΔR_H:

$$\Delta\omega = K_{T3} \frac{1 + T_K \cdot p}{1 + T_k \cdot K_K \cdot p} \cdot \frac{1}{Jp + F_\partial} \Delta h - \left(\frac{\partial M_s}{\partial R_H}\right)_0 \cdot \frac{1}{Jp + F_\partial} \Delta R_H - \left(\frac{\partial M_s}{\partial X_H}\right)_0 \cdot \frac{1}{Jp + F_\partial} \Delta X_H - \left(\frac{\partial M_s}{\partial i_f}\right)_0 \cdot \frac{1}{Jp + F_\partial} \Delta i_f$$

where, the shaft generator is paralleled with the power plant generators, the ME ACS equation shall be written as follows:

$$\Delta\omega = K_{T3}\cdot\frac{1+T_k.P}{1+T_k.K_k.P}\cdot\frac{1}{Jp+F_\partial}.\Delta h-$$
$$\left(\frac{\partial M_s}{\partial R_H}\right)_0\cdot\frac{1}{Jp+F_\partial}\Delta R_h-$$
$$\left(\frac{\partial M_s}{\partial R_H}\right)_0\cdot\frac{1}{Jp+F_\partial}\Delta X_h-\left(\frac{\partial M_s}{\partial i_f}\right)_0\cdot\frac{1}{Jp+F\partial}\Delta i_f-$$
$$a_1\cdot\frac{1}{Jp+F_\partial}\frac{\Delta H}{H_0}+a_4\cdot\frac{1}{Jp+F_\partial}f(t)-$$
$$(\Delta M_{sk}+\Delta Q)\frac{1}{Jp+F_\partial}$$

Where:

$$a_1=\frac{h_0.k_{T3}}{D\cdot 6.28\cdot n_{diz}};a_4=\frac{M_{eo}}{6.25\cdot n},\ \Delta M_{vint}=a_1\cdot\frac{\Delta H}{H_0}-a_4.f(t)$$

h_0, M_{eo} are the initial fuel pump rack position and the diesel engine torque conditions; n_{diz} = 428 rpm; J = 610, …, 678 kg. m^2 is the total moment of intertia of the ME, the flywheel, the shaft generator, the propeller shaft and the propeller; $\Delta H/H_0$ is the relative propeller pitch alteration; H_0 is the initial propeller pitch condition; f(t) is the oscillation of the propeller moment of resistance (Zhilenkov and Chernyi, 2015). Therefore, we equalize the linkage derivatives in Eq. 3 to zero and obtain:

$$\begin{cases}0=(1+s)\psi_q-R_{hd}{}^{i}{}_{d},\ 0=-(1+s)\psi_d-R_{Hq}{}^{i}{}_{q},0=u_f\\ -K_RR_fi_f,\ 0=-k_RR_{yd}i_{yd},0=-k_RR_{yq}i_{yq}\end{cases}\quad(17)$$

From the fourth and the fifth equations of the system Eq. 17, it follows: $i_{yd}=0$, $i_{yq}=0$. From the first and the fourth equations in Eq. 12 while factoring in $i_{yd}=0$, $i_{yq}=0$ and the Eq. 17, we obtain:

$$\{\psi_d=X_{dh}i_d+X_{ad}K_\omega K_ui_f,\psi_q=X_{qH}{}^{i}q\quad(18)$$

Generator torque equation:

$$M_2=i_d\psi_q-i_q\psi_d\quad(19)$$

Make a substitution in Eq. 17:

$$1+S=\frac{\omega}{\omega_c}\quad(20)$$

Where:
- ω = The rotor speed (rad/sec)
- ω_c = The stator field rotation speed (rad/sec)

From Eq. 17-20, we obtain a system:

$$\begin{cases}0=\left(\frac{\omega}{\omega_c}\right)\psi_q-R_{Hd}i_d,\ 0=-\left(\frac{\omega}{\omega_c}\right)\psi_d-R_{Hq}i_q\\ \psi_d=X_{dH}i_d+X_{ad}K_\omega K_\mu i_f,\ \psi_q=X_{qH}i_q\\ M_s=i_d\psi_q-i_q\psi_d\end{cases}\quad(21)$$

Equation 21 derived generator torque (relative units) equation is written as:

$$M_s=\left(i_fX_{ad}K_\mu K_\omega\right)^2\times R_{Hd}\frac{X_{qH}^2\left(\frac{\omega}{\omega_c}\right)^3+R_{SHdq}\left(\frac{\omega}{\omega_c}\right)}{\left[X_{qH}X_{dH}\left(\frac{\omega}{\omega_c}\right)^2+R_{SHdq}\right]^2}\quad(22)$$

In physical units, the equation is:

$$M_s=\left(i_fX_{ad}K_\mu K_\omega\right)^2R_{Hd}\times\frac{X_{qH}^2\left(\frac{\omega}{\omega_c}\right)^3+R_{SHdq}\left(\frac{\omega}{\omega_c}\right)}{\left[X_{qH}X_{dH}\left(\frac{\omega}{\omega_c}\right)^2+R_{SHdq}\right]^2}M_{HOM}\quad(23)$$

where, M_{HOM} is the nominal generator torque (Nm):

$$R_{SHdq}=R_{Hd}R_{Hq}$$

From Eq. 23, find the partial derivatives of the generator moment with respect to the rotation speed, the active and inductive load resistance and the excitation current.

The equation system Eq. 17-20 helps answer the question what is the operation condition of a SyG at $K_d\to\infty$, $K_q\to\infty$ Eq. 1. Find the generator momen M_s for $K_d\to\infty$, $i_{sld}\to 0$ (1). Substitute $i_d=0$ in Eq. 21:

$$\left\{0=\left(\frac{\omega}{\omega_c}\right)\psi_q-R_{Hd}0,\psi_q=X_{qh}i_q,M_s\ \ 0\cdot\Psi_qi_q\Psi_d\right.$$

Transformations result in $M_s=0$. The generator is in an idle mode at $K_d\to\infty$. This is confirmed by the classical electric machine theory. Find the generator moment M_s for $K_q\to\infty$, $i_{sld}\to 0(1)$. Substitute $i_q=0$ in Eq. 21:

$$\{\psi_q = X_{qH}0,\ M_э = i_d\Psi_q - 0\cdot\Psi_d$$

CONCLUSION

Transformations result in $M_э = 0$. This is confirmed by the classical electric machine theory: at $\Psi = +90°$(1) the generator current lags at 90° behind the EMF. The generator only feeds inductive current in the network. At $\Psi = -90°$ (1) the generator current is 90° ahead of the EMF. The generator only feeds in capacitative current in the network and withdraws reactive power there from.

It has been analytically proven that in storms, the shaft generator is desynchronized due to the variability in the main power plant cranking shaft rotation speed.

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