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Some Algebraic Results of Shadowing Property in Dynamical Systems

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Abstract: We concentrate in this study on the continues maps that have the shadowing property. In this study some algebraic and general properties of this concept are proved. The set P of continues maps that have the shadowing property (POTP) is subspace of a space of all continuous map are shown and (P, +) is an Abelian group where + is a binary operation defined on P.

Key words: Pseudo-orbit, POTP, shadowing property, subspace, group, concept

INTRODUCTION

The shadowing property has a major role in the general qualitative theory of dynamical systems. In recent years, it has been extensively developed to become a fundamental concept in dynamical systems that have many deep links with the concepts of stability and chaotic behavior with these systems. The dynamical system is often used to justify and correct the computer simulation of the system (Fakhari and Ghane, 2010).

In the dynamical system, the approximate orbit of the small distance errors is called a pseudo-orbit. Which depends on the study of the shadowing with the presence of true orbits near these orbits (Coppel, 1965).

MATERIALS AND METHODS

Preliminaries: Let $(\mathbb{R}^n, r)$ be a metric space with metric r and $h:\mathbb{R}^n\to\mathbb{R}^n$ be a continues map on $\mathbb{R}^n$, consider the dynamical system on $\mathbb{R}^n$ generated by the iterations of h that is $h^0 = id_z$ and $h^{j+1} = h^j \circ h$ for all $j\in\mathbb{N}_0$. We shall identify the map h with the corresponding dynamical system.

A sequence $\{z_j\}_{j=0}^{\infty} \subset \mathbb{R}^n$ is called a (true) orbit of h if $z_{j+1} = h(z_j)$, for all $j\in\mathbb{N}_0$. For $\rho>0$ a sequence $\{z_j\}_{j=0}^{\infty}\subset\mathbb{R}^n$ satisfying $r(h(z_j), z_{j+1})\leq\rho$ for all $j\in\mathbb{N}_0$ is called ρ-pseudo orbit of h. For $\epsilon>0$, a point $z\in \mathbb{R}^n$ is said ϵ-shadows a ρ-pseudo orbit $\{z_j\}_{j=0}^{\infty}$ if $r(h(z_j), z_{j+1})<\epsilon$ for $j\in\mathbb{N}_0$ (Pilyugin, 1999; Al-Badarneh and Karak, 2015).

Definition 2.1; Pilyugin (1999): The continues map h is said to has the pseudo-orbit tracing property POTP. (shadowing property) on $\mathbb{R}^n$ if for $\epsilon>0$ there exists $\rho>0$ such that for any ρ-pseudo orbit $\{z_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$ there is a point z that ϵ-shadows $\{z_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$. We recall the definition of subspace as:

Definition 2.2 (Stoll, 2013): Let C be a space of continuous maps, and let P be a subset of C. Then p is a subspace if:

- The zero map, $O:\mathbb{R}^n\to\mathbb{R}^n$ is in P
- If g and h are continues maps in P, then g+h is in P
- If h is continues map in P and $\alpha\in\mathbb{R}^n$ is a constant, then αh is in P

We recall the definition of abelian group as:

Definition 2.3 (Grigoryan and Tonev, 2006): A non-empty set of continues maps G is a group on which there is a binary operation $(g, h)\to g+h$ such that:

- If a maps g and h in G then g+h is also in G (closure)
- f+(g+h) = (f+g)+h for all maps f, g, h in G (associativity)
- There is an identity map O in G such that h+O = O+h = h for all map h in G (identity)
- if h map in G, then there is an inverse map h′ in G such that (inverse)

If for all g, h in G, g+h = h+g (commutative), then we say a group G abelian group.

RESULTS AND DISCUSSION

Main theorems: In this study, we prove the main results about continues maps that have shadowing property and display the algebraic properties of these maps.

Proposition 3.1: Let $(\mathbb{R}^n, r)$ be metric space, if $O:\mathbb{R}^n\to\mathbb{R}^n$ defined by O(z) be the zero map then O has the POTP.

Proof: For a $\in>0$ there exists $\rho = 1$ such that for any ρ-pseudo-orbit $\{z_j\}_{j=0}^{\infty}$ of O in $\mathbb{R}^n$ there is a point z = 1/2 that $\in$-shadows $\{z_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$ then $r(O^j(z), z_j)\leq\in$ for $j\in\mathbb{N}_0$. Hence, the zero map has the POTP.

Theorem 3.2: Let $(\mathbb{R}^n, r)$ be metric space, if $g,h:\mathbb{R}^n\to\mathbb{R}^n$ are continues maps in $\mathbb{R}^n$ have the POTP, then $g+h:\mathbb{R}^n\to\mathbb{R}^n$ also has this property.

Proof: Suppose that g has the POTP then for a $\in>0$ there exists $\rho>0$ such that for any ρ-pseudo-orbit $\{y_j\}_{j=0}^{\infty}$ of g in $\mathbb{R}^n$ there is a point y that $\in$-shadows $\{y_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$, then $r(g^j(y), y_j)\leq\in$, $j\in\mathbb{N}_0$. Since, h has the POTP then for a $\in>0$ there exists $\rho>0$ such that for any ρ-pseudo-orbit $\{z_j\}_{j=0}^{\infty}$ of h in $\mathbb{R}^n$ there is a point z that $\in$-shadows $\{z_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$, then $r(h^j(z), z_j)\leq\in$ for $j\in\mathbb{N}_0$.

Now, let $s\geq 1$ and $\in' = 2s\in$. Then for any ρ-pseudo-orbit $\{x_j\}_{j=0}^{\infty}$ of g+h in $\mathbb{R}^n$, there is a point x = y+z that $\in$-shadows $\{x_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$, since, for $j\in\mathbb{N}_0$, $r((g+h)^J(x), x_j) = r((g+h)^J(y+z), (y+z_j)_j)\leq s\ r\ (g^j(y)+h^j(z), y_j+z_j\leq s\ r\ (g(y)^j, y_j+h^j(z), z_j)2s\in = \in'$. Hence, g+h has the POTP.

Theorem 3.3: Let $(\mathbb{R}^n, r)$ be metric space, if $g+h:\mathbb{R}^n\to\mathbb{R}^n$ is continues map in $\mathbb{R}^n$ have the POTP then a maps $g:\mathbb{R}^n\to\mathbb{R}^n$ and $h:\mathbb{R}^n\to\mathbb{R}^n$ also has this property.

Proof: Suppose that g+h has the POTP then for a $\in>0$, $-\leq s$, $s'\leq 1$ and $s'' = s.s'$ there exists $\rho>0$ such that for any ρ-pseudo-orbit $\{x_j = y_j, z_j\}_{j=0}^{\infty}$ of g+h in $\mathbb{R}^n$ there is a point x = y+z that $\in$-shadows $\{x_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$, then $r((g+h)^j(x), x_j)\leq\in$ for $j\in\mathbb{N}_0$. Now:

$$r((g+h)^j (x), x_j) = r((g+h)^j (y+z), (y+z)_j)$$
$$= s\ r(g^j(y),+h^j(z), y_j+z_j) = s\ s' [\ r(g^j(y), y_j)+r(h^j(z), z_j)]$$
$$= s''\ r(g^j(y), y_j)+s''\ r(h^j(z), z_j) \leq\in /s''$$

Then $r(g^j(y), y_j)\leq\in/s''$ and $+s''r(h^j(z), z_j)\leq\in/s''$. Hence, g has the POTP for $\in' = \in/s''$ and $\rho>0$ such that for any ρ-pseudo-orbit $\{y_j\}_{j=0}^{\infty}$ of h in $\mathbb{R}^n$ there is a point y that $\in$-shadows $\{y_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$.

Also h has the POTP for $\in' = \in/s''$ and $\rho>0$ such that for any ρ-pseudo-orbit $\{z_j\}_{j=0}^{\infty}$ of h in $\mathbb{R}^n$ there is a point y that $\in$-shadows $\{z_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$.

Proposition 3.4: Let $(\mathbb{R}^n, r)$ be metric space, $h:\mathbb{R}^n\to\mathbb{R}^n$ defined by $h(z) = \alpha$ is a continuous map then h has the POTP.

Proof: For all $\alpha\in\mathbb{R}^n$ and for a $\in>0$ there exists $\rho = 1/2$ such that for any ρ-pseudo-orbit $\{z_j\}_{j=0}^{\infty} = \{z_1, z_2, z_3, ...]$ of h in $\mathbb{R}^n$ there is a point $z = \alpha$ that $\in$-shadows in $\mathbb{R}^n$, since $r(h^j(z), z_j)\leq 2\rho\leq\in$, for $j\in\mathbb{N}_0$. Hence, the h has the POTP.

Theorem 3.5: Let $(\mathbb{R}^n, r)$ be metric space, if $g, h:\mathbb{R}^n\to\mathbb{R}^n$ are continues maps in $\mathbb{R}^n$ have the POTP then $g, h:\mathbb{R}^n\to\mathbb{R}^n$ also has this property.

Proof: Suppose that g has the POTP then for a $\in>0$ there exists $\rho>0$ such that for any ρ-pseudo-orbit $\{y_j\}_{j=0}^{\infty}$ of g in $\mathbb{R}^n$ there is a point y that $\in$-shadows $\{y_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$, then $r\ (g^j(y), y_j)\leq\in$, $j\in\mathbb{N}_0$.

Since, h has the POTP then for a $\in>0$ there exists $\rho>0$ such that for any ρ-pseudo-orbit $\{z_j\}_{j=0}^{\infty}$ of h in $\mathbb{R}^n$ there is a point z that $\in$-shadows $\{z_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$, then $r(h^j(z), z_j)\leq\in$ for $j\in\mathbb{N}_0$. Now, let $s, s'\geq 1$ and $\in' = ss'\in^2$. Then for any ρ-pseudo-orbit $\{x_j\}_{j=0}^{\infty}$ of g.h in $\mathbb{R}^n$, there is a point x = y.z that $\in$-shadows $\{x_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$, since, for $j\in\mathbb{N}_0$, $r((g.h)^j(x), (x_j) = r((g.h)^j(y.z), (y.z_j)_j)$:

$$s[r((g^j(y).\ r\ (h^j\ (z), y_j.\ z_j)]$$
$$ss'\ [r((g^j(y),y_j).\ r\ (h^j\ (z), z_j)]\leq ss'\ \in^2 = \in'$$

Hence, g.h has the POTP.

Proposition 3.6: Let $(\mathbb{R}^n, r)$ be metric space, for any continues map $h:\mathbb{R}^n\to\mathbb{R}^n$ that has the POTP, the inverse of this map also has this property.

Proof: If $h:\mathbb{R}^n\to\mathbb{R}^n$ any continues map that has the POTP then for a $\in>0$ there exists $\rho>0$ such that for any ρ-pseudo-orbit $\{z_j\}_{j=0}^{\infty} = \{z_1, z_2, z_3, ...]$ of h in $\mathbb{R}^n$ there is a point z that $\in$-shadows $\{z_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$, then $r\ (h^j(z), z_j)\leq\in$, $j\in\mathbb{N}_0$. Now, let the inverse of h is $h' = -h(z)$ and there exists $\rho>0$ such that $\{y_j\}_{j=0}^{\infty} = \{z_1, -z_2, z_3, -z_4, ...]$ ρ-pseudo orbit of h' and there exists y = z that $\in$-shadows $\{y_j\}_{j=0}^{\infty}$ in $\mathbb{R}^n$, since, $r\ (h^j\ (y), y_j)\leq\in$, for $j\in\mathbb{N}_0$, Hence, h w' has the POTP.

Proposition 3.7: Let C a space of continuous maps, the set of all continuous maps that have POTP P are a sub space of C.

Proof: To prove, first by proposition 3.1. The zero map, $O:\mathbb{R}^n\to\mathbb{R}^n$ has the POTP. Second, by Theorem 3.2. for all continues maps $g, h:\mathbb{R}^n\to\mathbb{R}^n$ that have the POTP then g+h has this property. Third, for all continues maps $h:\mathbb{R}^n\to\mathbb{R}^n$ that has the POTP, then αh has this property, for $\alpha\in\mathbb{R}^n$, since, $\alpha h = \alpha.h$ when α is constant map by proposition 3.4. then, α has the POTP, then by Theorem 3.5. αh has the POTP.

Proposition 3.8: Let P the set of all continuous maps that have POTP and + is a binary operation defined on P, then (P, +) is an abelian group.

Proof: To prove, first, for all g, $h:\mathbb{R}^n\rightarrow\mathbb{R}^n$ that have the POTP, then by Theorem 3.2. $g+h:\mathbb{R}^n\rightarrow\mathbb{R}^n$ also has this property. Hence + is closure on P. Second, if f, g, h $g+h:\mathbb{R}^n\rightarrow\mathbb{R}^n$ have the POTP, then by continuity of maps $[f+(g+h)](z) = [f+(g+h)](z)$ for any $z\in\mathbb{R}^n$. Hence + is associativity on P. Third, by proposition 3.1. there is an identity map O in P such that, h+O = O+h = h for all map h in P. Hence + has identity on P. Forth, if $h:\mathbb{R}^n\rightarrow\mathbb{R}^n$ in P, then by proposition 3.6 there is $h':\mathbb{R}^n\rightarrow\mathbb{R}^n$ in P such that $h+h' = h'+h = O$. Hence, + has inverse on P. Finally, for all $h:\mathbb{R}^n\rightarrow\mathbb{R}^n$ in P, then g+h = h+g. Hence, + is commutative on P.

CONCLUSION

Researchers have added many concepts about the shadowing property and studied the relationship between them and the chaotic properties in the dynamical systems, see (Blank, 1989; Gu, 2007; Al-Shara'a and Abdul-Zahra, 2017; Al-Juboury, 2015).

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