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Modelling of Indenter Pressed into Heterogeneous Soil

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Abstract: Damage caused to soils during harvesting has extremely negative environmental consequences which can disrupt the reforestation. This situation threatens the entire ecosystem. In this regard, the goal to improve the environmental compatibility of logging machines has good prospects. At first, this implies considering some methods for calculating the soil deformation induced by the rolling motion of logging machines as the purpose of this study, since when the soil is deformed, its porosity, aeration, water permeability and microbiological activity decrease. This goal is achieved by the following methods: mathematical calculation, comparison, systematization as well as a computational experiment which resulted in a solution to 1000 combinations of random values. Results section provides some calculation methods for calculating the effect that an indenter has on a heterogeneous soil and for making calculations with regard to the linear/square/cube law of soil deformation.

Key words: Soil deformation, mathematical model, Bernstein-Letoshnev equation, logging machines, eco-efficiency, linear

INTRODUCTION

There are two well-known approaches to modelling the effect of movers on the soil. These approaches are based on two different ways to determine the stress-strain relationship between the soil and the mover deforming it.

The first one is based on the empirical and experimental dependencies of stress and soil settlement caused by a mover-indenter. By structure, these dependencies are alike those from the well-known Bernstein-Letoshnev equation (Bernstein, 1913). If the parameters characterizing the settlement curve are known, the Bernstein-Letoshnev equation allows getting results qualitatively and quantitatively consistent with those that can be get in field experiments. The approach is applied, for example by Anisimov *et al*. (2006a, b), Grigoriev *et al*. (2006, 2008, 2009, 2012, 2013a, b), Shapiro *et al*. (2008), Shapiro and Grigoriev (2006). The research problems often lie in the field of modeling the dynamics of a wheeled vehicle as a multi mass system. A detailed analysis of soil behavior at stress goes to the background, since, the basics of this approach imply that only integral indices of soil deformation can be obtained.

The second approach is based on the solutions made for a classical problem from soil mechanics regarding the process of pressing the indenter into a semi-space. It allows carrying out a more detailed analysis of soil behavior at stress and specifically analyzing the effect of a large number of mover's parameters on its interaction

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with the soil. This approach was applied to solve the problems of skidding effectiveness by Nikiforova *et al.* (2014), Khitrov *et al.* (2015, 2016) and Grigoriev (2006). A rigid indenter was taken as a model of a mover in order to model the mover-soil interaction based on the Bernstein-Letoshnev dependence.

Strong soil indentation can have serious environmental consequences (Ke *et al.*, 2016; Berezhnoi *et al.*, 2014). Since, significant damage to forest soils during logging operations disturbs forest regeneration, secondary forest productivity decreases and the hydrological regime of the territory and the structure of forest landscapes change (Sultanov, 2015; Siczek *et al.*, 2015). Under the weight of machines, soil compacts and mineralizes. As a result, its bulk density increases while such parameters as porosity, aeration, water permeability and microbiological activity decrease, thus, some ruts appear worn into the soil (Lai *et al.*, 2016). All these factors negatively affect the formation of future stands, reduce their productivity, contribute to the change in mesorelief and reduce the flotation ability of machines.

Thus, the purpose of this study is to consider some methods intended for calculating soil deformation processes that occur under the weight of logging machines.

MATERIALS AND METHODS

In the theoretical aspect, scientific studies and concepts generalizing the principles and mechanisms of soil transformation under the weight of logging machines were of great importance. We also carried out mathematical analysis and applied a system approach. A computer-aided experiment was carried out during the research.

RESULTS AND DISCUSSION

Soil and subsoil have a similar deformation module, calculated by:

$$E_0 = \frac{\omega b \cdot (q_2 - q_1) \cdot (1-\upsilon^2)}{Z_2 - Z_1} \quad (1)$$

Where:

ω = Coefficient depending on the indenter's shape
b = Indenter's width
Z_2 and Z_1 = Settlings made at stresses q_2 and q_1
υ = Poisson's ratio

Soil and subsoil are often considered as a viscoelastic medium, therefore, the equation of their deformation traditionally comprises of two components, elastic and viscous (Shapiro *et al.*, 2017):

$$E = E_E + E_\eta \quad (2)$$

Where:

E_E = Elastic component of deformation
E_η = Viscous component of deformation

The elastic component is calculated by Shapiro *et al.* (2017):

$$E_E = \frac{q}{E_1} \quad (3)$$

Where:

q = Effective normal stress
E_1 = Elasticity module

The equation of viscous component is as follows (Grigoriev *et al.*, 2013):

$$\frac{dE_\eta}{dt} = \frac{q}{\eta} \quad (4)$$

Where:

t = Contact time
η = Rheological parameter of soil

Equation 3 and 4 are used to build the viscoelastic Maxwell Model which differentially is represented by the following equation (Bazarov *et al.*, 2012):

$$\frac{dE}{dt} = \frac{dE_E}{dt} + \frac{dE_\eta}{dt} = \frac{q}{\eta} + \frac{1}{E} \cdot \frac{dq}{dt} \quad (5)$$

At the constant stress q, Eq. 5 is solved as follows (Bazarov *et al.*, 2012):

$$E = \frac{q}{\eta} t + \frac{q}{E_1} = \frac{q}{E_1} \cdot \left(1 + \frac{t}{\tau}\right) \quad (6)$$

where: $\tau = \eta / E_1$

Time during which the mover contacts with the soil affecting it is calculated by:

$$t = \frac{L}{V} \quad (7)$$

Where:

L = Length of mover's surface contacting the soil
V = Moving speed

If we substitute Eq. 7 in 6, we will get the following equation of time-dependent pattern of soil deformation:

$$E = \frac{q}{E_1} \cdot \left(1 + \frac{L}{\tau V}\right) \quad (8)$$

The stresses in a soil mass are known to decrease with depth, so, the stress distribution is limited by this parameter. Brining the module of general deformation to the matter of elasticity, dependence Eq. 2 then can be written as:

$$\frac{Z_2-Z_1}{H} = \frac{\rho_2-\rho_1}{\rho_0} = \frac{\omega b\cdot(q_2-q_1)\cdot(1-\upsilon^2)}{E_0 H} \quad (9)$$

Where:
H = Strain thickness
ρ_0 = Soil density at zero stress
ρ_1 = Soil density at stress q_1
ρ_2 = Soil density at stress q_2

If we consider the viscoelastic Maxwell model in the same manner as the Eq. 2, we will get the following equation of the medium state:

$$\frac{\rho_2-\rho_1}{\rho_0} = \frac{\omega b\cdot(q_2-q_1)\cdot(1-\upsilon^2)}{E_0 H}\cdot\left(1+\frac{L}{\tau V}\right) \quad (10)$$

The relationship between the stress q the settlement Z is calculated by the Bernstein-Letoshnev equation:

$$q = AZ^n \quad (11)$$

where A, n parameters characterizing soil resistance to deformation, determined by measurement depending on the type of soil, its state and the indenter's parameters. In the case of equal pressings:

$$q = AZ_1^n \quad (12)$$

In the case of cyclic stress:

$$q_N+q_1 = A\cdot(Z_N+Z_1)^n \quad (13)$$

where, N number of cycles. Based on Eq. 12 and 13 should be rewritten as (Anisimov *et al*., 2006):

$$q_N = AZ_N^n\cdot\left[\left(1+\frac{Z_1}{Z_N}\right)^n-\left(\frac{Z_1}{Z_N}\right)\right] \quad (14)$$

In this case, if we assume that Z_N>>Z_1, the dependence establishing the pattern of deformation induced by a cyclic stress will be:

$$q_N = AZ_N^n \; q_N = AZ_N^n \quad (15)$$

Then, equation of settlings made after the Nth stress can be written as:

$$Z_N = Z_1 N^{\frac{1}{n}} \quad (16)$$

The following relationship between the soil settlement and density is now revealed:

$$\frac{\rho_{N+1}-\rho_0}{\rho_0} = \frac{Z_N-Z_1}{H} \quad (17)$$

Based on Eq. 16, let us rewrite the Eq. 17 to:

$$\frac{\rho_{N+1}-\rho_0}{\rho_0} = \frac{Z_1}{H}\cdot\left(1+N^{\frac{1}{n}}\right) \quad (18)$$

The value for settlings can be found by:

$$Z_1 = \frac{\omega b q_1\cdot(1-\upsilon^2)}{E_0 H}\cdot\left(1+\frac{L}{\tau V}\right) \quad (19)$$

Therefore, equation of the state of soil under cyclic stress imposed by an indenter will be:

$$\frac{\rho_{N+1}-\rho_0}{\rho_0} = \frac{\omega b q_1\cdot(1-\upsilon^2)}{E_0 H}\cdot\left(1+\frac{L}{\tau V}\right)\cdot\left(1+N^{\frac{1}{n}}\right) \quad (20)$$

Brining the problem of pressing an indenter to the matter of elasticity, let's rewrite the Eq. 20 as follows (Grigoriev *et al*., 2013; Larin, 2007):

$$\frac{\rho_{N+1}-\rho_0}{\rho_0} = \frac{\omega b q_1\cdot(1-\upsilon^2)}{E_0 H}\cdot\left(1+N^{\frac{1}{n}}\right) \quad (21)$$

Based on the accepted elasticity module and the Bernstein-Letoshnev equation for the soil settlement, we will get:

$$Z_1 = \left(\frac{q_1}{A}\right)^{\frac{1}{n}} \quad (22)$$

and further we can write the next formula for the module of general deformation:

$$E_0 = A^{\frac{1}{n}} H q_1^{1-\frac{1}{n}} \quad (23)$$

If we substitute the Eq. 23 in the equations of the state of elastic Eq. 19 and viscous Eq. 20 soils, we will get the following Eq. 24 and 25:

$$\frac{\rho_{N+1}-\rho_0}{\rho_0}=\frac{\omega b q_1^{\frac{1}{n}}\cdot\left(1-\upsilon^2\right)}{A^{\frac{1}{n}}H^2}\cdot\left(1+\frac{L}{\tau V}\right)\cdot\left(1+N^{\frac{1}{n}}\right) \quad (24)$$

$$\frac{\rho_{N+1}-\rho_0}{\rho_0}=\frac{\omega b q_1^{\frac{1}{n}}\cdot\left(1-\upsilon^2\right)}{A^{\frac{1}{n}}H^2}\cdot\left(1+N^{\frac{1}{n}}\right) \quad (25)$$

At this point, researchers by the Anisimov and Bolshakov (1998) have empirically determined the exponential dependence of soil deformation induced by cyclic stress:

$$\frac{q_N}{q_1}=10^{\alpha\frac{Z_n}{Z_1}} \quad (26)$$

where, coefficient depending on the parameters of both the indenter and the soil. Equation 26 implies that the depth of soil settling, made under the Nth stress imposed by the indenter is calculated by:

$$Z_N=\frac{Z_1}{\alpha}\lg N \quad (27)$$

In this case, the relative soil shrinkage ratio under the cyclic stress can be calculated by:

$$\frac{Z_N+Z_1}{H}=\frac{Z_1\cdot\left(1+\frac{1}{\alpha}\lg N\right)}{H} \quad (28)$$

In this case, Eq. 24 applied to the viscoelastic Maxwell model will transform in to:

$$\frac{\rho_{N+1}-\rho_0}{\rho_0}=\frac{\omega b q_1\cdot\left(1-\upsilon^2\right)}{E_0 H}\cdot\left(1+\frac{L}{\tau V}\right)\cdot\left(1+\frac{1}{\alpha}\lg N\right) \quad (29)$$

Based on data obtained by a device intended for soil testing 1.61, we will get:

$$\frac{\rho_{N+1}-\rho_0}{\rho_0}=\frac{\omega b q_1^{\frac{1}{n}}\cdot\left(1-\upsilon^2\right)}{A^{\frac{1}{n}}H^2}\cdot\left(1+\frac{L}{\tau V}\right)\cdot\left(1+\frac{1}{\alpha}\lg N\right) \quad (30)$$

Based on Eq. 28, let us rewrite the Eq. 26 for the state of soil with regard to elasticity:

$$\frac{\rho_{N+1}-\rho_0}{\rho_0}=\frac{\omega b q_1\cdot\left(1-\upsilon^2\right)}{E_0 H}\cdot\left(1+\frac{1}{\alpha}\lg N\right) \quad (31)$$

If we apply the equation for calculating the module of deformation by Eq. 22, we will get the following Eq. 32:

$$\frac{\rho_{N+1}-\rho_0}{\rho_0}=\frac{\omega b q_1^{\frac{1}{n}}\cdot\left(1-\upsilon^2\right)}{A^{\frac{1}{n}}H^2}\cdot\left(1+\frac{1}{\alpha}\lg N\right) \quad (32)$$

These equations for the state of soil under the stress imposed by an indenter couple the soil density and settlement with the parameters of soil deformability A, n, α, υ, H as well as with the parameters dependent on the indenter L, b and q_1.

The considered approach allowed obtaining the most significant results in the field of modeling the interaction of logging machines with the soil. If the soil parameters A, n, α, υ, H are known, the calculated dependencies allow obtaining the integral indicators of the mover-soil interaction, the track depth and the soil resistance to deformation and without making time-consuming calculations necessary for solving the majority of problems in soil mechanics. At this point, this approach allowed us to draw and apply some schemes for solving the problems of dynamic logging systems which are already aggravated enough with mathematical difficulties related to the need in solving differential equations of oscillations in multi mass systems.

Besides if the soil parameters A, n, α, υ, H are known, the considered approach remains for now the only tested method for solving problems of cyclic-induced soil settlement, thus, the only tested method for improving the eco-efficiency of logging.

Nevertheless, the analyzed equations for the state of soil have been obtained for the case of a rigid non-deformable indenter (hard wheels). On the top of this, the parameters of the contact area L, b are taken as the basic data. Besides soil parameters A, n, α should be determined by measurement made as a feed-back from complex experiments which methodology differs in different researchers and has not been systematized to a single form.

As the methodology was designed to determine the integral indicators of mover-soil interaction, application field of this approach was narrowed. In case of studying the flotation ability of wheeled vehicles, mover's elasticity should be taken into account along with a more detailed analysis of the stress-strain state of the soil mass, since the latter parameter lays grounds for assessing the resistance of soil to deformation. The latter is required for calculating the traction-coupling properties of the mover.

At first, a diagram (Fig. 1) is drawn for determining the depth of soil settling at stress, imposed by an intender in

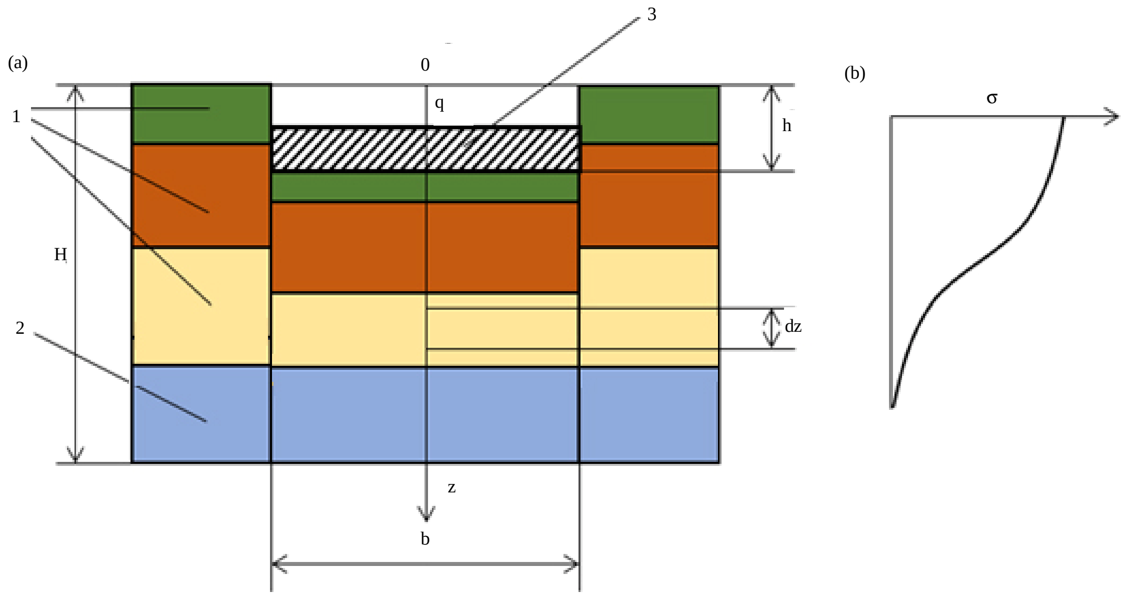

Fig. 1: Calculating the depth of soil settling induced by normal stress; a) General diagram: 1) Soil layers; 2) Untouched layer; 3) Indenter which effect is equal with the effect that a mover has and b) Drawing of normal stress distribution in soil

order to model the mover-soil interaction. The figure is supported with a drawing of stress distribution in the soil with depth.

In many studies, stress distribution σ in soil with depth is considered to be non-linear and calculated by:

$$\sigma = \frac{J \cdot q \cdot k_d}{1+(Az)^2} \tag{33}$$

Where:

J = Contact area coefficient taking into account its shape (intender's length-width ratio in other words, the average length and width of the contact area)

A = Coefficient of the neighboring untouched layer's effect on the compression strain distribution

k_d = Dynamic contact time coefficient (depends on the moving speed, rheological parameters of soil and the contact area size)

The J coefficient is calculated by formula written in accordance with the mechanics of contact interaction (Tertsagi, 1961):

$$J = \frac{0.03+1/b}{0.6+0.431/b} \tag{34}$$

The A coefficient is calculated by the empirical formula (Ageykin, 1981):

$$A = \frac{1}{0.64 \cdot b \cdot (1+b/H)} \tag{35}$$

Equation 35 includes the dynamic stress coefficient which is usually calculated on the back of the Maxwell's theory. According to this theory, stress in the elastic medium decreases in proportion to the effective stress (Kharkhuta and Ievlev, 1961):

$$\sigma_t = \sigma_0 \exp(1-t/t_p) \tag{36}$$

Where:

σ_t = Stress observed in specific moment in time

σ_0 = Initial stress

t = Time during which the soil is under tress (contact time)

t_p = Stress relaxation time

The dynamic coefficient is further calculated by the following Eq. 37 (Tertsagi, 1961):

$$k_? = \frac{\sigma_0 \int_0^t [\exp(1-t/t_p)]dt}{\sigma_0 \int_0^\infty [\exp(1-t/t_p)]dt} = 1-\exp(-t/t_p) \tag{37}$$

which can be replaced by more easy-to-use Eq. 38:

$$k_Д = t/(t+t_p) \tag{38}$$

without the loss of calculation accuracy. The time of relaxation time is measured for different types and states of soil, according to the standardized methodology (Grigoriev *et al*., 2013a, b).

The intender's length is determined by a layout geometry of the elastic wheel rolling motion (Tertsagi, 1961). Calculation is made by the following Eq. 38:

$$l = 2\sqrt{Dh_Z - h_Z^2} + \sqrt{D\cdot(h_Z+h)-(h_Z-h)^2} \tag{39}$$

Where:

h_Z = Deformation of an intender (maximum radial deformation of the wheel)

D = Wheel diameter

All calculation dependencies for values included in Eq. 35 were verified. At the first stage of calculation, we drew our attention to the compression of the primary soil layer, calculated by the following differential dependency (Tertsagi, 1961):

$$dh^* = \frac{\sigma}{E-\sigma}dz \tag{40}$$

The value of compression of all the layers is calculated by integrating the dependencies Eq. 40:

$$h^* = \int_{z_1}^{z_2} \frac{\sigma}{E-\sigma}dz, \tag{41}$$

In general terms, the integral Eq. 41 is represented by:

$$h^* = -\left.\frac{Jqk_d \operatorname{arctanh}\left(\frac{AEz}{\sqrt{(Jqk_d)\cdot E}}\right)}{A\sqrt{(Jqk_d)\cdot E}}\right|_{z=z_1}^{z=z_2} \tag{42}$$

As it follows from Fig. 1, integration limits z_2 and z_1 are equal with H-h and 0, respectively. At this point, we will get a transcendental equation of deformation h:

$$h^* = -\frac{Jqk_d \operatorname{arctanh}\left(\frac{AE\cdot(H-h^*)}{\sqrt{(Jqk_d)\cdot E}}\right)}{A\sqrt{(Jqk_d)\cdot E}} \tag{43}$$

At this point, there are several significant circumstances to be highlighted. The first is that at the time of this approach being developed, we tended to simplify the settlement h* calculation and to avoid the need to solve Eq. 43 in numbers. In this regard, it was suggested to assume the following (Tertsagi, 1961):

$$\frac{\sigma}{E-\sigma} \approx \frac{\sigma}{E} \tag{44}$$

In this case, equation for the integral 1.77 become simpler:

$$h^* = \frac{Jqk_d k_p ab \operatorname{arctg}\left(\frac{H-h^*}{Ab}\right)}{E} \tag{45}$$

but the value h* still cannot be expressed analytically. Since, the equation for soil settling is fundamental and is applied to solve most problems in modeling the effect of an elastic mover on the soil, further developments are based either on the graphical analytic methods or on additional assumptions.

In Forest Science, researches working according to the Ageykin's theory have took a different path. Mathematical models are built using the Eq. 43 without simplifications. Then, a computational experiment is carried out with a variation of basic data. At the end of the experiment, results are approximated. The dependencies obtained in this way are suitable for practical use and provide results that are close to the original numerical solutions. We agree that these results are significant but only partial problems in the field of logging machines have been solved on their back.

Another way of building mathematical models is about replacing the Eq. 43 and 45 by formulas that allow expressing the settlement h* directly. The method is applied by Tertsagi (1961) devoted to the interaction of a wheeled vehicle with clay/loamy/sandy-loam/sandy soils. As we recall, this method was not applied to the case of organo-mineral soils.

At the second stage of calculation, we took into account the non-linear pattern of soil deformation, conditioned by an increase in the depth of settling caused by soil sliding. The majority of researchers agree with Ageykin (1981) that this phenomenon is taken into account in a proper way by multiplying the linear deformation h* by an additional coefficient (Tertsagi, 1961):

$$h = k_p h^* \tag{46}$$

where, k_p soil sliding coefficient (Shapiro *et al*., 2008; Tertsagi, 1961):

$$k_p = \frac{p_S}{p_S - q\cdot k_d} \tag{47}$$

where, p_S ground bearing capacity. The bearing capacity of soil depends not only on its physical and mechanical properties but also on the indenter's parameters. The value of the bearing capacity can be calculated or measured on the ground of a settlement curve.

The further solution obviously becomes more complicated, since, the bearing capacity included in Eq. 47 is not an independent constant in Eq. 43 but varies depending on the indenter's size and the depth of settling.

In addition, a number of questions arise on determining the bearing capacity. It is commonly supposed that its value depends on such parameters as the physical and mechanical properties of soil, indenter's size-shape factor and on the stress direction. The latter factor is especially important when it comes to modeling the rolling motion of machines along the curved path.

According to Ageykin (1981), the general formula for calculating the bearing capacity of soil is the following (Tertsagi, 1961):

$$p_S = \frac{\pi}{2} p_{S0} \alpha \, \text{arctg} \frac{\pi \cdot (H-h)}{2b} \quad (48)$$

Where:

p_{S0} = Bearing capacity of soil mass of unlimited thickness
α = Limited thickness coefficient.

The α coefficient is calculated by the Eq. 49 (Tertsagi, 1961):

$$\alpha = \frac{\pi}{2} \text{arctg} \frac{\pi \cdot (H-h)}{2b} \quad (49)$$

Formula for calculating the bearing capacity of soil mass of unlimited thickness is the following:

$$p_{S0} = K_{\beta 1} I_1 X_1 b + K_{\beta 2} I_2 X_2 + X_3 h \quad (50)$$

where, X_1, X_2, X_3 coefficients of the physical and mechanical properties of soil (these coefficients depend on the internal friction angle and the specific cohesion); I_1, I_2 contact area size/shape coefficients; $K_{\beta 1}$, $K_{\beta 1}$ stress angle coefficients. The $K_{\beta 1}$ and $K_{\beta 2}$ coefficients are calculated by the following Eq. 51 (Volskaya, 2007):

$$K_{\beta 1} = \frac{\pi - 4\beta \, \text{tg}\, \phi}{\pi + 4\beta \, \text{tg}\, \phi} \quad (51)$$

$$K_{\beta 2} = \frac{3\pi - 2\beta}{3\pi + 2\beta} \quad (52)$$

Where:

φ = Internal friction angle
β = Stress angle calculated by Tertsagi (1961)

$$\beta = \arccos \frac{q}{\sqrt{q^2 + \tau^2}} \quad (53)$$

where, τ shearing stress. As we ended with Eq. 53, another issues arisen, there is a need to select adequate dependencies for calculating the shearing stresses that depend on the shear properties of soil and the mover's parameters (mover's width b, tire characteristics such as the geometric parameters of lugs, tread pattern design and saturation coefficient).

The resulting models can be modified by replacing certain formulas. This act does not require a change in the overall structure of the mathematical models. The unquestionable advantage of this approach is its universality. It allows obtaining results for a wide range of soil conditions and mover's parameters. The mover is also characterized by almost all the major parameters load, tire/tread characteristics, rigidity and tire inner pressure.

Soil parameters are parameters that have a strictly defined physical significance deformation modulus, specific cohesion, internal friction angle, shear modulus, specific weight, the Poisson's ratio and the deformable layer thickness. The methods for determining these values have been tested and standardized.

However, there is a problem laying behind this. Since the physical and mechanical properties of soil which is driven on by the logging machines, vary, drawing up the soil map of the logging area is harder to do but this is required for the theoretical developments to apply. In our opinion, we should turn at this to the experience of Foreign colleagues, who use only one soil parameter a cone index which is relatively easy to determine in the field using a hand-held cone penetrometer. In this case, theoretical dependences that allow correlating the conical index with the physical and mechanical properties of soil are known. There is a number of papers presenting experimental data confirming this possibility. Methods for determining the soil deformation modulus by the cone index have been developed and already tested in a number of cases. We believe that it would be relevant to carry out additional studies to establish the possibility of estimating by the cone index not only the modulus of deformation but also the specific cohesion, the internal friction angle and the shear modulus.

A large number of factors included in mathematical models as well as the structure of equations that belong to these models, cause computational difficulties. There are still no unified models and methods designed for assessing the flotation ability of wheeled logging machines. At this point, we mean models and methods that would take in sum the most significant factors at least for the case of homogeneous soil. If the basic settlement dependencies take into account the variability of the physical and mechanical properties of soil, the modeling problem will become even more complicated. Nevertheless, we insist on the need for this complication because logging machines interact with the heterogeneous soil which properties change under the cyclic stress while these machines run across the area.

The problem of pressing an intender into an elastic semi-space is solved in many works studying the flotation ability of machines in off-road conditions. The computational scheme that has become traditional for solving this problem is shown in Fig. 2. The resistance of medium is expressed through the module of general deformation E:

$$E = \frac{\sigma}{\varepsilon} \tag{54}$$

Where:
σ = Normal stress
ε = Relative compression strain

The law of stress distribution σ with depth is accepted with regard to its decrease (Tertsagi, 1961):

$$\sigma = \frac{Jp}{1+\left(z/ab\right)^2} \tag{55}$$

Where:
J = Parameter dependent on the indenter's geometry
z = Vertical coordinate, reckoned upward from the surface of the medium
p = Pressure on the soil
a = Deformable layer thickness
b = Indenter's width

The value of compression of the primary layer which initial thickness was z_0 is calculated by:

$$dh_L = \varepsilon dz_0 \tag{56}$$

The thickness of compressed primary layer is calculated by:

$$dz = (1-\varepsilon)dz_0 \tag{57}$$

From Eq. 54, 55 and 57 it follows that:

$$dh_L = \frac{\sigma}{E-\sigma}dz \tag{58}$$

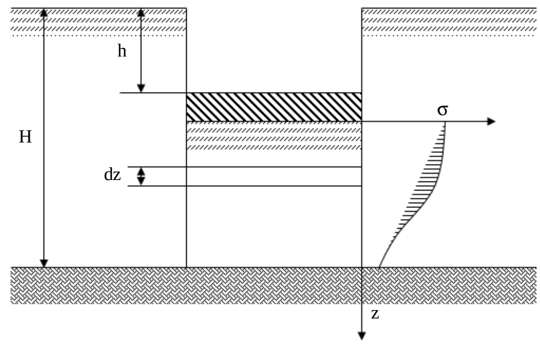

Fig. 2: Computational scheme for solving the problem of pressing and indenter in elastic semi-space

At this point, total compression strain is calculated by:

$$h_L = \int_0^{H-h_L} dh_L \tag{59}$$

Equation 59 includes the assumption E -σ≈E, thus:

$$h_L = \frac{Jpab\arctan\left(\frac{H-h_L}{ab}\right)}{E} \tag{60}$$

If thickness is unlimited, one shall assume that H→∞8, then:

$$h_L = \frac{1}{2}\frac{J\pi pab}{E} \tag{61}$$

If we integrate Eq. 59, assuming that the stress factor is as considerable as in the deformation modulus, we will get:

$$h_L = \frac{Jpab\arctan\left(\frac{E(H-h_L)}{ab\sqrt{E(E-Jp)}}\right)}{\sqrt{E(E-Jp)}} \tag{62}$$

Let us assume that H→∞ in the case of unlimited thickness, then:

$$h_L = \frac{1}{2}\frac{J\pi pab}{\sqrt{E(E-Jp)}} \tag{63}$$

Figure 3 and 4 show that calculations made by Eq. 60-63 are significant enough only in cases, when the bearing capacity of soil is relatively low (<0.3 MPa). However, logging machines are also used in swampy areas where soils have low bearing capacity, so, dependencies Eq. 62 and 63 seem to be more reasonable.

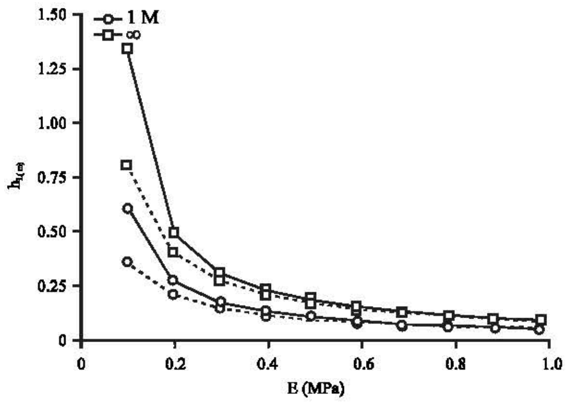

Fig. 3: Ordinary soil mass compression: dashed line, calculations by Eq. 60 and 61 and full line by Eq. 62 and 63

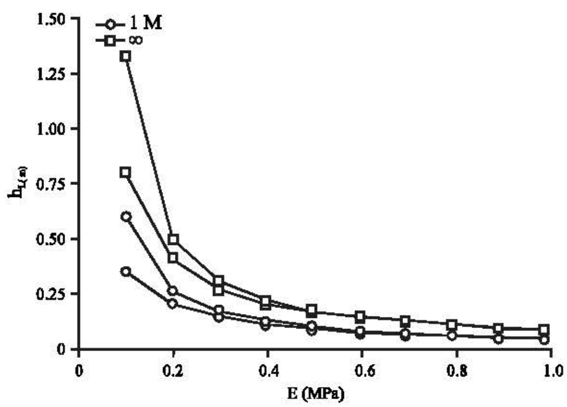

Fig. 4: Swamped soil mass compression: dashed line, calculations by Eq. 60 and 61 and full line by Eq. 62 and 63

We should note that Eq. 62 and 63 were obtained on the assumption that the deformation modulus remains constant in the z coordinate but this obviously is not always true. In addition, stress imposed on soil decreases with depth, so, the soil is compacted unevenly. Hence, even an initially homogeneous mass will become heterogeneous after the mover applies load to it. In this regard, let us consider the issue of modeling the effect of an indenter on soil with a variable deformation modulus.

Let us consider the case of pressing an indenter into the soil with a variable modulus of linear deformation. Let us assume that:

$$E = a_0 + a_1 z \tag{64}$$

where, a_0 and a_1 line function coefficients. We will take the a_0 and a_1 coefficients as the initial data. The total compression strain will be calculated with regard to Eq. 46, 48, 49 and 64:

$$h_L = \int_0^{H-h_L} \frac{(ab)^2 Jp}{(ab)^2 (a_0 + a_1 z - Jp) + a_1 z^3 + a_0 z^2} dz \tag{65}$$

If we integrate the right-hand side of Eq. 65, we will get a cumbersome expression that will be uneasy to analyze and apply.

In this case, the numerical solution of an integral equation seems more appropriate. A particular case is not hard to do solve on the back of modern software products. On the other hand, solving each subcase on a separate basis is inconvenient.

We propose a different method based on a computational experiment. The experiment is as follows. The program generates two values of a deformation modulus: one directly for the ground surface E_1 and one for the point located at a depth equal to the thickness of the deformable layer E_2. The value of deformable layer thickness H is also randomly generated. All three variables are distributed according to the law of equal density in the range from 0.5-5 MPa for modules and in the range from 0.3-0.8 m for the H. Based on the numerical values obtained, we will calculate the a_0 and a_1 coefficients included in Eq. 64:

$$\begin{cases} a_0 = E_1 \\ a_1 = \dfrac{E_2 - E_1}{H} \end{cases} \tag{66}$$

Other initial data: J = 0.7, p = 80 MPa, a = 0.4, b = 0.7 m. As the numerical solution to Eq. 65 is found, the h_L value is substituted in Eq. 62 to find E.

Thus, we get a deformation modulus E_{eq} for a homogeneous soil mass, settled at the certain depth h_L. If all else is equal, h_L included in the obtained modulus will be equal with the value of settlement included in the deformation modulus obtained for the heterogeneous soil mass.

The experiment ended with solutions made for 1000 combinations of random variables E_1, E_2, H and thus, with a_0 and a_1. As we processed the calculation results, we were able to establish the dependence of the equivalent deformation modulus on a_0, a_1, H:

$$E_{eq} = 0.95a_0 + 0.45a_1 H \tag{67}$$

The results obtained by comparing data of the computational experiment with those calculated by Eq. 67 are shown in Fig. 5. The coefficient of determination for Eq. 67 is $R^2 = 0.9773$.

Thus, if the soil deformation modulus vary with depth according to a linear law, the depth of soil settling will be calculated by Eq. 62 which deformation modulus is calculated by Eq. 67.

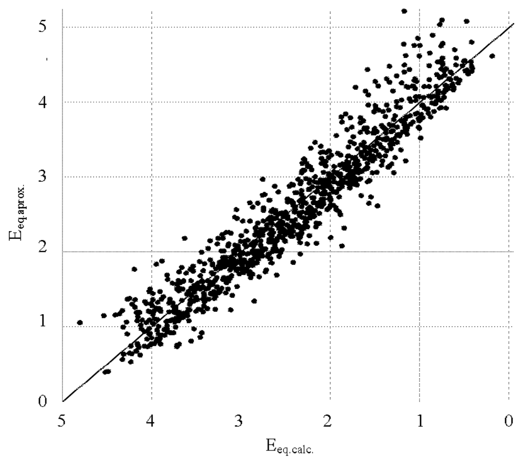

Fig. 5: Correlation between data of the computational experiment and data calculated by Eq. 67

Next, let us consider the case, when an indenter is pressed in the soil mass with a variable deformation modulus that varies according to the square law. Let us assume that:

$$E = b_0+b_1z+b_2z^2 \tag{68}$$

where, b_0, b_1 and b_2 coefficients of the quadratic function. We will take the b_0-b_2, coefficients as the initial data. The total compression strain will be calculated by Eq. 59 with regard to Eq. 53, 54, 56 and 68:

$$h_L = \int_0^{H-h_L} \frac{(ab)^2 Jp}{z^2\left(b_0+b_1z+b_2z^2\right)+(ab)^2\left(b_0+b_1z+b_2z^2-Jp\right)}dz \tag{69}$$

Let us go in for the computational experiment once again. Now, the program generates three values: one directly for the ground surface E_1; one for the point located at a depth equal to the half-value thickness of the deformable layer E_2; one for the point located at a depth equal to the thickness of the deformable layer E_3. The value of deformable layer thickness H is also randomly generated. All three variables are distributed according to the law of equal density in the range from 0.5-5 MPa for modules and in the range from 0.3-0.8 m for the H. Based on the numerical values obtained, we will calculate the b_0-b_2, coefficients included in Eq. 68:

$$\begin{cases} b_0 = E_1 \\ b_1 = -\dfrac{3E_1-4E_2+E_3}{H} \\ b_2 = \dfrac{2\cdot(E_1-2E_2+E_3)}{H^2} \end{cases} \tag{70}$$

Other initial data: J = 0.7, p = 80 MPa, a = 0.4, b = 0.7 m. As the numerical solution to Eq. 69 is found, the h_L value is substituted in Eq. 64 to find E.

Thus, we get a deformation modulus E_{eq} for a homogeneous soil mass, settled at the certain depth h_L. If all else is equal, h_L included in the obtained modulus will be equal with the value of settlement included in the deformation modulus obtained for the heterogeneous soil mass. In this case, however, modulus obtained for the heterogeneous soil mass will vary with depth according to the square law.

The experiment ended with solutions made for 1000 combinations of random variables E_1-E_3, H and thus with b_0, b_1 and b_2. As we processed the calculation results, we were able to establish the dependence of the equivalent deformation modulus on b_0-b_2 and H:

$$E = 0.966b_0+0.0191b_0b_1+0.00396b_0b_2+0.229b_1H+0.0543b_2H \tag{71}$$

The results obtained by comparing data of the computational experiment with those calculated by Eq. 71 are shown in Fig. 6. The coefficient of determination for Eq. 67 is R^2 = 0.9094.

Thus, if the soil deformation modulus vary with depth according to the square law, the depth of soil settling will be calculated by Eq. 62 which deformation modulus is calculated by Eq. 71.

Next, let us consider the case, when an indenter is pressed in the soil mass with a variable deformation modulus that varies according to the cube law. Let us assume that:

$$E = c_0+c_1z+c_2z^2+c_3z^3 \tag{72}$$

where, c_0-c_3 coefficients of the 3rd degree polynomial function. We will take the c_0-c_3, coefficients as the initial data. The total compression strain will be calculated by Eq. 59 with regard to Eq. 53, 54, 56, 72:

$$h_L = \int_0^{H-h_L} \frac{a^2b^2Jp}{z^5c_3+z_4c_2+z^3\left(c_3a^2b^2+c_1\right)+z^2\left(c_2a^2b^2+c_0\right)+zc_1a^2b^2+a^2b^2\left(c_0-Jp\right)}dz \tag{73}$$

As before, let us go in for the computational experiment. Now, the program generates four values: one directly for the ground surface E_1; one for the point located at a depth that equals one-third of the deformable layer thickness E_2; one for the point located at a depth that equals two-thirds of the deformable layer thickness E_3 and one for the point located at a depth equal to the thickness of the deformable layer E_4. The value of deformable layer thickness H is also randomly generated.

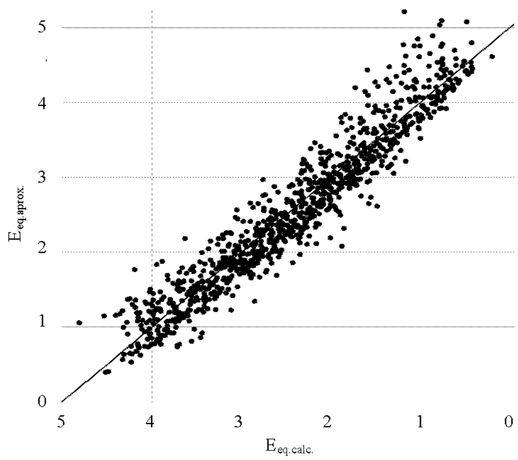

Fig. 6: Correlation between data of the computational experiment and data calculated by Eq. 71

All variables are distributed according to the law of equal density in the range from 0.5-5 MPa for modules and in the range from 0.3-0.8 m for the H. Based on the numerical values obtained, we will calculate the c_0-c_3 coefficients included in Eq. 72:

$$\begin{cases} c_0 = E_1 \\ c_1 = -\frac{1}{2}\cdot\frac{11E_1-18E_2+9E_3-2E_4}{2H} \\ c_2 = \frac{9}{2}\cdot\frac{2E_1-5E_2+4E_3-E_4}{H^2} \\ c_3 = -\frac{9}{2}\cdot\frac{E_1-3E_2+3E_3-E_4}{H^3} \end{cases} \quad (74)$$

Other initial data: J = 0.7, p = 80 MPa, a = 0.4, b = 0.7 m. As the numerical solution to Eq. 73 is found, the h_L value is substituted in Eq. 62 to find E.

Thus, we get a deformation modulus E_{eq} for a homogeneous soil mass, settled at the certain depth h_L. If all else is equal, h_L included in the obtained modulus will be equal with the value of settlement included in the deformation modulus obtained for the heterogeneous soil mass. In this case, however, modulus obtained for the heterogeneous soil mass will vary with depth according to the cube law.

The experiment ended with solutions made for 1000 combinations of random variables E_1-E_4 H and thus with c_0-c_3. As we processed the calculation results, we were able to establish the dependence of the equivalent deformation modulus on c_0-c_3 and H:

$$\begin{aligned} E = {} & 1.05c_0+0.124c_1+0.023c_2-0{,}0103c_3-0.0000139c_0c_1c_2- \\ & 0.0000151c_0c_1c\text{-}0.00000141c_0c_2c_3-0.0325c_0c_1H+ \\ & 0.0189c_0c_2H+0.00762c_0c_3H-7.54\cdot10^{-9}c_1c_2c_3 \\ & +0.000183c_1c_2H-0.00000738c_2c_3H-0.109c_0H+0.000212c_1c_2 \\ & +0.00023c_1c_3+0.092c_1H+0.0000157c_2c_3+0.0727c_2H+0.0412c_3H \end{aligned} \quad (75)$$

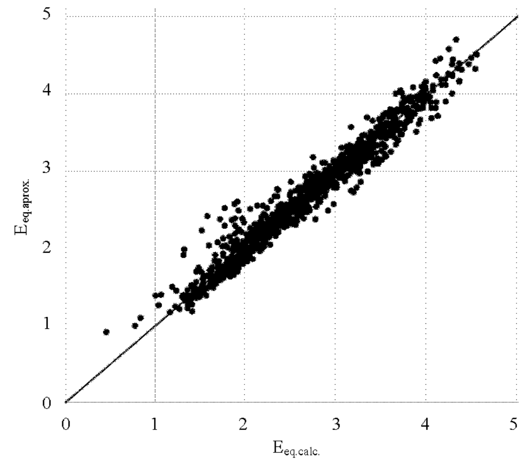

Fig. 7: Correlation between data of the computational experiment and data calculated by Eq. 75

The results obtained by comparing data of the computational experiment with those calculated by Eq. 75 are shown in Fig. 7. The coefficient of determination for Eq. 75 is $R^2 = 0.9552$.

Thus, if the soil deformation modulus vary with depth according to the cube law, the depth of soil settling will be calculated by Eq. 62 which deformation modulus is calculated by Eq. 75.

CONCLUSION

Simulations on the effect that logging machines have on the heterogeneous soil mass with a deformation modulus varying with depth according to a known law can be made with a homogeneous soil mass settling at the same depth as the heterogeneous soil mass does if all else is equal. The deformation modulus of such a homogeneous soil mass is called the equivalent deformation modulus.

Thus, when modeling the effect that an indenter has on the heterogeneous soil, numerical solutions to the integral equation that expresses its penetration into the heterogeneous soil mass can be replaced by one expressing the penetration into the homogeneous soil mass Eq. 62. The latter equation includes the equivalent deformation modulus. This practice is much simpler in terms of computing.

If the soil deformation modulus vary with depth according to a linear law, the equivalent deformation modulus is calculated by Eq. 67 which was found during the computational experiment.

If the soil deformation modulus vary with depth according to the square law, the equivalent deformation modulus is calculated by Eq. 71 which was found during the computational experiment. If the soil deformation modulus vary with depth according to the cube law, the equivalent deformation modulus is calculated by Eq. 75.

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