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Calculation of the Aberration Optimum Balance for Circular Synthetic Apertures

Adnan Falh Hassan and Ban Hussein Ali
Department of Physics, Faculty of Science, University of Kufa, Kufa, Iraq

Abstract: It is well known that, the synthetic apertures of different shapes are good single a aperture alternative which is used in different optical system to improve its efficiency. However, the issue of the aberration is a considerable challenge. In this research, the optical defocusing error and the spherical aberration of third and fifth order were calculated for circular synthetic apertures system. The evaluated aberration would balance the self-aberration already existed within the optical system which is difficult to reduce it. Therefore, this would lead to a system which is to high extent is free of the aberration. The mathematical calculations were conducted to calculate the value of aberration that balance the intrinsic aberration, the resulted data were acceptable by supposing the number of apertures equal one (single aperture) which agree with previous research.

Key words: Aberration, synthetic aperture, point spread function, optimum balance, spherical aberration, Iraq

INTRODUCTION

A fundamental issue that associated with lenses is the poor image quality. The deviation of the actual image from the ideal one with respect to the actual size, shape and position is well-known by "Aberration", aberration can be caused from the inherent shortcoming of a lens and not caused by certain faults in the construction of the lens. This includes the irregularity in the surface of the lens. The kind of aberration that would be resulted from the variation of refractive index within the range of wavelength of light is chromatic aberration. However, the other types of aberrations are also, resulted when monochromatic aberration. It is important to note that, lens aberrations are resulted from laws of refraction to the spherical surfaces and not necessary caused by the defective construction of lenses whose surface are not spherical (Subrahmanyam *et al*., 2015). The images with aberration are fundamentally diffraction patterns of the stop of the aperture can be considered as a phase mask whose values represent the aberration (Lipson *et al*., 2010). Instrument cannot produce image without aberration for an extended object. In this context, the best that can be performed practically is to make sure that the aberration which may be produced is made as small as possible. However, the latter may be at the expense of making others larger (Born and Wolf, 1999). All rays that are originating from an object were perfect, meaning that all the rays emerging from a single object point can be converted into a single image point when the magnification of the system was a constant of the optical system which it is independent of that ray being considered. Due to real optical system, non-paraxial rays also, take part in formation of image the actual, images depart from the ideal images. The departure results in what are known as aberration. The wide range of space optical applications have encouraged the researchers to consider the optical system which having synthetic apertures. Given that these aperture have some problems such as the way that the aperture are distributed, the cost of fabrication the weight and moment of inertial, the pore mentioned problems can be overcome by administering some kind of loss in optical performance for these optical system. However, there are many advantages and application that might be associated with such synthetic apertures (Harvey *et al*., 1986). The common interest in the synthetic aperture optical system can be divided into three main types namely, astronomic telescoped which would a simple field of view, scan application which need huge field of view, finally, sending laser beam application and targets tracking using telescope. Synthetic apertures can be define as a collection of individual optical systems for large single aperture which are known as mosaic or segmented mirror. The synthetic apertures is an imaging system for independent optical system which formal together image field. The gool behind the synthetic aperture design to reduce the aberration and to narrowing Point Spread Function (PSF), so, we can obtain maximum energy in the center (Fender, 1984). The effect of optical aberration on an array of synthetic aperture was investigated by Hooker when he found that the rank of low aberration reduces the PSF more that high rank aberration.

MATERIALS AND METHODS

Thoery: It should be noted that different ranking aberration balance is considered as one of the important

Corresponding Author: Adnan Falh Hassan, Department of Physics, Faculty of Science, University of Kufa, Kufa, Iraq

points in diffraction theory. The idea behind the balancing is to reduce the effect of aberration by choosing a group of high ranking aberration to contradict (oppose) the influence of law ranking aberration (Shannon, 1985). Which would lead to the best image quality. The method which is used in the above have been employed by Marechal to find out the minimum value of mean square deviation in wave front. So, it is possible to obtain the maximum central intensity value. This is according to strehl criterion. The mean square deviation in wave front (contrast) according to Wang *et al*. (2008):

$$V = \frac{1}{A}\iint_{area} w^2 dA - \left[\frac{1}{A}\iint_{area} w dA\right]^2 \tag{1}$$

Where:

A = Pupil area
dA = Element ray differential area
w = Aberration function

When using polar coordinate, so, Eq. 1 can be written as:

$$V = \frac{1}{A}\iint_{r\varphi}\left[w(r,\varphi)\right]^2 r dr d\varphi - \left[\frac{1}{A}\iint_{r\varphi} w(r,\varphi) r dr d\varphi\right]^2 \tag{2}$$

Marchel has clarified that the normalized maxima intensity for light distribution in the image of point source for the system that having a low aberration quantity when using circular exist pupil as follow:

$$I = (1 - 2\pi^2 V/\lambda) \tag{3}$$

RESULTS AND DISCUSSION

The resulted data are presented in this study. This includes the results from optimum balance calculation with defocusing error, third and fifth rank spherical aberration.

Optimum balance calculation with defocusing error: The defocusing error can expressed as Wang *et al*. (2008):

$$w = w_{20} r^2 \tag{4}$$

So, by substituting Eq. 4 by Eq. 2, we can obtain:

$$V = \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^{\frac{1}{\sqrt{N}}} (w_{20} r^2)^2 r dr d\varphi - \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^{\frac{1}{\sqrt{N}}} (w_{20} r^2) r dr d\varphi\right]^2\right] \tag{5}$$

The integration limit of Eq. 5 is for an array of synthetic aperture which a ready derived by Falhhassan (2004) by solving the above integration it is possible to obtain:

$$V = \frac{4 N w_{20}^2 - 3 w_{20}^2}{12 N^4} \tag{6}$$

When (Rayleigh) condition for the maximum intensity I = 0.8 which is given in Eq. 3, so:

$$w_{20} \leq \frac{\sqrt{12}\lambda N^2}{\sqrt{180}\sqrt{4N-3}} \tag{7}$$

by substituting Eq. 6 by Eq. 7, Eq. 8 will be given as follow:

$$w_{20} \leq \frac{\sqrt{12}\lambda N^2}{\sqrt{180}\sqrt{4N-3}} \tag{8}$$

The equation represents the allowed defocus error value in the optical system. In term number of synthetic aperture (N) to verify Eq. 8. We suppose the number of synthetic aperture one (N = 1), thus, W_{20} = 0.25 •. Where this value highly agree with Marechal for a single circular aperture whose area is •.

When inputting different values of synthetic apertures, we obtain the values of optimum balance for defocusing error when synthetic apertures having different numbers as shown in Table 1. These values are calculated for the first time.

Optimum balance calculation with third ranked spherical aberration: The third ranked spherical aberration is given by DiMarzio (2011):

$$w = w_{20} r^2 + w_{40} r^4 \tag{9}$$

By substituting Eq. 9 in Eq. 2, we obtain:

$$V = \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^{\frac{1}{\sqrt{N}}} (w_{20} r^2 + w_{40} r^4)^2 r dr d\varphi - \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^{\frac{1}{\sqrt{N}}} (w_{20} r^2 + w_{40} r^4) r dr d\varphi\right]^2\right] \tag{10}$$

Table 1: The focus error balance for different numbers of apertures

No.of apertures	Defocusing aberration
N = 2	W_{20} = 0.481 •
N = 3	W_{20} = 0.77 •
N = 4	W_{20} = 1.1457 •
N = 6	W_{20} = 2.028 •

So, by solving the integrals Eq. 11 can be obtained as follows:

$$V = \frac{4N-3}{12N^4} w_{20}^2 + \frac{9N-5}{45N^6} w_{40}^2 + \frac{3N-2}{6N^5} w_{20}\, w_{40} \quad (11)$$

To obtain the minimum value for the contrast equation, take partial differential for Eq. 11 and equating it to zero (•V/•w_{20} = 0) which represents the best focus:

$$\frac{\partial v}{\partial w_{20}} = \frac{4N-3}{6N^4} w_{20} + \frac{3N-2}{6N^5} w_{40} = 0 \quad (12)$$

or by optimum ratio:

$$\frac{w_{20}}{w_{40}} = \frac{2-3N}{4N-3} \quad (13)$$

To verify Eq. 13, we suppose the number of apertures equal one (N = 1) (i.e, single circular aperture), we can obtain:

$$\frac{w_{20}}{w_{40}} = -1\lambda \quad (14)$$

This would agree with Marechal . Thus, for different number of apertures it is possible to obtain the following optimum ratio as shown in Table 2.

By substituting the value of W20 from Eq. 14 in Eq. 11, the contrast value can be calculated as a function of third ranked spherical aberration and the number of synthetic apertures:

$$V = \left[\frac{(2-3N)^2}{12N^6(4N-3)} + \frac{9N-5}{45N^6} - \frac{(3N-2)^2}{6N^6(4N-3)}\right] w_{40}^2 \quad (15)$$

When N = 1, we obtain:

$$V = \frac{w_{40}^2}{180} \quad (16)$$

By substituting the value of V from Eq. 16 in Eq. 7, we have W20• -•, W40• -1• when N takes different numbers, we will have the following value of optimum balance for 3rd spherical aberration when synthetic apertures having different number as shown in Table 3.

Optimum balance calculation using 5th ranked spherical aberration: The 5th spherical aberration is given by Shannon *et al*. (1993):

$$w = w_{20}r^2 + w_{40}r^4 + w_{60}r^6 \quad (17)$$

Table 2: Defocusing values and optimum ratio for different numbers of synthetic apertures

No. of apertures	Optimum ratio w_{20}/w_{40}	W_{20}
N = 2	-0.4 •	-0.4 • W_{40}
N = 3	-0.259 •	-0.259 • W_{40}
N = 4	-0.152 •	0.152 • W_{40}
N = 6	-0.1269 •	0.1269

Table 3: The 3rd spherical aberration balance for different numbers of apertures

No. of apertures	V	W_{40}	W_{20}
N = 1	-0.1646 w^2_{40}	-1 •	1 •
N = 2	-0.00833 w^2_{40}	-0.8166 •	0.326 •
N = 3	-0.001244 w^2_{40}	-2.1126 •	0.528 •
N = 4	-0.0003129 w^2_{40}	-4.213 •	0.640 •
N = 6	-0.000390 w^2_{40}	-3.772 •	0.478 •

So, by substituting Eq. 17 in Eq. 2, we obtain:

$$V = \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^{\frac{1}{\sqrt{N}}} (w_{20}r^2 + w_{40}r^4 + w_{60}r^6)^2\, r\,dr\,d\varphi - \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^{\frac{1}{\sqrt{N}}} (w_{20}r^2 + w_{40}r^4 + w_{60}r^6)\, r\,dr\,d\varphi\right]^2\right] \quad (18)$$

By solving the integrals, we can obtain:

$$V = \left[\frac{1}{3N^3} - \frac{1}{4N^4}\right] W_{20}^2 + \left[\frac{1}{5N^5} - \frac{1}{9N^6}\right] W_{40}^2 + \left[\frac{1}{7N^7} - \frac{1}{16N^8}\right] W_{60}^2 + \left[\frac{1}{2N^4} - \frac{1}{3N^5}\right] W_{20} W_{40} + \left[\frac{2}{5N^5} - \frac{1}{4N^6}\right] W_{20} W_{60} + \left[\frac{1}{3N^6} - \frac{1}{6N^7}\right] W_{40} W_{60} \quad (19)$$

When the number of aperture equal one (N = 1)and by using the two relations •v/•w_{20} and •v/•w_{40} = 0, we get:

$$\frac{w_{20}}{w_{60}} = \frac{3}{5} \quad \frac{w_{40}}{w_{60}} = \frac{3}{2}$$

So,

$$V = \frac{w_{60}^2}{2800} \quad (20)$$

By substituting Eq. 20 in 7, we get W_{60} = 3.944 •, W_{40} = -5.9 • and W_{20} = 2.36 •. When the number of apertures is something different the one we get values of optimum balance for Wth spherical aberration when synthetic apertures having different numbers as shown in Table 4.

The values of the calculated aberration had reduced the amount of aberration to its highest level in the optical system that has synthetic apertures.

Table 4: The 5th spherical aberration balance for different numbers of apertures

N	V	w_{40}/w_{60}	w_{20}/w_{60}	W_{60}	W_{40}	W_{20}
1	$w^2_{60}/2800$	3/-2	3/5	3.944 •	-5.911 •	2.36 •
2	$17\ w^2_{60}/358400$	27/-40	21/200	34.223 •	-23.100 •	3.593 •
3	$11\ w^2_{60}/38782800$	17/-38	13/285	139.954 •	-62.61 •	6.383 •
4	$1\ w^2_{60}/26214400$	7/-40	57/2240	381.622 •	-127.77 •	9.71 •
6	0.001W60	184/-41	31/2760	2.288 •	-10.267 •	0.025 •

CONCLUSION

Optical aberration of synthetic apertures can rapidly degrade image quality. No single lens is free from all the aberration. It is not possible to minimize all the aberrations simultaneously in a lens system. The value of calculated aberration in this research balances the intrinsic aberration already existed in optical system which have multiple apertures which lead to the lowest amount of aberration.

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