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Some Structures of Čech Fuzzy Soft Closure Spaces

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Abstract: The concept of Čech fuzzy soft closure space (ČF-scs for short) is very recently defined and its basic properties are introduced by Majeed. In this study, we continue the study of ČF-scs. We show that every ČF-scs gives a parameterized family of Čech fuzzy closure spaces (ČF-fcss, for short). Furthermore, some properties of fuzzy soft neighborhood of a fuzzy soft point are introduced. Finally, the notion of fuzzy soft exterior (respectively, boundary) in ČF-scs is introduced and its basic properties are discussed, supported by counter examples.

Key words: Fuzzy set, fuzzy soft set, fuzzy soft point, Čech fuzzy soft closure operator, fuzzy soft exterior, fuzzy soft boundary

INTRODUCTION

The principal idea of fuzzy set was presented by Zadeh (1965). Fuzzy sets were introduced to provide means to represent situations mathematically which give rise to ill-defined classes, i.e., collections of objects for which there is no exact criteria for membership. By Molodtsov (1999) started a novel idea of soft set theory which is a totally new approach for displaying ambiguity and uncertainty. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov (1999). Maji *et al*. (2001) were combined fuzzy setsand soft sets to introduce the concept of fuzzy soft sets. Later in 2011, Tanay and Kandemir (2011) were gave the concept of topological structure based on fuzzy soft sets.

Czech (1966) introduced the notion of Čech closure spaces (X, C) where C: P (X)→P (X) is a mapping satisfying C (∅) = ∅, A⊆C (A) and C (A∪B) = C (A)∪C (B), the mapping C called Čech closure operator on X, it is similar to a topological closure operator, excluding that it is not required to be idempotent. That is Čech closure operator comply just three of the four Kuratowski closure axioms. By Mashhour and Ghanim (1985) introduced the concept of ČF-fcss when they substitute sets by fuzzy sets in the definition of Čech closure space which stats as (An operator C: $I^X \to I^X$ is said to be Čech fuzzy closure operator (ČF-fco for short) on X, if C satisfied the following three axioms: (C1) $C(\bar{0}) = \bar{0}$, (C2)$\mu \leq C(\mu)$ and (C3) $C(\mu \vee \rho) = C(\mu \vee C(\rho))$. Recently, by Gowri and Jegadeesan (2014) using the notion of soft sets to introduced and investigation soft Čech closure spaces, the soft closureoperator in that sense was defined from the power set P (X_{FA}) of X_{FA} to itself (where F_A is a soft set over the universe set X with the set of parameter K and A⊆K). Also, in the same year, Krishnaveni and Sekar (2014) introduced and study Čech soft closure spaces (where the soft closure operator here defined from the set of all soft sets over X to itself). Very recently, Majeed (2018) employ the fuzzy set theory to define and study the notion of Čech fuzzy soft closure spaces (ČF-scs's for short) which is a generalization to Čech soft closure spaces that given by Krishnaveni and Sekar (2014).

The aim of this study is to complete the study of ČF-scs'sin order to obtain new properties for them. In this study, some properties of this spaces are introduced, such as, we show that the union of two Čech fuzzy soft closure operators is also Čech fuzzy soft closure operator (Theorem 3.1). However, the intersection is not. Also, we obtained a very important relationship between ČF-scs's and ČF-fcss. That is, every ČF-scs gives a parameterized family of ČF-fcss (Theorem 3.5). In addition, some properties of fuzzy soft neighborhood of a fuzzy soft point are introduced. In this study, the notion of fuzzy soft exterior (respectively, boundary) of a fuzzy soft sets in ČF-scs is defined and give the properties of its. Also, we find several deviations for some results in exterior and boundary that are hold in both ordinary and soft topological spaces but not in ČF -scss. These deviations are clarified by giving several examples.

MATERIALS AND METHODS

Preliminaries: We expect that the reader is knows about the usual notions and most basic ideas of fuzzy set theory. Throughout our study, X will refer to the initial universe, I = [0, 1], I_0 = (0, 1], I^Xbe the family of all fuzzy sets of X and K the set of parameters for X.

A fuzzy soft set λ_A on X is a mapping from K-I^X, i.e., λ_A: K→I^X where λ_A (h) $\neq \bar{0}$ if h∈A⊆K and λ_A (h) = $\bar{0}$ if h∉A⊆K where $\bar{0}$ is the empty fuzzy set on X. The family of all fuzzy soft sets over X denoted by F_{ss}(X, K) (Roy and Samanta, 2012; Varol and Aygun, 2012). Let λ_A, μ_B∈F_{ss} (X, K), then λ_A is called a fuzzy soft subset of μ_A, denoted by $\lambda_A \subseteq \mu_B$, if $\lambda_A(h) \leq \mu_B(h)$ for all h∈K. Also λ_A and μ_B are said to be equal, denoted by $\lambda_A = \mu_B$ if $\lambda_A \subseteq \mu_B$ and $\mu_B \subseteq \lambda_A$. The union (respectively, intersection) of λ_A and μ_A, denoted by $\lambda_A \cup \mu_B$ (respectively, $\lambda_A \cap \mu_B$) is the fuzzy soft set $\sigma_{(A\cup B)}$ defined by $\sigma_{(A\cup B)}$ (h) = $\lambda_A(h) \vee \mu_B(h)$, (respectively is the fuzzy soft set $\sigma_{(A\cap B)}$ defined by $\sigma_{(A\cap B)}$ (h) = $\lambda_A(h) \wedge \mu_B(h)$) for all h∈K. The constant fuzzy soft sets taking, respectively, values $\bar{0}$ and $\bar{1}$ at every h∈K are denoted by $\bar{0}_K$ and $\bar{1}_K$, respectively. For the fuzzy soft set λ_A in X, $\bar{1}_K$ -λ_A will stand for the complement of λ_A is the fuzzy soft set defined by $(\bar{1}_K - \lambda_A)(h) = \bar{1} - \lambda_A(h)$ for each h∈K, Its clear that $\bar{1}_K - (\bar{1}_K - \lambda_A) = \lambda_A$ (Varol and Aygun, 2012).

According to the concept of Atmaca and Zorlutuna a fuzzy soft set λ_A∈F_{ss}(X, K) is called fuzzy soft point, denoted by x_t^h,if there exist x∈X and h∈K such that $\lambda_A(h)(x)$ = t(0<t≤) and 0 otherwise for all y∈X-{x}.The fuzzy soft point x_t^h is said to be belongs to the fuzzy soft set x_t^h, denoted by λ_A if for the element $\lambda_A(h)(x)$ (see (Atmacaand and Zorlutuna, 2013).

Definition 2.1; Majeed (2018): An operator θ: F_{ss} (X, K)→F_{ss}(X, K) Is called Č ech fuzzy soft closure operator (Č-fsco for short) on X if the following axioms are satisfied:

- (C1) θČ($\bar{O}_K$)
- (C2) $\lambda_A \subseteq \theta(\lambda_A)$, for all $\lambda_A \in F_{ss}(X, K)$
- (C3) $\theta(\lambda_A \cup \mu_B) = \theta(\lambda_A) \cup \theta(\mu_A)$, for all λ_A, $\mu_B \in F_{ss}(X, K)$

The triple (X,θ, K) is called a ĈF-fscs

A fuzzy soft set λ_A is said to be closed fuzzy soft set (closed-fss, for short) in (X, θ, K) if λ_A = (λ_A). A fuzzy soft set λ_A is said to be an open fuzzy soft (open-fss, for short) set if $\bar{1}_K$-λ_A is a closed-fss.

Proposition2.2; Majeed (2018): Let (X, θ, K) be a ČF-scs and λ_A, μ_B∈F_{ss}(X, K) such that $\lambda_A \subseteq \mu_B$, then $\theta(\lambda_A) \subseteq \theta(\mu_B)$.

Definition 2.3; Majeed (2018): Let (X, θ, K)be a ČF-scs and let $\lambda_A \in F_{ss}$(X, K). The interior of λ_A,denoted by Int (λ_A) is defined as (λ_A) = $\bar{1}_K$-(θ($\bar{1}_K$-λ_A)).

Proposition 2.4; Majeed (2018): Let (X, θ, K) be a ČF-scs and let λ_A, $\mu_B \in F_{ss}$(X, K). Then:

- Int ($\bar{O}_K$) $\bar{O}_K$ and Int ($\bar{1}_K$) = $\bar{1}_K$
- Int (λ_A) ⊆ λ_A
- Int ($\lambda_A \cap \mu_B$) = Int (λ_A)∩Int (μ_B)
- If $\lambda_A \subseteq \mu_B$, then Int($\lambda_A$)⊆Int ($\mu_B$)
- λ_A is an open-fss⇔Int(λ_A) = λ_A
- Int (λ_A)∪Int (μ_B)⊆Int ($\lambda_A \cup \mu_B$)

Definition 2.5; Majeed (2018): A fuzzy soft set λ_A in a ČF-scs (X, θ, K) is said to be fuzzy soft neighborhood of a fuzzy soft point x_t^h, if x_t^h $\tilde{\in}$ Int (λ_A)

Definition 2.6; Majeed (2018): Let (X, θ, K) be a ČF-scs. A fuzzy soft point x_t^h is said to be a fuzzy soft interior point of a fuzzy soft set λ_A, if there exists an open-fss μ_B such that x_t^h $\tilde{\in}$ $\mu_B \subseteq \lambda_A$

Definition 2.7; Majeed (2018): Let V be a non-empty subset of X, then $\bar{V}_K$ denotes the fuzzy soft set $\bar{V}_K$ over X for which V(h) = $\bar{1}_V$ for all h ∈ K (where $\bar{1}_V$: X → 1 such that $\bar{1}_V$ (x)c = 1 if x ∈ V and $\bar{1}_V$ (x) = 0 if x ∉ V)

Theorem 2.8; Majeed (2018): Let (X, θ, K) be a ČF-scs,V ⊊ X and let θ_V: F_{ss}(V, K)→F_{ss}(V, K) defined as θ_V (λ_A) = $\bar{V}_K$ ∩ θ (λ_A) Then θ_V is a ČF-sco. The triple (V,θ_V, K) is said to be Č ech fuzzy soft closure subspace (ČF-sc subspace, for short) of (X, θ, K).

RESULTS AND DISCUSSION

Some properties of Čech fuzzy soft closure spaces: In this study, some properties of ČF-scs's are studied such as the union and intersection of two ČF-fscos θ_1, $\theta_2$on X. In addition, a very important result is obtained, that is, every ČF-scs (X, θ, K) (respectively, ČFss-sc subspace (V, θ_V, K) of (X, θ, K)) gives a parameterized family of ČF-fcs's (respectively, ČF-fc subspaces of Č-fcs) in the sense by Mashhour and Ghanim (1985), we give an example to show the converse does not hold. Finally, the properties of fuzzy soft neighborhood of a fuzzy soft point are investigated. Firstly, we show the union of two F-scos θ_1, θ_2 on X is also F-scos on X.

Theorem 3.1: Let (X, θ_1, K) and (X, θ_2, K) be ČF-scs's over the same universe X and the set of parameters K. Definite θ_1, ∪ θ_2: F_{ss}(X, K) → F_{ss}(X, K) given by for each λ_A ∈ F_{ss}(X, K):

$$(\theta_1 \cup \theta_2)(\lambda_A) \cup \theta_2(\lambda_A) \qquad (1)$$

Then θ_1, ∪ θ_2 is a Č-fsco on X and (X, θ_1, ∪ θ_1, K) is a ČF-scs.

Proof: We must show $\theta_1,\cup\theta_2$ satisfies the axioms (C1-C3) of Definition 2.1 Now:

- (C1) $(\theta_1, \cup \theta_1)(\bar{0}_K) = \theta_1(\bar{0}_K) \cup \theta_2(\bar{0}_K) = \theta_1(\bar{0}_K)$
- (C2) Let $\lambda_A \in F_{ss}(X, K)$ Since, $\theta_1$and θ_2 are Č-fsco'son X. This implies $\lambda_A \subseteq$ and $\lambda_A \subseteq \theta_2(\lambda_A)$. It follows $\lambda_A \subseteq \theta_1(\lambda_A) \cup \theta_2(\lambda_A) = (\theta_1 \cup \theta_2)(\lambda_A)$
- (C3) Let λ_A, $\mu_B \in F_{ss}(X, K)$. Then

$$\begin{aligned}(\theta_1 \cup \theta_2)(\lambda_A \cup \mu_B) &= \theta_1(\lambda_A \cup \mu_B) \cup \theta_2(\lambda_A \cup \mu_B)\\ &= (\theta_1(\lambda_A) \cup \theta_1(\mu_B)) \cup (\theta_2(\lambda_A) \cup \theta_2(\mu_B))\\ &= (\theta_1(\lambda_A) \cup \theta_2(\lambda_A)) \cup (\theta_1(\mu_B)) \cup \theta_2(\mu_B))\\ &= (\theta_1 \cup \theta_2)(\lambda_A) \cup (\theta_1 \cup \theta_2)(\mu_B)\end{aligned}$$

Hence, $\theta_1 \cup \theta_2$ is a Č-fsco on X. Thus, $(X, \theta_1, \cup\theta_1, K)$ is a ČF-scs

Remark 3.2: The Intersection of two ČF-fscos on X need not to be ČF-fsco on X. The following example clarify that.

Example 3.3: Let X = {a, b}and let $(\lambda_A)_1, (\lambda_A)_2 \in F_{ss}(X, K)$ such that $(\lambda_A)_1 = \{(h_1, a_{0.2})\}$ and $(\lambda_A)_2 = \{(h_2, a_{0.3}) \vee b_{0.4}\}$ Define Č-fscos $\theta_1 \cup \theta_2$ $F_{ss}(X, K)$ as follows:

$$\theta_1(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K,\\ \{(h_1, a_{0.3})\} & \text{if } \lambda_A \subseteq (\lambda_A)_1 \\ \bar{1}_K & \text{otherwise}\end{cases}$$

And:

$$\theta_2(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K,\\ \{(h_2, a_{0.3} \vee b_{0.5})\} & \text{if } \lambda_A \subseteq (\lambda_A)_2 \\ \bar{1}_K & \text{otherwise}\end{cases}$$

Then, (X, θ_1, K) and (X, θ_2, K) are Č-scs's on X. Define $\theta_1 \cap \theta_2$ $F_{ss}(X, K)$ as follows: $(\theta_1 \cap \theta_2)(\lambda_A) = \theta_1(\lambda_A) \cap \theta_2(\lambda_A)$, for all $\lambda_A \in F_{ss}(X, K)$. It is clear that $\theta_1 \cap \theta_2$ satisfies the axioms (C1) and (C2) of Definition 2.1, however $\theta_1 \cap \theta_2$ does not satisfies the axiom (C3). That is if we consider $(\lambda_A)_1 (\lambda_A)_2$ $F_{ss}(X, K)$, then $(\theta_1 \cap \theta_2)((\lambda_A)_1 \cup (\lambda_A)_2) = \bar{1}_K$ and $(\theta_1 \cap \theta_2)((\lambda_A)_1) \cup (\theta_1 \cap \theta_2)(\lambda_A)_2) = \{(h_1, a_{0.3})\}$. $\{(h_2, a_{0.3}) \vee b_{0.5}\}$. Hence, $(\theta \cap \theta_2)((\lambda_A)_1) \cup (\lambda_A)_2) \neq (\theta \cap \theta_2)((\lambda_A)_1) \cup (\theta_1 \cap \theta_2)((\lambda_A)_2)$.

Remark 3.4: As we mentioned that $F_{ss}(X, K)$ denoted to the family of all fuzzy soft sets which are defined on the universe setX and a set of parameters K. That means$F_{ss}(X, K) = \{\lambda_A, \lambda_A: K \to 1_X\}$. It follows That, for each parameter $h \in K$, the family $\{\lambda_A(h), \lambda_A \in F_{ss}(X, K)\}$ 1^X. The following theorem show that each ČF-scs gives a parameterized family of ČF-fcss in the sense of Mashhour and Ghanim (1985).

Theorem 3.5 Let (X, θ, K) be a ČF-scs. For each $h \in K$, define $\theta_h: I^X \to I^X$ as follows :

$$\theta_h(\mu) = \inf_{\mu = (\lambda_A)(h)} \{(\theta(\lambda_A))(h)\} \tag{2}$$

for each $\mu \in I^X$. Then θ_h is a ČF-fco on X.

Proof: For any $h \in K$, we must show θ_h satisfies the axioms of ČF-co in (Mashhour and Ghanim, 1985), (C1) since, θ is a ČF-sco, then by (C1) of definition 2.1, we have $\theta(\bar{0}k)$. That is mean $(\theta(\bar{0}_K))(h) = \bar{0}$ for each $h \in K$. This implies $\theta_h(\bar{0}) = \inf_{\bar{0} = (\lambda_A)(h)} \{(\theta(\lambda_A))(\ )\} = \bar{0}$.

(C2) We must show $\mu \leq \theta_h(\mu)$ for all $\mu \in I^X$. Since, θ is a ČF-sco, then by (C2) of definition 2.1, we have $\lambda_A \subseteq \theta(\lambda_A)$ for each $\lambda_A \in F_{ss}(X, K)$. This mean $(\lambda_A)(h) \leq (\theta(\lambda_A))(h)$ for all $h \in K$. So, for any $\mu \in I^X$, such that $\mu = (\lambda_A)(h)$ we have $\mu \leq \inf_{\mu = (\lambda_A)(h)} \{(\theta(\lambda_A))(h)\}$. Thus, $\mu \leq \theta_h(\mu)$.

(3) Let $\mu, \rho \in I^X$. Then:

$$\begin{aligned}&\theta_h(\mu) V \theta_h(\rho) = \inf_{\mu = (\lambda_A)(h)}\\ &\{(\theta(\lambda_A))(h)\} V \inf_{\rho = (\mu_B)(h)} \{(\theta(\mu_B))(h)\} =\\ &\inf_{\mu V \rho = (\theta(\lambda_A))(h) V (\theta(\mu_B))(h)} \{(\theta(\lambda_A))(h) V (\theta(\mu_B))(h)\} =\\ &\inf_{\mu V \rho = (\theta(\lambda_A))(h) V (\theta(\mu_B))(h)} \{(\theta(\lambda_A) U \theta(\mu_B))(h)\} = \theta_h(\mu v p)\end{aligned}$$

Thus, θ_h is a ČF-co on X for each $h \in K$. Hence, (X, θ_h) is a ČF-cs on X for each $h \in K$.

Remark 3.6: Now we discuss the converse of above theorem is it hold? That mean, if we have a collection of ČF-cos θ_{hi} on X such that each one corresponding a parameter $h \in K$. Is this collection gives a ČF-sco on X. We answer about this question through the following example.

Example 3.7: Let X = {a, b, c}, K = {h_1, h_2} and let $\lambda_A, \mu_B \in F_{ss}(X, K)$ such that $\lambda_A = \{(h_1, b_1 v\ c_1), (h_1, a_1 v\ b_1)\}$ and $\mu_B = \{(h_1, a_1), (h_2, a_1 v\ b_1)\}$. Define Č-fco's θ_{h1}, θ_{h2}: $I^X \to I^X$ as follows:

$$\theta_{h1}(\mu) = \begin{cases} \bar{0} & \text{if } \mu = \bar{0},\\ b_1 V c_1 & \text{if } \mu \leq (\lambda_A)(h_1),\\ a_1 & \text{if } \mu \leq (\mu_B)(h_1),\\ \bar{1} & \text{otherwise}\end{cases}$$

And:

$$\theta_{h2}(\mu)=\begin{cases}\bar{0} \text{ if } \mu=\bar{0},\\ a_1 V b_1 \text{ if } \mu\leq(\lambda_B)(h_2),\\ \bar{1} \text{ otherwise}\end{cases}$$

Then (X, θ_{h1}) and (X, θ_{h1}) are ČF-fcss on X. Now, define fuzzy soft closure operator θ: F_{ss} (X, K)→F_{ss} (X, K) as follows:

$$\theta(\rho_C)=\{(h_1,\theta_{h_1}((\rho_C)(h_1))),(h_2,\theta_{h_2}((\rho_C)(h_2)))\}$$

This implies:

$$\theta(\rho_C)=\begin{cases}\bar{0}_K & \text{if } \rho_C=\bar{0}_K,\\ \{(h_1,b_1Vc_1),(h_2,a_1Vb_1)\} & \text{if } \rho_C\subseteq\lambda_A,\\ \{(h_1,a_1),((h_2,a_1Vb_1))\} & \text{if } \rho_C\subseteq\mu_B,\\ \bar{1}_K & \text{otherwise}\end{cases}$$

The fuzzy soft operator θ is not Č-sco, since:

$$\theta(\lambda_A\cup\mu_B)=\bar{1}_K\neq$$
$$\{(h_1,a_1Vb_1Vc_1),(h_2,a_1Vb_1)\}=\theta(\lambda_A)\cup\theta(\mu_B)$$

Theorem 3.8: Let (X, θ, K) be a ČF-scs and $\lambda_A \in F_{ss}$ (X, K). If λ_A is a closed-(respectively, open-) fss in (X, θ, K) then (λ_A) (h) is aclosed (respectively, open) fuzzy set in (X, θ_h), for all h∈K.

Proof: Let λ_A by a closed-fss in (X, θ, K). Then $\theta(\lambda_A) = \lambda_A$ which means ($\theta(\lambda_A)$) (h) = (λ_A) (h) for all h∈K. Now, we want to prove (λ_A) (h) is aclosed fuzzy set in (X, θ_h). So, we must prove ($\theta_h(\lambda_A)$) (h) = (λ_A) (h). Now:

$$\theta_h((\lambda_A)(h))=\inf_{(\lambda_A)(h)=(\mu_B)(h)}\{(\theta(\mu_B))(h)\}=$$
$$\inf_{(\lambda_A)(h)=(\mu_B)(h)}\{(\theta(\lambda_A))(h),(\theta(\mu_B))(h)\}=$$
$$\inf_{(\lambda_A)(h)=(\mu_B)(h)}\{(\lambda_A)(h),(\theta(\mu_B))(h)\}=$$
$$(\lambda_A)(h),\text{because}(\theta(\lambda_A))(h)=$$
$$(\lambda_A)(h)\text{and}(\mu_B)(h)\subseteq(\theta(\mu_B)(h))$$

Hence, (λ_A) (h) is a closed fuzzy set in (X, θ_h) for all h∈K.

Now, if λ_A is an open-fss. Then $\theta(\bar{1}_K-\lambda_A)=\bar{1}_K-\lambda_A$. By using the same idea of proof the first part we get the result.

Theorem 3.9: Let (X, θ, K) be a ČF-scs, let V⊆X and (V, θ_V, K) be a ČF-sc subspace of (X, θ, K). Then (V, $(\theta_v)(_h)$) is a Č-fc subspace of a Č-fcs (X, θ_h) for each h∈K.

Proof: For each h∈K. Define $(\theta_V)_h$:I^V→I^V such that (μ) for each $\mu\in I^V$

$$(\theta_v)_h(\mu)=\bar{1}_v\wedge\theta_h(\theta)$$

To show (θ_V)h satisfies (C1-C3) of the definition of ČF-co, (C1):

$$(\theta_v)_h(0)=\bar{1}_v\wedge\theta_h(\bar{0})=\bar{1}_v\wedge\bar{0}=\bar{0}$$

(C2) Let $\mu\in I_V$. Then $\mu\leq\bar{1}_V$ and $\mu\leq\theta_h$ (μ). This implies $\mu\leq\bar{1}_V\wedge\theta_h(\mu)=(\theta_V)_h(\mu)$ Hence, $\mu\leq(\theta_V)_h(\mu)$. (C3) Let μ_1 $\mu_2\in I^V$, then:

$$(\theta_V)_h(\mu_1V\mu_2)=\bar{1}_V\wedge\theta_h(\mu_1V\mu_2)=$$
$$\bar{1}_V\wedge(\theta_h(\mu_1)V\theta_h(\mu_2))=$$
$$(\bar{1}_V\wedge\theta_h(\mu_1))V(\bar{1}_V\wedge\theta_h(\mu_2))=$$
$$(\theta_V)_h(\mu_1)V(\theta_V)_h(\mu_2)$$

Hence, $(\theta_V)_h$ is a ČF-co operator on V and (V,$(\theta_V)_h$) is a Č-fc subspace of a ČF-cs (X,θ_h) for each h∈K.

Now, some properties of fuzzy soft neighborhood of a fuzzy soft point are given in the next theorem and propositions.

Theorem 3.10: Let (X, θ, K) be a ČF-scs $\lambda_A\in F_{ss}$ (X, K) and x_t^h be a fuzzy soft pointover X. Then:

- Each $x_t^h\in F_{ss}$ (X, K) has a fuzzy soft neighborhood
- If λ_A and μ_B are fuzzy soft neighborhood of x_t^h, then $\lambda_A\cap\mu_B$ is also, a fuzzy soft neighborhood of x_t^h
- If λ_A is fuzzy soft neighborhood of x_t^h and $\lambda_A\subseteq\mu_B$, then μ_B is also, a fuzzy soft neighborhood of x_t^h

Proof: For any $x_t^h\in F_{ss}$ (X, K), $x_t^h \tilde{\in} \bar{1}_K$ and by proposition 2.4 (1), we have $x_t^h\tilde{\in}\text{Int}(\bar{1}_K)$. Thus, $\bar{1}_K$ is fuzzy soft neighborhood for any x_t^h. Let λ_A and μ_B are fuzzy soft neighborhoods of x_t^h then $x_t^h\tilde{\in}\text{Int}(\lambda_A)$ and $x_t^h\tilde{\in}\text{Int}(\mu_B)$. This implies $x_t^h\tilde{\in}\text{Int}(\lambda_A)$. By proposition 2.4 (3) we get $x_t^h\tilde{\in}\text{Int}(\lambda_A\cap\mu_B)$. This implies $\lambda_A\cap$ μ_B is a fuzzy soft neighborhood of x_t^h. Let λ_A be a fuzzy soft neighborhood of x_t^h and $\lambda_A\subseteq\mu_B$. Then $x_t^h\tilde{\in}\text{Int}(\lambda_A)$. Since, $\lambda_A\subseteq\mu_B$ then by Proposition 2.4(4), Int (λ_A)⊆Int (μ_B). Thus, $x_t^h\tilde{\in}\text{Int}(\mu_B)$. Hence, μ_B is a fuzzy soft neighborhood of x_t^h.

Proposition 3.11: Let (X, θ, K) be a ČF-sec.-scs. For any open-fss λ_A in (X, θ, K), λ_A is a fuzzy soft neighborhood of each $x_t^h \tilde{\in} \lambda_A$.

Proof: Let λ_A be an open-fss. Then Int (λ_A), this yield for each $x_t^h \tilde{\in} \lambda_A$ we have $x_t^h \tilde{\in} \mathrm{Int}(\lambda_A)$. Therefore, λ_A is a fuzzy soft neighborhood for each $x_t^h \tilde{\in} \lambda_A$.

Proposition 3.12: Let (X, θ, K) be a ČF-sec, $\lambda_A \in F_{ss}$ (X, K) and x_t^h be a fuzzy soft point over X.If x_t^h a fuzzy soft interior point of λ_A then x_t is a fuzzy interior point of (λ_A) (h) in (X, θ_h) for each h∈K.

Proof: For each h∈K, (λ_A) (h) $\in I^x$ If x_t^h is a fuzzy soft interior point of λ_A, then there exists an open-fss μ_B in (X, θ, K) such that $x_t^h \tilde{\in} \mu_B \subseteq \lambda_A$. This means that $x_t \in$ (μ_B) (h) ⊆ (λ_A) (h). Since, μ_B is an open-fss in (X, θ, K). Then by theorem 3.8 (μ_B) (h) is an open fuzzy set in (X, θ_h) and $x_t \in$ (μ_B) (h). This implies x_t is a fuzzy interior point of (λ_A) (h) in (X, θ_h).

Fuzzy soft exterior and fuzzy soft boundary of fuzzy soft sets inech fuzzy soft closure spaces: This section is devoted to define and investigate the notion of fuzzy soft exterior (respectively, boundary) of fuzzy soft sets in ČF-scs's and give the relationships between them and the ČF ech fuzzy soft closure θ, (respectively, interior Int) of fuzzy soft sets, this study supported by several examples.

Definition 4.1: Let (X, θ, K) be a ČF-css and $\lambda_A \in F_{ss}$ (X, K). The fuzzy soft exterior of λ_A, denoted by ext λ_A is defined as:

$$\mathrm{ext}(\lambda_A) = \mathrm{Int}(\bar{1}_K - \lambda_A) \tag{3}$$

Definition 4.2: Let (X, θ, K) be a ČF-scs and $\lambda_A \in F_{ss}$ (X, K). A fuzzy soft point x_t^h is called a fuzzy soft exterior point of λ_A, if there exists $\mu_B \in F_{ss}$ (X, K) such that $x_t^h \tilde{\in}$ Int (μ_B)⊆ $\bar{1}_K - \lambda_A$. Some basic properties of fuzzy soft exterior are given in the next proposition.

Proposition 4.3: Let (X, θ, K) be a ČF-scs and λ_A $\mu_B \in F_{ss}$ (X, K). Then:

- If $\lambda_A \subseteq \mu_B$, then ext ($\mu_B$)⊆ext ($\lambda_A$)
- ext ($\lambda_A \cup \mu_B$) = ext (λ_A)∩ext (μ_B)
- ext (λ_A)∪ ext (μ_B) ⊆ ext ($\lambda_A \cap \mu_B$)
- λ_A is a closed-fss if and only if (λ_A) = $\bar{1}_K - \lambda_A$

Proof: Let $\lambda_A \mu_B \in F_{ss}$ (X, K) such that $\lambda_A \subseteq \mu_B$. Then $\bar{1}_K - \mu_B \subseteq \bar{1}_K - \lambda_A$ by Proposition 2.4(4), we have Int ($\bar{1}_K - \mu_B$) ⊆Int ($\bar{1}_K - \lambda_A$) which is mean ext (μ_B) ⊆ext (λ_A).

$$\mathrm{ext}(\lambda_A \cup \mu_B) = \mathrm{Int}(\bar{1}_K - (\lambda_A \cup \mu_B)) =$$
$$\mathrm{Int}(\bar{1}_K - \lambda_A \cap \bar{1}_K - \mu_B) = \mathrm{Int}(\bar{1}_K - \lambda_A) \cap \mathrm{Int}(\bar{1}_K - \mu_B) =$$
$$\mathrm{Int}(\bar{1}_K - \lambda_A) \cap (\bar{1}_K - \mu_B) = \mathrm{ext}(\lambda_A) \cap \mathrm{ext}(\mu_B)$$

$$\mathrm{ext}(\lambda_A) \cup \mathrm{ext}(\mu_B) = \mathrm{Int}(\bar{1}_K - \lambda_A) \cup \mathrm{Int}(\bar{1}_K - \mu_B) \subseteq$$
$$\mathrm{Int}(\bar{1}_K - \lambda_A \cup \bar{1}_K - \mu_B) = \mathrm{Int}(\bar{1}_K - (\lambda_A \cap \mu_B)) =$$
$$\mathrm{ext}(\lambda_A \cap \mu_B)$$

Suppose λ_A is a closed-fss, then $\bar{1}_K - \lambda_A$ is an open-fss. Now, ext (λ_A) = Int $(\bar{1}_K - \lambda_A) = \bar{1}_K - \lambda_A$. Conversely, suppose ext (λ_A) $\bar{1}_K - \lambda_A$. To prove (λ_A) is a closed-fss. From (Eq. 4.1) and hypothesis, we get ext (λ_A) = Int $(\bar{1}_K - \lambda_A) = \bar{1}_K - \lambda_A$. That is mean $\bar{1}_K - \lambda_A$ an open-fss which implies λ_A is a closed-fss.

Remark 4.4: Let (X, θ, K) be a ČF-sec and $\lambda_A \in F_{ss}$ (X, K) then:

- ext (λ_A)∩Int (λ_A)≠ $\bar{0}_K$
- The equality of proposition Eq. (4.3) and (3) is not true in general

The following example explain remark Eq. (4.4) (1).

Example 4.5: Let X = {a, b}, K = {h_1, h_2}. Define $(\lambda_A)_1$, $(\lambda_A)_2$, $(\lambda_A)_3$ and $(\lambda_A)_4 \in F_{ss}$ (X, K) as follows:

$$(\lambda_A)_1 = \{(h_1, a_{0.5} v b_{0.2}), (h_2, a_{0.5} v b_{0.7})\}$$
$$(\lambda_A)_2 = \{(h_1, a_{0.5} v b_{0.8}), (h_2, a_{0.5} v b_{0.3})\}$$
$$(\lambda_A)_3 = \{(h_1, a_{0.5} v b_{0.2}), (h_2, a_{0.5} v b_{0.3})\}$$
$$(\lambda_A)_4 = \{(h_1, a_{0.5} v b_{0.8}), (h_2, a_{0.5} v b_{0.7})\}$$

Define Č-fsco θ: F_{ss} (X, K)→F_{ss} (X, K) as follows:

$$\theta(\lambda_A) = \begin{cases} \bar{0}_K & \text{if } \lambda_A = \bar{0}_K \\ (\lambda_A)_1 & \text{if } (\lambda_A)_3 \subset \lambda_A \subset (\lambda_A)_1, \\ (\lambda_A)_2 & \text{if } (\lambda_A)_3 \subset \lambda_A \subset (\lambda_A)_2, \\ \lambda_A & \text{if } \lambda_A \subseteq (\lambda_A)_3, \\ (\lambda_A)_4 & \text{if } \lambda_A = \lambda_A \subset (\lambda_A)_1, (\lambda_A)_2, (\lambda_A)_4, \\ \bar{1}_K & \text{otherwise} \end{cases}$$

Consider $(\lambda_A)_1$ then Int ($(\lambda_A)_1$) = {(h_1, $a_{0.5}$, $Vb_{0.2}$), (h_2, $a_{0.5}$, $Vb_{0.3}$)} and ext ($(\lambda_A)_1$) = {(h_1, $a_{0.5}$, $Vb_{0.2}$), (h_2, $a_{0.5}$, $Vb_{0.3}$)}. Thus its clear that Int (λ_A)∩ext (λ_A)≠ $\bar{0}_K$. The next example explain remark Eq. (4.4) (2).

Example 4.6: Let X = {a, b}, K = {h_1, h_2}. Define Č-fsco θ: F_{ss} (X, K)→F_{ss} (X, K) as follows:

$$(\lambda_A)=\begin{cases}\bar{0}_K & \text{if } \lambda_A=\bar{0}_K,\\ \{(h_1,a_{t_1}Vb_1)\} & \text{if } \lambda_A=\{(h_1,a_{t_1}):t_1\in I_0\},\\ \{(h_1,a_1Vb_2)\} & \text{if } \lambda_A=\{(h_1,b_{t2}):t_2\in I_0\},\\ \{(h_1,a_1Vb_1)\} & \text{if } \lambda_A=\{(h_1,a_tVb_{t2}):t_1,t_2\in I_0\},\\ \{(h_2,a_{t_3}Vb_1)\} & \text{if } \lambda_A=\{(h_1,a_{t_3}):t_3,\in I_0\},\\ \{(h_2,a_1Vb_{t_4})\} & \text{if } \lambda_A=\{(h_2,a_{t_4}):t_3,\in I_0\},\\ \{(h_1,a_1Vb_1)\} & \text{if } \lambda_A=\{(h_2,a_{t_3}Vb_{t_4}):t_3,t_4\in I_0\},\\ \theta(h_1,\lambda_A(h_1))\cup\theta(h_1,\lambda_A(h_1)) & \text{otherwise}\end{cases}$$

Then, (x, θ, k) is a ČF-scs. Let, λ_A = {(h_1, $a_{0.6}$)} and μ_B = {(h_1, $a_{0.6}$)}. Then, ext ($\lambda_A \cap \mu_B$) = $\bar{1}_K$. On the other hand ext (λ_A) = {(h_1, $a_{0.4}$), (h_2, a_1Vb_{t4})} and ext (μ_B) = {(h_1, $a_{0.4}$), (h_2, a_1Vb_1)}. Thus, It is clear that ext ($\lambda_A \cap \mu_B$) = $\bar{1}_K \not\subset$ {(h_1, $a_{0.4}Vb_{0.4}$), (h_2, a_1Vb_1)} = ext (λ_A) ∪ ext (μ_B).

Definition 4.7: Let (×, θ, k) be a ČF-scs and $\lambda_A \in F_{ss}$ (X, K). The fuzzy soft boundary of λ_A denoted by λ_A is defined as:

$$\text{Bd}(\lambda_A)=\theta(\lambda_A)\cap\theta(\bar{1}_K\text{-}\lambda_A) \tag{4}$$

The next remark includes several deviations for some results in exterior and boundary that are hold in both ordinary and soft topological spaces but not in ČF-scs's.

Remark 4.8: Let (x, θ, k) be a ČF-scs and $\lambda_A \in F_{ss}$ (X, K). Then:

- Bd (λ_A)∩ext (λ_A) ≠ $\bar{0}_K$
- Bd (λ_A)∩Int (λ_A) ≠ $\bar{0}_K$
- θ (λ_A)≠ Bd (λ_A)∪Int (λ_A)
- θ (Bd (λ_A))≠ Bd (λ_A)

Next, we introduce several examples to explain the above remark. First we give an example to explain part 1 and 2 in remark 4.8.

Example 4.9: In Example 4.5. Consider the fuzzy soft set $(\lambda_A)_1$. Then Bd $(\lambda_A)_1$ = {(h_1, $a_{0.5}Vb_{0.8}$), (h_2, $a_{0.5}Vb_{0.7}$)} and ext $((\lambda_A)_1)$ = {(h_1, $a_{0.5}Vb_{0.8}$), (h_2, $a_{0.5}Vb_{0.3}$)} Then it follows Bd $((\lambda_A)_1)$∩Int $((\lambda_A)_1)$≠ $\bar{0}_K$. Also, Int $((\lambda_A)_1)$ = {(h_1, $a_{0.5}Vb_{0.2}$), (h_2, $a_{0.5}Vb_{0.3}$)}. Thus, it is clear that Bd $((\lambda_A)_1)$∩Int $((\lambda_A)_1)$ ≠ $\bar{0}_K$. The next example explain part (3) in remark 4.8.

Example 4. 0: In example 4.6. Let λ_A = {(h_1, $a_{0.5}Vb_1$), (h_2, a_1Vb_1)} ∈ F_{ss} (X, K). Then θ (λ_A) = $\bar{1}_K$, Bd (λ_A) = {(h_1, $a_{0.5}Vb_1$)} and Int (λ_A) = {(h_1, $a_{0.5}$), (h_2, a_1Vb_1)}. This implies Bd (λ_A)∪Int (λ_A) = {(h_1, $a_{0.5}Vb_1$), (h_2, a_1Vb_1)} which is not equal toθ (λ_A) = $\bar{1}_K$. Finally, we introduce an example to explain part (4) in remark 4.8.

Example 4.11: Let X = {a, b}, K {h_1, h_2}. Define ČF-scoθ: F_{ss} (X, K)→F_{ss} (X, K) as follows:

$$\theta(\lambda_A)=\begin{cases}\bar{0}_K & \text{if } \lambda_A=\bar{0}_K,\\ \{(h_1,a_{0.4})\} & \text{f } \lambda_A\subseteq\{(h_1,a_{0.3})\}\\ \bar{1}_K & \text{otherwise}\end{cases}$$

Then, (X, θ, K) be ČF-scs. Let λ_A = {(h_1, $a_{0.7}Vb_1$), (h_2, a_1Vb_1)}. Then Bd (λ_A) = {(h_1, $a_{0.4}$)}. And θ (Bd (λ_A)) = $\bar{1}_K$ ≠ Bd (λ_A). Some properties of fuzzy soft boundary in ČF-scs's are introduced in the next.

Proposition 4.12: (X, θ, K) be a ČF-scs and $\lambda_A \in F_{ss}$ (X, K). Then:

- $\bar{1}_K$ -Bd (λ_A) = Int ($\bar{1}_K$ -λ_A)∪ Int (λ_A)
- Bd (λ_A∪ μ_B)⊆ Bd (λ_A) ∪ Bd (μ_B)
- Bd (λ_A∩ μ_B)⊆ {Bd (λ_A)} ∩ {Bd (μ_B)∩ θ(λ_A)}

Proof:

$$\bar{1}_K\text{-Bd}(\lambda_A)=\bar{1}_K\text{-}\left(\theta(\lambda_A)\cap\theta(\bar{1}_K\text{-}\lambda_A)\right)=$$
$$\left(\bar{1}_K\text{-}\theta(\lambda_A)\right)\cup\left(\bar{1}_K\text{-}\theta(\bar{1}_K\text{-}\lambda_A)\right)=\bar{1}_K\text{-}$$
$$\left(\bar{1}_K\text{-}\left(\text{Int}(\bar{1}_K\text{-}\lambda_A)\right)\right)\cup\text{Int}(\lambda_A)=$$
$$\text{Int}(\bar{1}_K\text{-}\lambda_A)\cup\text{Int}(\lambda_A)$$

By using the definition of the fuzzy soft boundary, we have:

$$\text{Bd}(\lambda_A\cup\mu_B)=\theta(\lambda_A\cup\mu_B)\cap\theta\left((\bar{1}_K\text{-}\lambda_A)\cap(\bar{1}_K\text{-}\mu_B)\right)=$$
$$\{\theta(\lambda_A)\cup\theta(\mu_B)\}\cap\theta\left((\bar{1}_K\text{-}\lambda_A)\cap(\bar{1}_K\text{-}\mu_B)\right)=$$
$$\subseteq\{\theta(\lambda_A)\cup\theta(\mu_B)\}\cap\{\theta(\bar{1}_K\text{-}\lambda_A)\cap\theta(\bar{1}_K\text{-}\mu_B)\}=$$
$$\{\theta(\lambda_A)\cap\theta\{(\bar{1}_K\text{-}\lambda_A)\cap\theta(\bar{1}_K\text{-}\mu_B)\}\}\cup$$
$$\{\theta(\mu_B)\cap\{\theta(\bar{1}_K\text{-}\lambda_A)\cap\theta(\bar{1}_K\text{-}\mu_B)\}\}$$
$$\{\text{Bd}(\lambda_A)\cap\theta(\bar{1}_K\text{-}\lambda_A)\}\cup\{\text{Bd}(\mu_B)\cap\theta(\bar{1}_K\text{-}\lambda_A)\}$$
$$\subseteq\text{Bd}(\lambda_A)\cup\text{Bd}(\mu_B)$$

Similar of part (2).

Theorem 4.13: Let (X, θ, K) be a ČF-scs and (λ_A) ∈ F_{ss} (X, K). If λ_A∩Bd (λ_A) = $\bar{0}_K$ then λ_A is an open-fss.

Proof: Let $\lambda_A \in F_{ss}$ (X, K), we must prove θ ($\bar{1}_K$ -λ_A) = $\bar{1}_K$ -λ_A. Since, $\lambda_A \cap$Bd (λ_A) = $\bar{0}_K$, then $\lambda_A \cap(\lambda_A)\cap\theta$ ($\bar{1}_K$ -λ_A) = $\bar{0}_K$. This implies $\lambda_A \cap \theta$ ($\bar{1}_K$ -λ_A) = $\bar{0}_K$ which means θ ($\bar{1}_K$ -λ_A)$\subseteq$ $\bar{1}_K$ -λ_A. On the other hand $\bar{1}_K$ -$\lambda_A \subseteq \theta$ ($\bar{1}_K$ -λ_A). It follows, θ ($\bar{1}_K$ -λ_A) = $\bar{1}_K$ - λ_A and hence we get λ_A is an open-fss. The converse of Theorem 4.13 is not true in general as the following example show.

Example 4.14: In Example 4.5. Consider the fuzzy soft set $(\lambda_A)_3$ = {(h_1, $a_{0.5}$V$b_{0.2}$), (h_2, $a_{0.5}$V$b_{0.3}$)}. Then $(\lambda_A)_3$ is an open-fssbecause Int $(\lambda_A)_3$ = $(\lambda_A)_3$. But $(\lambda_A)_3 \cap$Bd ($(\lambda_A)_3$) = $\lambda_A)_3 \neq \bar{0}_K$.

Theorem 4.15: Let (X, θ, K) be a ČF-scs and $\lambda_A \in F_{ss}$ (X, K). If λ_A is a cosed-fss, then Bd (λ_A) $\subseteq$ λ_A.

Proof: Let λ_A be a closed-fss, then θ (λ_A) = λ_A. Now, Bd (λ_A) = $\cap\theta$ ($\bar{1}_K$ λ_A)$\subseteq\theta$(λ_A) = λ_A. That is Bd (λ_A) $\subseteq$ λ_A. The converse of the above theorem is not hold as we show in the next example.

Example 4.16: In example 4.5. Let λ_A = {(h_1, $a_{0.9}$V$b_{0.9}$), (h_2, $a_{0.9}$V$b_{0.9}$)} $\in F_{ss}$ (X, K). Then Bd λ_A = {(h_1, $a_{0.1}$V$b_{0.1}$), (h_2, $a_{0.1}$V$b_{0.1}$)} $\subset$ λ_A. But λ_A is not a closed-fss in X because θ (λ_A) = $\bar{1}_K \neq \lambda_A$.

Theorem 4.17: Let (X, θ, K) be a ČF-scs and $\lambda_A \in F_{ss}$ (X, K). If Bd (λ_A), then λ_A is a closed-fss in X.

Proof: Let $\lambda_A \in F_{ss}$ (X, K). First we prove that λ_A is a closed-fss. Since, Bd (λ_A) = $\bar{0}_K$ implies θ (λ_A)$\cap\theta$ ($\bar{1}_K$ -λ_A) $\bar{0}_K$. It follows θ (λ_A)$\subseteq$ $\bar{1}_K$ -θ ($\bar{1}_K$ -λ_A) = Int (λ_A) $\subseteq$ λ_A. Thus we get θ (λ_A)$\subseteq\lambda_A$ and from (C2) of Definition 2.1, $\lambda_A\subseteq\theta$ (λ_A). Hence, θ (λ_A) = λ_A and this mean λ_A is a closed-fss.

Now, we prove λ_A is an open-fss. Since, Bd (λ_A) = $\bar{0}_K$, then θ (λ_A)$\cap\theta$ ($\bar{1}_K$ -λ_A) = $\bar{0}_K$, implies θ (λ_A)$\cap\theta$ $\bar{1}_K$ -Int (λ_A) = $\bar{0}_K$ (by applying θ($\bar{1}_K$ -λ_A) = $\bar{1}_K$ Int (λ_A)). From the prove of the first part of theorem we have λ_A is a closed-fss, this implies $\lambda_A\cap$ $\bar{1}_K$ -Int (λ_A) = Int $\bar{0}_K$ which implies $\lambda_A\cap$ $\bar{1}_K$ Int (λ_A). Hence, λ_A = Int (λ_A). So, λ_A is an open-fssin X. Thus, λ_A is a open-fssin X . The converse of above Theorem is not hold. The next example explain that.

Example 4.17: In example 4.5. Consider the fuzzy soft set $(\lambda_A)_3$ = {(h_1, $a_{0.5}$V$b_{0.2}$), (h_2, $a_{0.5}$V$b_{0.3}$)}. It is clear that $(\lambda_A)_3$ is a closed-fss in X. But Bd $(\lambda_A)_3$ = $(\lambda_A)_3 \neq \bar{0}_K$

CONCLUSION

In addition, some properties of fuzzy soft neighborhood of a fuzzy soft point are introduced. In this study, the notion of fuzzy soft exterior (respectively, boundary) of a fuzzy soft sets in ČF-scs is defined and give the properties of its. Also, we find several deviations for some results in exterior and boundary that are hold in both ordinary and soft topological spaces but not in ČF-scs's. These deviations are clarified by giving several examples.

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