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Fuzzy KU-Semi-Groups and Investigate Some Basic Properties

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Abstract: In this research, we introduce the concept of fuzzy KU-semigroup and investigate some basic properties. Also, fuzzy S-ideals, fuzzy k-ideals and fuzzy P-ideals of a KU-semigroup are studied and few important properties are obtained. Furthermore, few results of fuzzy S-ideals of a KU-semigroup under homomorphism are discussed.

Key words: KU-algebra, KU-semigroup, fuzzy S-ideal, fuzzy k-ideals, fuzzy P-ideas, Iraq

INTRODUCTION

Prabpayak and Leerawat (2009 a, b) introduced a new algebraic structure which is called a KU-algebra. They studied a homomorphism of KU-algebras and investigated some properties. Kareem and Elaf (2017) introduced the concept of KU-semigroup and defined some types of ideals in this concept and the relations between these types are discussed. The fuzzy set was initiated by Zadeh (1965). Since, then this concept has been appliedm in many distinct branches of mathematics such as groups, vector space, topological spaces, rings and modules. The notion of fuzzy KU-ideals of KU-algebras was introduced by Mostafa and Kareem (2014) and Mostafa *et al.* (2011) and investigated several basic properties. Many Mathematicians have studied “A fuzzy” for some algebraic structures with semi group (Khan *et al.*, 2014; Roh *et al.*, 2000; Williams and Husain, 2007). In this study, we study fuzzy ideals of KU-semigroup and investigate some properties. We give some relations between fuzzy S-ideals and fuzzy k-ideals. Also, few results of fuzzy S-ideals of a KU-semigroup under homomorphism are discussed. The image and the pre-image of fuzzy k-ideals in KU-semigroups are defined. Finally, the product offuzzy k-ideals to product KU-semigroups are discussed.

Definitions and basic concepts: In this part, we present some definitions and background about a KU-algebra and fuzzy KU-algebra.

Definition 2.1; Prabpayak and Leerawat (2009a, b): Algebra (X,*, 0) is called a KU-algebra if it satisfies the following axioms: (ku_1) $(x*y)*[(y*z))*(x*y)] = 0$, (ku_2) $x*0 = 0$, (ku_3)$0*x = x$, (ku_4)$x*y = 0$ and $y*x = 0$ implies $x = y$ and (ku_5) $x*x = 0$.

On a KU-algebra X, we can define a binary relation ≤ by putting $x\leq y \Leftrightarrow y*x = 0$. Then (X, ≤) is a partially ordered set and 0 is its smallest element. Thus, (X,*,0) satisfies the following conditions. For all x, y, z∈X, we that ($ku_{1\backslash}$) $(y*z)*(x*z)\leq(x*y)$, ($ku_{2\backslash}$) $0\leq x$, ($ku_{3\backslash}$) $x\leq y$, $y\leq x$ implies $x = y$ and ($ku_{4\backslash}$) $y*x\leq x$.

Definition 2.2; Prabpayak and Leerawat (2009): A non-empty subset S of a KU-algebra (X,*, 0) is called KU-sub algebra of X if x*y∈S whenever x, y,∈S.

Definition 2.3; Prabpayak and Leerawat (2009): A non-empty subset I of a KU-algebra (X, *, 0) is called an ideal of X if for any x, y∈X, then:

- 0∈I and
- x*y, x∈I imply that y∈I

Definition 2.4; Kareem and Elaf (2017): A KU-semigroup is anonempty set X with 2 binary operations*, ° and constant 0 satisfying the following axioms:

- (X,*, 0) is a KU-algebra
- (X, ∘) is a semigroup
- The operation ∘ is distributive (on both sides) over the operation*, i.e., $x\circ(y*z) = (x\circ z)$ and $(x*y)\circ z = (x\circ z)*(y\circ z)$ for all x, y, z∈X

Example 2.5; Kareem and Elaf (2017): Let X = {0, 1, 2, 3} be a set. Define*-operation and °-operation by the following tables. Then (X,*,°, 0) is a KU-semigroup.

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*	0	1	2	3
0	0	1	2	3
1	0	0	0	2
2	0	2	0	1
3	0	0	0	0

∘	0	1	2	3
0	0	0	0	0
1	0	1	0	1
2	0	0	2	2
3	0	1	2	3

Definition 2.6; Kareem and Elaf (2017): A non-empty subset A of X is called a sub KU-semigroup of X, if x*y, x∘y∈A for all x, y∈A.

Definition 2.7; Kareem and Elaf (2017): A non-empty subset A of a KU-semigroup X is called an S-ideal of X, if:

- A is an ideal of X
- For all x∈X, a∈A, we have x∘a∈A and a∘x∈A

Definition 2.8; Kareem and Elaf (2017): A non-empty subset A of a KU-semi group X is called a k-ideal of X, if:

- A is an KU-ideal of X
- For all x∈X, a∈A, we have x∘a∈A and a∘x∈A

Definition 2.9; Kareem and Elaf (2017): A non-empty subset A of a KU-semigroup X is called a P-ideal of X, if (p_1) For any x, y, z∈X, z*(x*y)∈A and z*x∈A⇒z*y∈A. (p_2) For all x∈X, a∈A, we have x∘a∈A and a∘x∈A.

Definition 2.10; Kareem and Elaf (2017): Let A_1 and A_2 be two S-ideals in a KU-semigroup X. The product $A_1 \times A_2 = \{(a, b): a \in A_1, b \in A_2\}$ and the binary operations "∘" and "*" on $A_1 \times A_2$ are define by the following: for all $(a_1, b_1), (a_2, b_2) \in A_1 \times A_2$. $(a_1, b_1)\circ(a_2, b_2) = (a_1\circ a_2, b_1\circ b_2)$, $(a_1, b_1)*(a_2, b_2) = (a_1*a_2, b_1*b_2)$.

Definition 2.11; Kareem and Elaf (2017): Let X and be two KU-semigroups. A mapping f: X→X' is called a KU-semigroup homomorphism if f(x*y) = f(x)*f(y) and f(x∘y) = f(x)∘f(y) for all x, y∈X. The set {x∈X: f(x) = 0} is called the kernel of f and denote by Ker f moreover, the set {f(x)∈X': x∈X} is called the image of f and denote by im f.

We review some fuzzy logic concepts. A fuzzy set μ in a set X is a function μ: X→[0, 1]. For t∈[0,1], the set U(μ, t) = {x∈X: μ(x)≥t} is called a level set of μ.

Theorem 2.12; Mostafa *et al.* (2011): In KU-algebra X. The following axioms are satisfied. For all x, y, z∈X:

- x≤y imply y*z≤x*z
- x*(y*z) = y*(x*z) for all x, y, z∈X
- ((y*x)*x)≤y

Definition 2.13; Mostafa *et al.* (2011): Let I be a nonempty subset of a KU-algebra X. Then I is said to be a KU-ideal of X, if (I_1) 0∈I and (I_2) ∀x, y, z∈X, x*(y*z)∈I and y∈I imply that x*z∈I.

Definition 2.14; Mostafa *et al.* (2011): A fuzzy set μ in a KU-algebra X is called a fuzzy sub-algebra of X if μ(x*y)≥ min {μ(x), μ(y)} for all x, y∈X.

Definition 2.15; Mostafa *et al.* (2011): Let X be a KU-algebra, a fuzzy set μ in X is called a fuzzy ideal of X if it satisfies the following conditions. (f_1)μ(0)≥μ(x) for all x∈X and (f_2)∀x, y∈X, μ(y)≥ min {μ(x*y), μ(x)}.

Definition 2.16; Mostafa *et al.* (2011): Let X be a KU-algebra, a fuzzy set μ in X is called a fuzzy KU-ideal of X if it satisfies the following conditions (FI_1)μ(0)≥μ(x) for all x∈X and (FI_2)∀x, y, z∈X, μ(x*z)≥ min {μ(x*(y*z)), μ(y)}.

Lemma 2.17; Mostafa *et al.* (2011): If μ is a fuzzy ideal of KU-algebra X and x≤y, then μ(x)≥μ(y).

Definition 2.18; Bhattacharye and Mukheriee (1985): Let μ and β be two fuzzy subsets of X. The product μ and β is defined by the following (μ×β) (x, y) = min {μ(x), β(y)}, ∀x, y∈X.

Definition 2.19; Bhattacharye and Mukheriee (1985): If β is a fuzzy subset of a set X. The strongest fuzzy relation on X that is a fuzzy relation on β is μ_β given by $\mu_\beta(x, y)$ = min {β(x), β(y)}, ∀x, y∈X.

Some properties of fuzzy ideals in KU-semigroups: In this study, we start with the fuzzification of some types of ideals in a KU-semigroup.

Defintion 3.1: A fuzzy set μ in is calleda fuzzy sub KU-semigroup of X if it satisfies the following condition: for all x, y∈X.

- μ(x*y)≥min {μ(x), μ(y)}
- μ(x∘y)≥min {μ(x), μ(y)}

Example 3.2: Let X = {0, a, b, c} bea set. Define*-operation. and ∘-operation by the following tables.

*	0	a	b	c
0	0	a	b	c
a	0	0	0	c
b	0	a	0	c
c	0	0	0	0

∘	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Then $(X,*,\circ,0)$ is a KU-semigroup. Define μ: $X\to[0,1]$ by $\mu(0) = 0.8$, $\mu(a) = \mu(b) = 0.6$, $\mu(c) = 0.4$. Then by routine calculation, we canprovem that μ is a fuzzy sub KU-semigroup of X.

Theorem 3.3: A fuzzy set μ of X is a fuzzy sub KU-semigroup if and only if for every $0\leq t\leq 1$, the level set $U(\mu, t)$ is either empty or a sub KU-semigroup of X.

Proof: Suppose μ is a fuzzy subKU-semigroup of X and $U(\mu,t)\neq 0$, then for any x, $y\in U(\mu, t)$, we have $\mu(x*y)\geq\min\{\mu(x), \mu(y)\} = t$. Thus, $x*y\in U(\mu, t)$. Also, $\mu(x\circ y)\geq\min\{\mu(x), \mu(y)\} = t$. Thus, $x\circ y\in U(\mu, t)$. Hence, $U(\mu, t)$ is a sub KU-semigroup of X.

Conversely, take $t = \min\{\mu(x), \mu(y)\}$, for all x, $y\in X$. Since, $U(\mu, t)\neq 0$ is a sub KU-semigroup of X, we have $\mu(x*y)\geq t = \min\{\mu(x), \mu(y)\}$ and $\mu(x\circ y)\geq t = \min\{\mu(x), \mu(y)\}$. Hence, μ is a fuzzy sub KU-semigroup.

Defintion 3.4: A fuzzy set μ in X is called a fuzzy S-ideal of X if it satisfies the following condition: for all x, $y\in X$. (S_1) $\mu(0)\geq\mu(x)$, $(S_2)\mu(y)\geq\min\{\mu(x*y), \mu(x)\}$. $(S_3)\mu(x\circ y)\geq\min\{\mu(x), \mu(y)\}$.

Example 3.5: Let X = {0, a, b, c} be a set in example 3.2. We define a fuzzy subset μ by the following $\mu(0) = 0.6$ and $\mu(x) = 0.2$, for any $x\neq 0$. Then it is easy to show that μ is a fuzzy S-ideal of X.

Example 3.6: Let X = {0, a, b} be a set. Define*-operation and ∘-operation by the following tables.

*	0	a	b
0	0	b	b
a	0	0	a
b	0	a	0

∘	0	a	b
0	0	0	0
a	0	a	0
b	0	0	b

Then $(X,*,\circ,0)$ is a KU-semigroup. Definea fuzzy set μ: $X\to[0, 1]$ by $\mu(0) = t_0$, $\mu(a) = \mu(b) = t_1$ where $t_0>t_1$. Then by routine calculation we can prove that μ is a fuzzy S-ideal of X.

Defintion 3.7: A fuzzy set μ in X is called a fuzzy k-ideal of X if it satisfies the following condition: for all x, y, $z\in X$. (k_1) $\mu(0)\geq\mu(x)$, (k_2) $\mu(x*z)\geq\min\{\mu x*(y*z), \mu(y)\}$. (k_3) $\mu(x\circ y)\geq\min\{\mu(x), \mu(y)\}$.

Example 3.8.: Let X = {0, a, b, c, d} be a set. Define*-operation and ∘-operation by the following tables.

*	0	a	b	c	d
0	0	a	b	c	d
a	0	0	b	c	d
b	0	a	0	c	d
c	0	a	0	0	d
d	0	0	0	0	0

∘	0	a	b	c	d
0	0	0	0	0	0
a	0	0	0	0	0
b	0	0	0	0	b
c	0	0	0	b	c
d	0	a	b	c	d

Then $(X,*,\circ,0)$ is a KU-semigroup. Definea fuzzy set μ: $X\to[0, 1]$ by $\mu(a) = \mu(0) = 0.4$, $\mu c = \mu(b) = 0.2$, $\mu(d) = 0.1$. Then by routine calculation we canprove that μ is a fuzzy k-ideal of X.

Theorem 3.9: Let X be a KU-semigroup. A fuzzyset μ in X is a fuzzy k-ideal of X if and only if μ is a fuzzy S-ideal of X.

Proof: Let μ be a fuzzy k-ideal of a KU-semigroup X and x, $y\in X$. Then by definition 3.7 $\mu(x*z)\geq\min\{\mu(x*(y*z)), \mu(y)\}$, we put x = 0, implies that $\mu(z)\geq\min\{\mu(y*z), \mu(y)\}$. And from defintion 3.7 $(k_3)\mu(x\circ y)\geq\min\{\mu(x), \mu(y)\}$, it follows that μ is a fuzzy S-ideal.

Conversely, let μ be a fuzzy S-ideal of a KU-semigroup X and x, y, $z\in X$, then by. Defintion 3.4 $\mu(x*z)\geq\min\{\mu(y*(x*z)), \mu(y)\}$. By Th.2.2 (2) $\mu(x*z)\geq\min\{\mu(x*(y*z)), \mu(y)\}$ and by definition 3.4 (S_3) $\mu(x\circ y)\geq\min\{\mu(x), \mu(y)\}$. Thus, μ is a fuzzy k-ideal of X.

Theorem 3.10: Let A be a nonempty subset of a KU-semigroup X and μ be a fuzzy set in X define as follows:

$$\mu(x) = \begin{cases} t_1 & x\in A \\ t_2 & \text{otherwise} \end{cases}$$

where $t_1>t_2$ such that $t_1, t_2\in[0, 1]$. Then μ is a fuzzy k-ideal of X if and only if A is a k-ideal of X.

Proof: Assume that μ is a fuzzy k-ideal of. Since, $\mu(0)\geq\mu(x)$ for all $x\in X$, we have $\mu(0) = t_1$ and so, $0\in A$. Let x, y, $z\in A$ such that, $x*(y*z)\in A$ and $y\in A$. Using (k_2), we know that $\mu(x*z)\geq\min\{\mu(x*(y*z)), \mu(y)\} = t_1$ and thus, $\mu(x*z) = t_1$. Hence, $x*z\in A$ and let x, $y\in A$ by using (k_3) we have $\mu(x\circ y)\geq\min\{\mu(x), \mu(y)\} = t_1$. Hence, $x\circ y\in A$, similarly, $y\circ x \in A$. Then A is k-ideal of X.

Conversely, suppose that A is ak-ideal of X. Since, $0\in A$, it follows that $\mu(0) = t_1\geq\mu(x)$ for all $x\in X$. Let x, y, $z\in X$. If $y\notin A$ and $x*z\in A$, then clearly $\mu(x*z)\geq\min\{\mu(x*(y*z)), \mu(y)\}$.

Assume that $y\in A$ and $x*z\notin A$. Then by definition 2.8 (i), we have $x*(y*z)\notin A$. Therefore, $\mu(x*z) = t_2 = \min\{\mu(x*(y*z)), \mu(y)\}$. And let $x\in A$, if $y\notin A$ and $x\circ y\in A$, then

$\mu(x \circ y) = t_1 \geq \min\{\mu(x), \mu(y)\}$. Assume that $x \notin A$, $y \in A$ and $x \circ y \notin A$. Then $\mu(x \circ y) = t_2 = \min\{\mu(x), \mu(y)\}$. Hence, μ is a fuzzy k-ideal of X.

Theorem 3.11: Let μ bea fuzzy set ina KU-semigroup X. Then μ is a fuzzy k-ideal if and only if for every $0 \leq t \leq 1$, the nonempty level set $U(\mu, t)$ of μ is a k-ideal of X.

Proof: Suppose that μ is a fuzzy k-ideal of X and $U(\mu, t) \neq 0$ for any $t \in [0, 1]$, then there exists $x \in U(\mu, t)$ and so, $\mu(x) \geq t$. It follows from (k_1) that $\mu(0) \geq \mu(x) \geq t$, so that, $0 \in U(\mu, t)$. Let x, y, $z \in X$ be such that, $x*(y*z) \in U(\mu, t)$ and $y \in U(\mu, t)$, then $\mu(x*(y*z) \geq t$ and $\mu(y) \geq t$. Using (k_2), we have that $\mu(x*z) \geq \min\{\mu(x*(y*z)), \mu(y)\} \geq \min\{t, t\} = t$. So, $(x*z) \in U(\mu, t)$. Let x, $a \in U(\mu, t)$ then $\mu(x) \geq t$, $\mu(a) \geq t$ and so, $\mu(x \circ a) \geq \min\{\mu(x), \mu(a)\} = \min\{t, t\} = t$ which implies that $x \circ a \in U(\mu, t)$. Similarly, $a \circ x \in U(\mu, t)$, therefore, $U(\mu, t)$ is k-ideal of X.

Conversely, assume that a nonempty level set $U(\mu, t)$ of μ is ak-ideal of X for every $t \in [0, 1]$. For any $x \in X$, if $\mu(x) = t$, then $x \in U(\mu, t)$ and since, $0 \in U(\mu, t)$ in that case $\mu(0) \geq t = \mu(x)$, so that, $\mu(0) \geq \mu(x)$ for all $x \in X$. Now, we need to show that, μ satisfies (k_2) and (k_3). If not, then there exist a, b, $c \in X$ such that $\mu(a*c) \leq \min\{\mu(a*(b*c)), \mu(b)\}$. If we take $t_0 = 1/2(\mu(a*c) + \min\{\mu(a*(b*c)), \mu(b)\})$, the nwe have $\mu(a*c) < t_0 < \min\{\mu(a*(b*c)), \mu(b)\}$. Hence, $(a*(b*c)) \in U(\mu, t_0)$ and $b \in U(\mu, t_0)$ but $a*c \notin U(\mu, t_0)$ which means that $U(\mu, t)$ is not k-ideal of X. This is contradiction.

And let a, $b \in U(\mu, t_0)$ such that, $\mu(a \circ b) \leq \min\{\mu(x), \mu(x), \mu(a)\}$, then by taking $t_0 = 1/2(\mu(a \circ b)) + \min\{\mu(x), \mu(a)\})$ then we have $\mu(a \circ b) < t_0 \min\{\mu(x), \mu(a)\}$. Hence, x, $a \in U(\mu, t_0)$ but $a \circ b \notin U(\mu, t_0)$ which means that $U(\mu, t_0)$ is not k-ideal of X. This is contradiction. Therefore, μ is a fuzzy k-ideal of X.

Defintion 3.12: A fuzzy set μ in X is called a fuzzy P-ideal of X if it satisfies the following condition: for all x, y, $z \in X$. $(P_1) \mu(0) \geq \mu(x)$, $(P_2) \mu(z*y) \geq \min\{\mu(z*(x*y)), \mu(z*x)\}$, $(P_3) \mu(x \circ y) \geq \min\{\mu(x), \mu(y)\}$.

Example 3.13: Let X = {0, a, b, c, d} be a set in example 3.8. We define a fuzzy subset μ by the following $\mu(0) = 0.5$ and $\mu(a) = \mu(b) = \mu(c) = 0.2$, $\mu(d) = 0.1$. Then it is easy to show that μ is a fuzzy P-ideal of X.

Theorem 3.14: Every fuzzy P-ideal of X is a fuzzy S-ideal of X.

Proof: Let μ be a fuzzy P-ideal of X. Then by (P_2), we have $\mu(z*y) \geq \min\{\mu(z*(x*y)), \mu(z*x)\}$, put z = 0, we get $\mu(0*y) \geq \min\{\mu(0*(x*y)\}, \mu(0*x)\}$. Thus, $\mu(y) \geq \min\{\mu(x*y), \mu(x)\}$. Hence, μ is a fuzzy S-ideal of X. The following example proves that the converse of this theorem is not true.

Example 3.15: Let X = {0, a, b} be a set. Define*-operation and∘-operation by the following tables.

*	0	a	b
0	0	b	b
a	0	0	a
b	0	a	0

∘	0	a	b
0	0	0	0
a	0	a	0
b	0	0	b

Then $(X, *, \circ, 0)$ is a KU-semigroup. Define a fuzzy set μ: $X \rightarrow [0, 1]$ by $\mu(0) = 0.5$, $\mu(a) = 0.1$, $\mu(b) = 0.3$. Then by routine calculation we can prove that μ is a fuzzy S-ideal of X but nota fuzzy P-ideal since, $\mu(0*a) = 0.1 \leq \min\{\mu(0*(b*a)), \mu(0*b)\} = 0.3$.

Corollary 3.16: Every fuzzy P-ideal of X is a fuzzy k-ideal.

Proof 3.9; Theorem 3.14: The proof is complete.

Theorem 3.17: Let μ and β be fuzzy k-ideals of a KU-semigroup X. Then $\mu \times \beta$ is a fuzzy k-ideal of $X \times X$.

Proof: For any $(x, y) \in X \times X$, we have the following $(\mu \times \beta)(0,0) = \min\{\mu(0), \beta(0) \geq \min\{\mu(x), \beta(y)\} = (\mu \times \beta)(x, y)$. Let (x_1, x_2), (y_1, y_2), $(z_1, z_2) \in X \times X$, then $(\mu \times \beta)(x_1*z_1, x_2*z_2) = \min\{\mu(x_1*z_1), \beta(x_2*z_2)\} \geq \min\{\min\{\mu(x_1*(y_1*z_1)), \mu(y_1)\}, \min\{\beta(x_2*(y_2*z_2)), \beta(y_2)\} = \min\{\min\{\mu\ x_1*(y_1*z_1)), \beta(x_2*(y_2*z_2))\}, \min\{\mu(y_1), \beta(y_2)\}\} = \min\{(\mu \times \beta)(x_1*(y_1*z_1), x_2*(y_2*z_2)), \{\mu \times \beta(y_1, y_2)\}\}$. Let (x_1, x_2), $(y_1, y_2) \in X \times X$, then, we have $\mu \times \beta\ (x_1 \circ y_1, x_2 \circ y_2) = \min\{\mu(x_1 \circ y_1), \beta(x_2 \circ y_2)\} \geq \{\min\{\mu(x_1), \mu(y_1)\}, \min\{\beta(x_2), \beta(y_2)\} \geq \{\min\{\mu(x_1), \beta(x_2)\}, \min\{\mu(y_1), \beta(y_2)\}\} \geq \min\{\mu \times \beta(x_1, x_2), \mu \times \beta(y_1, y_2)\}$. Hence, $\mu \times \beta$ is a fuzzy k-ideal of $X \times X$.

Lemma 3.18: For a given fuzzy subset β of a KU-semigroup X, let μ_β be the strongest fuzzy relation on X. If μ_β is a fuzzy k-ideal of $X \times X$, then $\beta(x) \leq \beta(0)$ for all $x \in X$.

Proof: Since, μ_β is a fuzzy k-ideal of $X \times X$, it follows from definition 3.7 (k_1) that $\mu_\beta(x, x) = \min\{\beta(x), \beta(x)\} \leq \min\{\beta(0), \beta(0)\}$, then $\beta(x) \leq \beta(0)$.

Theorem 3.19: Let β be a fuzzy subset of X and μ_β be a strongest fuzzy relation on X. Then β is a fuzzy k-ideal of X if and only if μ_β is a fuzzy k-ideal of $X \times X$.

Proof: Suppose that β is a fuzzy k-ideal of X. For all $y \in X$, then from $(S_1) \mu_\beta(0, 0) = \min\{\beta(0), \beta(0)\} \geq \min\{\beta(x), \beta(y)\}$

$= \mu_\beta (x, y)$. For any $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in X \times X$, we have from (S_2) $\mu_\beta(x_1 * z_1, x_2 * z_2) = \min \{\beta(x_1 * z_1), \beta(x_2 * z_2)\} \geq \min \{\min \{\beta(x_1 * (y_1 * z_1)), \beta(y_1)\}, \min \{\beta(x_2 * (y_2 * z_2)), \beta(y_2)\} = \min \{\min \{\beta(x_1) * (y_1 * z_1)), \beta(x_2 * (y_2 * z_2))\}, \min \{\beta(y_1)\}(y_2)\}\} = \min \{\mu_\beta[(x_1 * (y_1 * z_1), (x_2 * (y_2 * z_2)], \mu_\beta(y_1, y_2)\}$.

For any $(x_1, x_2), (y_1, y_2) \in X \times X$, then we have $\mu_\beta[(x_1, x_2) \circ (y_1, y_2)] = \mu_\beta(x_1 \circ y_1, x_2 \circ y_2) = \min \{\beta(x_1 \circ y_1), \beta(x_2 \circ y_2)\} \geq \{\beta(x_1), \beta(x_2)\} = \mu_\beta(x_1, x_2)$. Hence μ_β is a fuzzy k-ideal of $X \times X$.

Conversely, suppose that μ_β is a fuzzy k-ideal of $X \times X$. Then for all $(x, y) \in X$ we have $\min \{\beta (0), \beta (0)\} = \mu_\beta (0, 0) \geq \mu_\beta (x, y) = \min \{\beta(x), \beta(y)\}$. It follows that $\beta(0) \geq \beta(x)$. For all $(x_1, x_2), (y_1, y_2), (z_1, z_2)$ in $X \times X$, then we have $\{\beta(x_1 * z_1), \beta(x_2 * z_2)\} = \mu_\beta(x_1 * z_1, x_2 * z_2) \geq \min \{\mu_\beta\{x_1 * (y_1 * z_1)\}, \mu_\beta(y_1)\}, \min \{\mu_\beta\{x_2 * (y_2 * z_2)\}, \mu_\beta(y_2)\} = \min \{\mu_\beta[x_1 * (y_1 * z_1), x_2 * (y_2 * z_2)], \mu_\beta(y_1, y_2)\} = \min \{\min \{\beta(x_1 * (y_1 * z_1)), \beta(x_2 * (y_2 * z_2)\}, \min \{\beta(y_1), \beta(y_2)\}\} = \{\min \{\beta(x_1 * (y_1 * z_1)), \beta(y_1)\}, \min \{\beta(x_2 * (y_2 * z_2), \beta(y_2)\}\}$.

For any $(x_1, x_2), (y_1, y_2) \in X \times X$, then we have $\beta[(x_1, x_2) \circ (y_1, y_2)] = \beta(x_1 \circ y_1, x_2 \circ y_2) = \mu_\beta(x_1 \circ y_1, x_2 \circ y_2) \geq \min \{\beta(x_1 \circ y_1), \beta(x_2 \circ y_2)\} = \min \{\min \{\beta(x_1), \beta(y_1)\}, \min \{\beta(x_2), \beta(y_2)\} = \min \{\{\beta(x_1), \beta(x_2)\}, \min\{\beta(y_1), \beta(y_2)\}\}$. Hence, β is a fuzzy k-ideal of X.

Image (pre-image) of fuzzy k-ideals under homomorphism: In this study, we introduce the concepts of the image and the pre-image of. Fuzzy k-ideals in a KU-semigroup X under homomorphism. Also, we prove that the products of fuzzy k-ideals are a fuzzy k-ideal of a KU-semigroup X.

Definition 4.1: Let f be a mapping from X-Y. If μ is a fuzzy subset of X, then the fuzzy subset B of Y defined by:

$$f(\mu)(y) = B(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x), \text{if } f^{-1}(Y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 \qquad\qquad\qquad\qquad \text{otherwise} \end{cases}$$

is said to be the image of μ under f. Similarly, if B is a fuzzy subset of Y, then the fuzzy subset defined by $\mu(x) = B(f(x))$ for all $x \in X$ is called the pre-image of B unde f.

Theorem 4.2: A homomorphism pre-image of a k-fuzzy k-ideal is also a fuzzy k-ideal.

Proof: Let f: $X \rightarrow X'$ be a homomorphism of KU-semigroups, B be a fuzzy k-ideal of X' and μ be the pre-image of μ B under f. It follows that $\mu(x) = B(f(x))$, for all $x \in X$:

For any $x \in X$, then $\mu(0) = B(f(0)) \geq B(f(x)) = \mu(x)$. Let x, y, $z \in X$, then $\mu(x * z) = B(f(x * z) = B(f(x) *' f(z)) \geq \min \{B(f(x) *' (f(y) *' f(z)), B(f(y))\} = \min \{B(f(x * (y * z))), B(f(y))\} = \min \{\mu(x * (y * z)), \mu(y)\}$. Let x, y, $z \in X$, then $\mu(x \circ y) = B(f(x \circ y) = B(f(x) \circ f(y)) \geq \min \{B(f(x)), B(f(x)), B(f(y))\} = \min \{\mu(x), \mu(y)\}$. Hence, the pre-image of B under f is a fuzzy k-ideal of X.

Definition 4.3: A fuzzy subset μ of X has a sup property if for any subset T of X, there exist $t_0 \in T$ such that $\mu(t_0) = \sup\limits_{t \in T} \mu(t)$.

Theorem 4.4: Let f: $X \rightarrow X'$ be a homomorphism between two KU-semi groups X and X'. For every fuzzy k-ideal in μ, then $f(\mu)$ is a fuzzy k-ideal of X'.

Proof: By definition 4.1, $B(y') = f(\mu)(y') = \sup\limits_{x \in f^{-1}(y')} \mu(x)$ for all $y' \in X'$. Let f: $X \rightarrow X'$ be a homomorphism of KU-semigroups and μ be a fuzzy k-ideal o X with sup property and B bethe image of μ under f. Since, μ is a fuzzy k-ideal of, X and then we have $\mu(0) \geq \mu(x)$ for all $x \in X$. Note that $0 \in f^{-1}(0')$ where 0 and 0' are the zero of X and X', respectively. Thus, $B(0') = \sup\limits_{x \in f^{-1}(f')} \mu(t) = \mu(0) \geq \mu(x)$ for all $x \in X$ which implies that $B(0') \geq \sup\limits_{x \in f^{-1}(f')} \mu(t) = B(x')$ for any $x' \in X'$. For any x', y', $z' \in X'$, let $x_0 \in^{f-1}(x')$, $y_0 \in^{f-1}(y')$ and $z_0 \in f^{-1}(z')$ be such thatand $\mu(x0 * z0) = \sup\limits_{t \in f^{-1}(x' * z')} \mu(t), \mu(y0) = \sup\limits_{t \in f^{-1}(y')} \mu(t)$ then:

$$B(x' * z') = \sup_{t \in f^{-1}(x' * z')} \mu(t) =$$
$$\mu(x_0 * z_0) \geq \min\{\mu(x_0 * (y_0 * z_0)), \mu(y_0)\} =$$
$$\min\left\{\sup_{t \in f^{-1}(x' * (y' * z'))} \mu(t), \sup_{x \in f^{-1}(y')} \mu(t)\right\} =$$
$$\min\{B(x' * (y' * z')), B(y')\}$$

For any x', $y' \in X'$, let $x_0 \in^{f-1}(x')$ and $y_0 \in f^{-1}(y')$ be such that:

$$\mu(x_0 \circ y_0) = \sup_{x \in f^{-1}(x' \circ y')} \mu(t)$$

and then:

$$B(x' \circ y') = \sup_{x \in f^{-1}(x' \circ y')} \mu(t) =$$
$$\mu(x_0 \circ y_0) \geq \min\{\mu(x_0), \mu(y_0)\} =$$
$$\min\left\{\sup_{x \in f^{-1}(x')} \mu(t), \sup_{x \in f^{-1}(y')} \mu(t)\right\} = \min\{B(x'), B(y')\}$$

Hence, B is a fuzzy k-ideal of Y.

CONCLUSION

We have studied the concept of fuzzy in KU-semigroup. We discussed few properties of this concept and we investigate some related results. Some, types of fuzzy ideals in KU-semigroup are studied and the product of fuzzy k-ideals to product of KU-semigroup are established. The notion of a homomorphism of a KU-semigroup is discussed. The main purpose of our future research is to study of a generalization of a fuzzy KU-semigroup such as bipolar of a fuzzy KU-semigroupand interval value of a fuzzy KU-semigroup. Also, we can introduce the notion of graph for a KU-semigroup.

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