

Reliability Test of Inverse Distance Weighting (IDW) Method by Linier Limits

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Abstract: The Inverse Distance Weighting (IDW) is a multipoint interpolation method used in predicting the missing data as well as that can be used to count the average regional rain fall. According to Prasasti, it is necessary to include more data in order to obtain more significant interpolation result. So, to improve the accuracy of IDW method interpolation result, it is conducted by adding auxiliary points linearly between adjacent stations which then are called as IDW linier method. The reliability test is conducted by comparing the calculation results of linier IDW method by exact surface field and mathematic equation as well as conducted the application reliability test in the field, namely by comparing the calculation results with the observed data. Results of the application reliability test in exact field by mathematic equation of $X+Y+Z/10 = 12$ by using 3-5 rain stations, show that there is an average accuracy increase of IDW linear method of 2.21% from the original IDW method. Meanwhile, the reliability test in the field, Sidoarjo Regency and Mojokerto Regency show that the linear IDW method has an average accuracy increase of 5.13%. This accuracy increase is caused by the addition of some information and increasing close of information to the expected points, namely the auxiliary points.

Key words: Reliability, IDW, Linier limit, Linier method, application, auxiliary points

INTRODUCTION

The amount of average rainfall in an area can be set correctly if it is known the amount of rain points in each coordinate in the area or called as Averaging Area Rainfall (AAR) which the closer the rain point coordinate, so, the better the results will be (Soewarno, 2000).

AAR calculation is an interpolation effort of rain data from rain measurement location points in each watershed coordinate (Br, 1993), so, the AAR calculation is a calculation to find out missing point value in a rain area.

Currently, it has been developed the AAR calculation with multipoint interpolation such as nearest neighbour, Krigging and IDW methods in which to obtain the average rainfall in an area, the value in each grid/coordinate at watershed must be known and then it is obtained the averaged without calculating on the spatial relationship model between the rain station and affecting factors.

Among the methods, the most frequently used is IDW method because it is a quite simple method and can present quite appropriate results.

The IDW method is a method to predict missing data of some measured values (Multipoint interpolation). This IDW method can be used for multi sciences (universal) but in this case, it is used for hydrology mainly in issue of rain fall data processing (Lam, 1983).

It is greatly necessary for a simple and applicative method but can create more accurate calculation. Therefore, it is necessary to develop IDW method by taking data in the closest position to the expected points. The issues are:

- How is to determine the coordinate of Linear limit auxiliary point?
- How is to determine the amount of rain in the Linear limit auxiliary point?
- How is the reliability of IDW method by Linear limit than by its original method?

MATERIALS AND METHODS

IDW method: In predicting the amount of expected points in IDW method, position of the points must be in the measured data scope. Therefore, the resulted data must

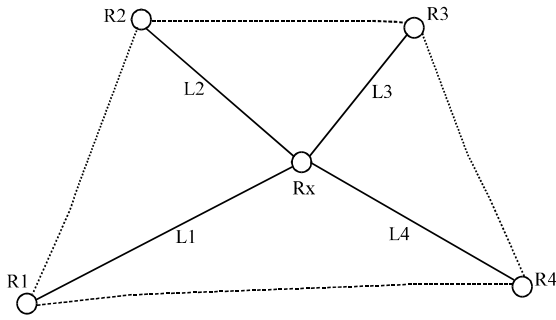


Fig. 1: The position data that is looked for its value (Rx) with 4 rain station data

not be under the minimum limit or above the maximum data (Isotrophic) (Ginanjar, 2015). In its use, IDW method takes 3 or more measured data minimally which the more data will present better performance. Because it will be more able to describe the position of expected points. The example of IDW method formulation form using 4 data is as follow Fig. 1 (Pande and Al-Mashidani, 2008):

$$R_x = \frac{\sum_{i=1}^n \frac{R_i}{L_i^p}}{\sum_{i=1}^n \frac{1}{L_i^p}} = \frac{\frac{R_1}{L_1^p} + \frac{R_2}{L_2^p} + \dots + \frac{R_n}{L_n^p}}{\frac{1}{L_1^p} + \frac{1}{L_2^p} + \dots + \frac{1}{L_n^p}}$$

Where:

R_x = Rain fall that is looked for its X station value

R_i = Rain fall of comparing station

L_i = Distance of comparing station on X

n = The number of comparing station

p = Orde

St = Station

Such as seen in IDW formula, there is $p =$ orde stating the orde level namely between 1 until 4 but in this study, the used one is 1 orde.

Theoretical approach of IDW method with linear limit:

There are some reasons why it is necessary to add auxiliary points in linear limit in predicting the missing data, namely as follow:

- The more data will give more description on the location in expected points
- The closer information will present more description on the expected points

Therefore, it is necessary to conduct innovation on the calculation of IDW method by: adding some data in the form of auxiliary points, adding data between the closest rain fall station and linear model, this addition is addressed to:

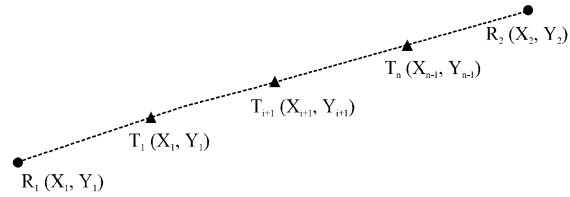


Fig. 2: Determination of auxiliary points between 2 stations

- Give limit value (control) in the system
- Multiply calculation input data

Addition of auxiliary station between 2 adjacent stations:

The addition of auxiliary station is given between 2 adjacent stations such as seen in Fig. 2 which the addition in each auxiliary point is put between in the middle of stations, so that, there are 2 same segments. The addition of following auxiliary point will divide 2 same segments between stations, so that, the number of segments is always even. Coordinate of additional auxiliary point (T_i) is determined by the following mathematic equation:

$$X_i = \frac{(n-i) \cdot X_1 + i \cdot X_2}{n} \quad Y_i = \frac{(n-i) \cdot Y_1 + i \cdot Y_2}{n}$$

Meanwhile, the number of auxiliary points is determined by the following interpolation equation:

$$T_i = R_i + i \frac{\|R_i - R_{i+1}\|}{n}$$

Where:

X_i = Abscissa of the ith auxiliary point

Y_i = Ordinate of the ith auxiliary point

T_i = The amount of the ith auxiliary station

R_1, R_2 = Rainfall station in tips of linear lines

n = The number of divider segment by even numbers = 2, 4, 8, ..., 2(n-1)

I = Numeric started from 1 until $n = 1, 2, 3, \dots, i < n$

The description of proposed auxiliary station is as follow in Fig. 3.

Methodology: To know the reliability of a method, so, it is necessary to conduct a test on an exact field, used as comparison. Testing in this case is conducted in horizontal field with mathematic equation. The testing pattern is as follow: model testing on the change of the number of rain fall, namely by using 3-5 rain stations, the model testing in point 1 is by adding the auxiliary points as follow:

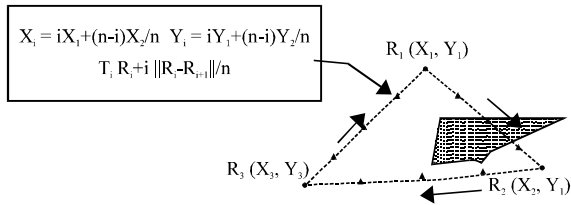


Fig. 3: Position of rain station and addition of auxiliary points in linear limit

- Adding 1 auxiliary station located in the middle of each linear line
- Adding 2 auxiliary stations distributed evenly in each linear line
- Adding 3 auxiliary stations distributed evenly in each linear line

To evaluate the reliability of calculation results, it can be used Mean Percentage Error (MPE) (Bambang and Sutanta, 2009):

$$MPE = \frac{\sum_{i=1}^n \frac{(Y_i - \bar{Y}_i)}{\bar{Y}_i}}{n} \times 100$$

Where:

- \bar{Y}_i = Actual value of data group
- Y_t = Predicted value of data group
- N = Number of data

RESULTS AND DISCUSSION

Reliability test of linear IDW method in exact field $X+Y+Z/10 = 12$: The testing on this case is conducted in horizontal field by mathematics equation $X+Y+Z/10 = 12$. The original and linear IDW method are used simultaneously to calculate the area set up, meanwhile, the station that has been known its value, then it is used to predict on the value of the area.

Testing using 3 rain stations: The form of testing using 3 rain stations can be seen in the following Fig. 4. The use of linear limit by using 3 stations, Table 1 can increase the average of 4.82%. The greatest increase is in addition of 2 auxiliary points caused by the closer auxiliary points to the target area.

Testing using 4 rain stations: The form of testing using 4 rain stations can be seen in the following Fig. 5 and Table 2 and 3. The use of linear limit by using 4 stations can increase the average of 0.59%.

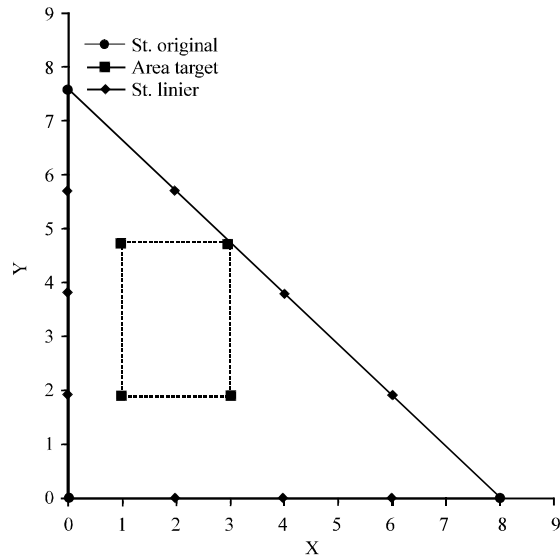


Fig. 4: IDW method testing with positions of 3 rain stations and target area for 3 types of additions of 1-3 auxiliary points

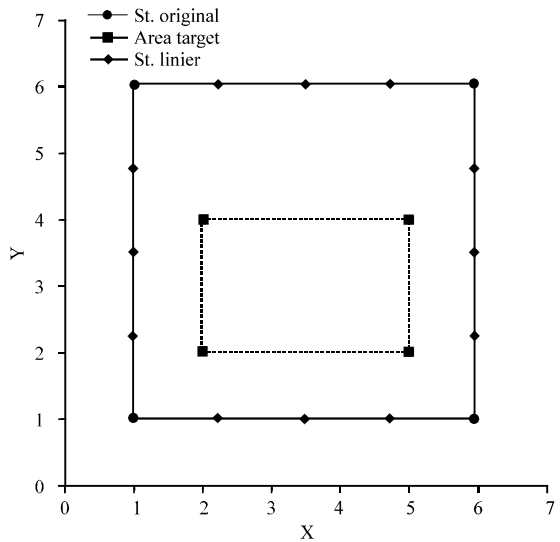


Fig. 5: IDW method testing with positions of 4 rain stations and target area for 4 types of additions of 1-3 auxiliary points

Testing using 5 rain stations: The use of linear limit by using 5 stations can increase the average of 1.22%. The accuracy increase in 1 point and 2 is greater than 3 auxiliary points, this is because 1 and 2 auxiliary points are more focus or closer to target area, meanwhile in 3 auxiliary points, it is only spreading (Fig. 6).

Table 1: Error comparison between original and linear IDW methods on the exact field using additional of 1-3 auxiliary points

X	Y	R exact	IDW original	Err (%)	1 aux.point	Err (%)	2 aux. point	Err (%)	3 aux. point	Err (%)
1	2	90.00	87.77	2.48	68.93	23.41	75.35	16.27	80.59	10.46
1	3	80.00	79.30	0.87	69.95	12.56	74.01	7.49	75.41	5.73
1	4	70.00	71.85	2.65	71.30	1.86	68.32	2.40	70.90	1.29
1	5	60.00	64.96	8.27	69.41	15.69	64.43	7.38	65.15	8.59
2	2	80.00	82.23	2.79	66.67	16.67	71.35	10.81	75.04	6.20
2	3	70.00	76.25	8.93	64.73	7.52	68.05	2.78	70.54	0.77
2	4	60.00	70.53	17.55	63.65	6.09	63.22	5.37	66.13	10.22
2	5	50.00	65.13	30.27	63.44	26.88	56.43	12.86	60.53	21.05
3	2	70.00	76.25	8.93	64.73	7.52	68.05	2.78	70.30	0.43
3	3	60.00	72.58	20.97	58.89	1.86	64.23	7.05	65.39	8.9
3	4	50.00	68.77	37.54	54.62	9.23	59.97	19.94	60.75	21.51
3	5	40.00	65.01	62.52	56.81	42.01	51.28	28.20	59.18	47.94
Err. average (%) =			16.98		14.28		10.28			11.93
Increase/decrease					2.70		6.7			5.05

Table 2: Error comparison between original and linear IDW methods on the exact field using additional of 1-3 auxiliary points

X	Y	R exact	IDW original	Err (%)	1 aux.point	Err (%)	2 aux. point	Err (%)	3 aux. point	Err (%)
2	2	80.00	69.37	13.29	60.16	24.80	63.62	20.48	64.80	19.00
2	3	70.00	60.41	13.70	60.16	14.05	60.69	13.30	60.47	13.61
2	4	60.00	53.90	10.16	57.40	4.34	55.85	16.91	55.71	7.15
3	2	70.00	60.41	13.70	60.16	14.05	60.69	13.30	60.47	13.61
3	3	60.00	55.15	8.08	54.80	8.67	54.98	8.36	54.99	8.35
3	4	50.00	50.00	0.00	50.00	0.00	50.00	0.00	50.00	0.00
4	2	60.00	53.00	10.16	57.40	4.34	55.85	6.91	55.71	7.15
4	3	50.00	50.00	0.00	50.00	0.00	50.00	0.00	50.00	0.00
4	4	40.00	44.85	12.12	45.20	13.00	45.02	12.54	45.01	12.52
5	2	50.00	50.00	0.00	50.00	0.00	50.00	0.00	50.00	0.00
5	3	40.00	46.10	15.25	42.60	6.51	44.15	10.37	44.29	10.72
5	4	30.00	39.59	31.96	39.84	32.79	39.31	31.02	39.53	31.75
Err. average (%) =					10.70		10.21		10.27	9.84
Increase/decrease =							0.49		0.43	0.86

Table 3: Error comparison between original and linear IDW methods on the exact field using additional of 1-3 auxiliary points

X	Y	R exact	IDW original	Err (%)	1 aux.point	Err (%)	2 aux. point	Err (%)	3 aux. point	Err (%)
1	2	90.00	72.79	19.12	77.32	14.09	80.90	10.12	77.71	13.66
1	3	80.00	68.61	14.24	71.71	10.36	71.50	10.63	71.06	11.17
1	4	70.00	66.57	4.90	64.44	7.94	65.20	6.86	64.71	7.56
2	2	80.00	69.21	13.49	70.47	11.91	68.88	13.90	70.26	12.17
2	3	70.00	64.32	8.11	65.33	6.67	65.11	6.99	65.08	7.03
2	4	60.00	60.00	0.00	60.15	0.26	61.05	1.75	60.00	0.00
3	2	70.00	64.23	8.24	65.53	6.38	63.67	9.04	65.29	6.73
3	3	60.00	60.00	0.00	60.19	0.32	59.36	1.06	60.00	0.00
3	4	50.00	55.68	11.35	55.00	9.99	54.86	9.72	54.92	9.85
4	2	60.00	60.00	0.00	60.17	0.28	59.28	1.20	60.00	0.00
4	3	50.00	55.77	11.53	54.87	9.73	53.75	7.50	54.71	9.43
4	4	40.00	50.79	26.98	49.76	24.41	49.53	23.83	49.74	24.35
Err. average (%) =				9.83		8.53		8.55		8.76
Increase/decrease =						1.30		1.28		1.07

Reliability test of linear IDW method in application in the field: The testing is conducted to know the reliability of linear IDW method in an actual field data which the uncertainty level is greater. The testing is conducted by using some stations simultaneously to find out values in a station that has been observed its rain value. Reliability test of linear IDW method in field application in Sidoarjo Regency (Fig. 7) and Mojokerto Regency (Fig. 8).

Reliability test of linear IDW method in field application in Sidoarjo Regency: Based on the calculation results, the comparison of error average on the applications of

original linear IDW methods with additional of 1-3 auxiliary points can be stated that the use of linear IDW method can increase the average accuracy of 5.5% (Table 4 and 5).

Reliability test of linear IDW method in field application in Mojokerto Regency: Based on the calculation results, the comparison of error average on the applications of original linear IDW methods with additional of 1-3 auxiliary points can be stated that the use of linear IDW method can increase the average accuracy of 9.53% (Table 6 and 7).

Table 4: Coordinate data and number of expected rain in Sidoarjo Regency

Missing data	Data	X	Y	R
Prambon	26/3/2001	673	9176.5	19
Data: St. Cepilples (20), Watu Tulis (13), Ketawang (18), Gedang Rowo (40) Kludan	27/3/2001	688	9170	20
Data: St. During Bedug (13), Putat (20), Kedung Cangkring (20), Porong (15)	16/3/2001	686	9178	26
Bakalan	16/3/2001	690	9182	55
Data: St. Durung Bedug (25), Klagen (7), Sumpul (47) Sruri	16/3/2001	690	9182	55
Data: St. Karang Nongko (43), Kategan (40), Kemlaten (14), Sedati Banjar (15), Kemantren (76) Ketawang	16/3/2001	679.5	9175.5	18
Data: St. Ponokawan (20), Karang Nongko (14), Kremlung (10), Gedang Rowo (40)				

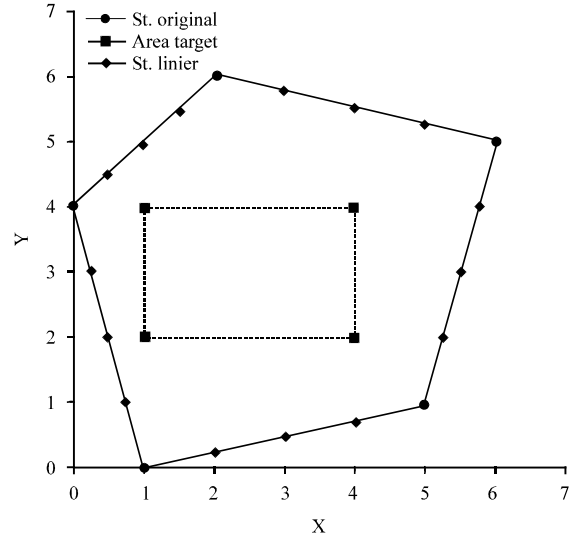


Fig. 6: IDW method testing with positions of 5 rain stations and target area for 3 types of additions of 1-3 auxiliary points

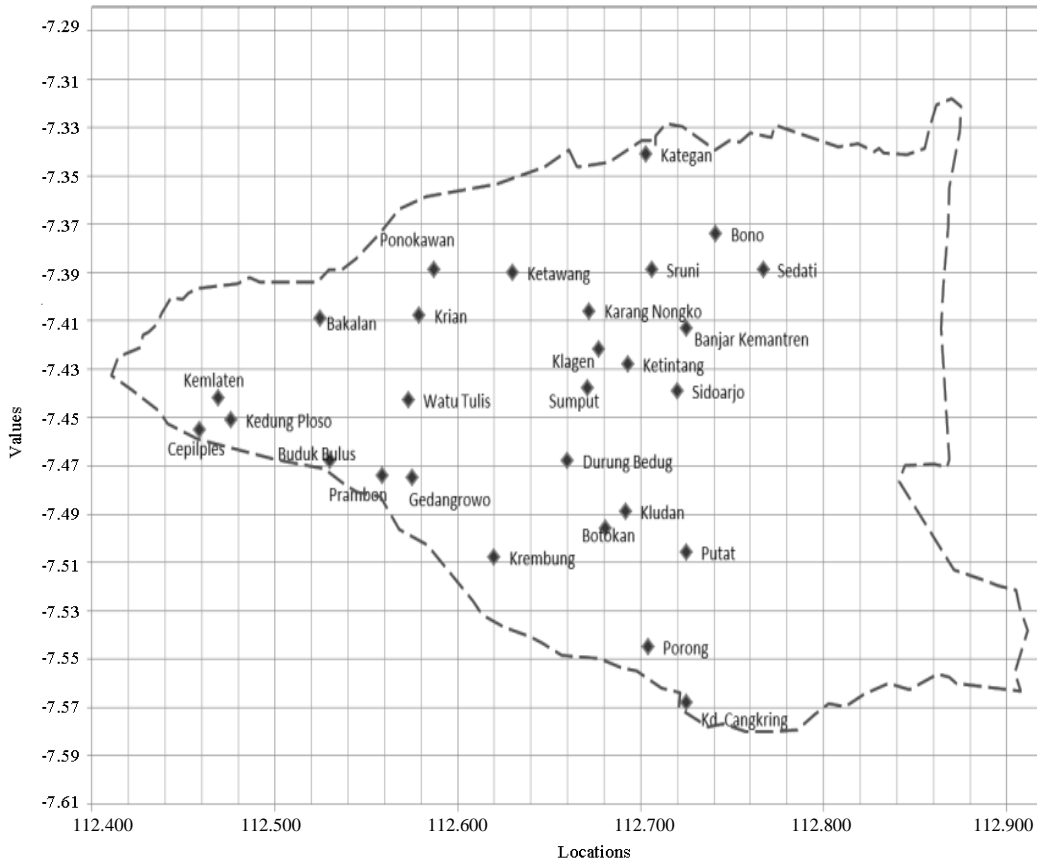


Fig. 7: Location of testing in Sidoarjo Regency

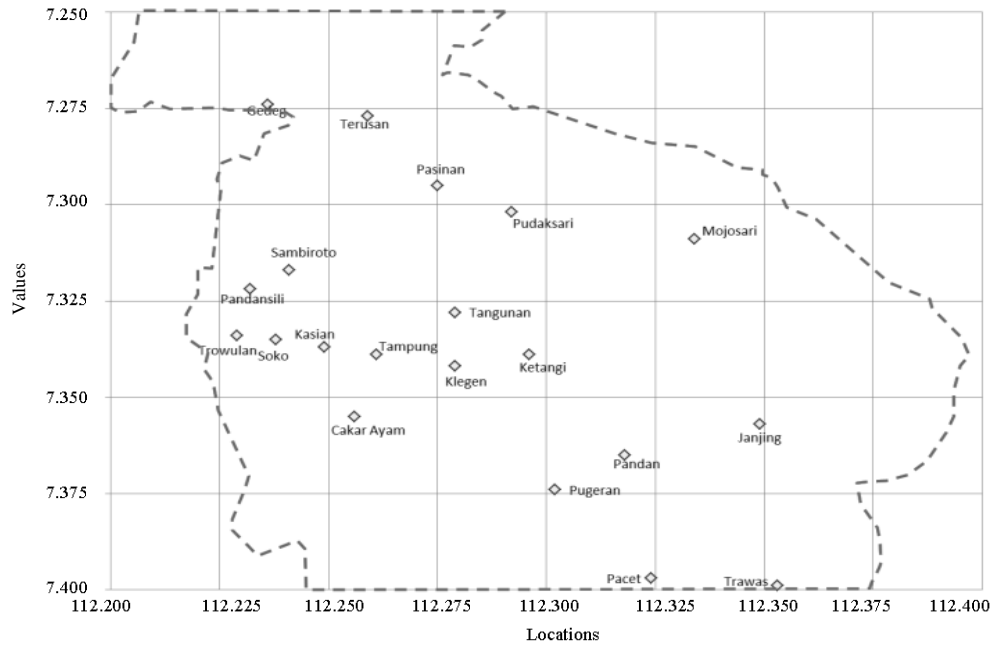


Fig. 8: Location of testing in Mojokerto Regency

Table 5: Comparison of average error of original and linear IDW method applications by additional of 1-3 auxiliary points in Sidoarjo Regency

X	Y	R exact	IDW original	Err (%)	1 aux.point	Err (%)	2 aux. point	Err (%)	3 aux. point	Err (%)
673.00	9176.50	19.00	24.05	26.56	19.70	3.66	20.18	6.21	22.81	20.03
688.00	9170.00	20.00	17.63	11.85	17.99	10.06	18.99	5.06	17.24	13.78
690.00	9182.00	45.00	45.18	0.41	44.71	0.65	42.80	4.90	42.93	4.60
679.50	9175.50	26.00	26.15	0.57	26.41	1.58	26.11	0.44	26.14	0.52
679.50	9175.50	18.00	21.51	19.52	16.66	7.44	16.92	6.00	19.67	9.29
Err. average (%) =				11.78		4.68		4.52		9.64
Increase/decrease =					7.10		7.26		2.14	

Table 6: Coordinate data and number of expected rain in Mojokerto Regency

Missing data	Data	X	Y	R
Tampung	26/3/2001	112.261	7.339	57
Data: St. Caker Ayam (30), Kasihan (70), Tangunan (70), Klegen (57)				
Kasihan	27/3/2001	112.249	7.337	30
Data: St. Tamoung (57), Caker Ayam (30), Sbr Soko (20), Sambiroto (46)				
Klegen	16/3/2001	112.279	7.342	45
Data: St. Caker Ayam (19), Tampung (40), Tangunan (35), Ketangi (84)				
Tanganan	16/3/2001	112.279	7.328	35
Data: St. Puduksari (3), Tampung (40), Klegen (65), Ketangi (84)				
Pandan	16/3/2001	112.318	7.365	66
Data: St. Ketangi (84), Janjing (50), Pacet (52), Pugeran (95)				

Table 7: Comparison of error average on the applications of original and linear IDW methods with additional of 1-3 auxiliary points in Mojokerto Regency

X	Y	R exact	IDW original	Err (%)	1 aux.point	Err (%)	2 aux. point	Err (%)	3 aux. point	Err (%)
7.365	112.318	66.00	75.44	14.31	63.05	4.47	64.07	2.93	70.54	6.88
7.328	112.279	35.00	53.26	52.16	42.93	22.65	46.05	31.59	45.72	30.64
7.342	112.279	45.00	46.29	2.87	44.40	1.33	44.17	1.84	48.19	7.08
7.337	112.249	30.00	37.66	25.54	32.26	7.55	32.18	7.26	38.65	28.84
7.339	112.261	57.00	53.66	5.87	57.40	0.70	57.93	1.63	54.74	3.96
Err. average (%) =				20.15		7.34		9.05		15.48
Increase/decrease =					12.81		11.10		4.67	

CONCLUSION

Based on reliability test results of original and linear IDW method in exact field, the contour surface and field application can be concluded as follow:

Based on the reliability test results on the application of exact field with mathematic equation of $X+Y+Z/10 = 12$, there is an average accuracy increase of 2.29% from original IDW method.

Based on the reliability test results on the IDW method application in Sidoarjo and Mojokerto Regencies using linear limit, there is an average accuracy increase of 7.52% from original IDW method. This accuracy increase is caused by the addition of some information in linear IDW and that the information is closer to the expected points, namely the auxiliary points.

RECOMMENDATIONS

Based on the reliability test result, the IDW method with linear limit in general can increase the calculation result of IDW method though it is in small number, this is caused by addition of auxiliary stations in each linear line in the same number and also, same distance. On the other side, the additional auxiliary points create mutual balance of auxiliary points for each other. Therefore, it is recommended to conduct further study using additional auxiliary points that are truly closer to the expected points, so that, it can more increase the calculation result of IDW method.

The reliability test result of IDW method with linear limit is better than original method. This is because it does not use the furthest point namely the main data. This is because the furthest point presented small effect weight in the expected point, therefore, it is recommended to use the closest auxiliary points to the expected point areas.

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