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Extended Medium Domination Number of a Jahangir Graph

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Abstract: The medium domination number of a Jahangir graph $J_{m,n}$ was introduced by M. Ramachandran and N. Parvathi. Recently Mahadevan *et al.*, introduced the concept of extended medium domination number of a graph. edom (u, v) is the sum of number of u-v paths of length less than or equal to 3. ETDV (G) = Σedom (u, v) for u, v∈V(G) is the total number of vertices that dominate every pair of vertices. The extended medium domination number of G is EMD(G) = ETDV(G)/(p2) where, p is the number of vertices in G. Here, we generalize the extended medium domination number of Jahangir graph, sunflower graph and windmill graph.

Key words: Extended medium domination number, sunflower, extended, vertices, dominate, graph

INTRODUCTION

Extended medium domination number plays an vital role in various branches of computer communication network. For all the basic definitions and the formula for edom(G), ETDV(G) and EMD(G) are as by Ramya *et al.* (2012). Ramachandran and Parvathi (2015) found the medium domination number for Jahangir graph. By Mahadevan *et al.* (2015), we found extended medium domination number for various types of graphs. Motivated by the above in this study, we establish this number for Jahangir graph, sunflower and windmill graph (Buckley and Harary, 1990; Vargor and Dundar, 2011).

Example 1.1: From Fig. 1, it can be verified that ETDV(G) = 63; EMD(G) = 63/15.

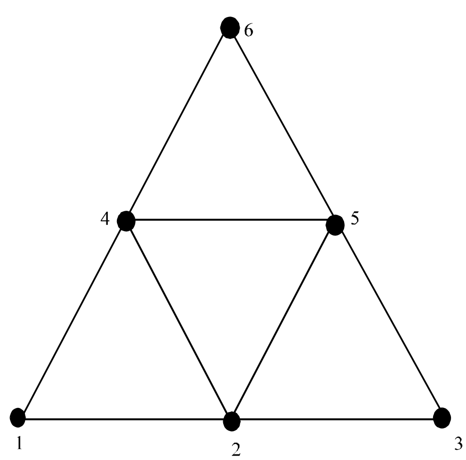

Fig. 1: Sunflower and windmill graph

Definition 1.2: Windmill graph Wd(k, j) is an undirected graph constructed for j, k≥2 by joining j copies of the complete graph K_k at a shared vertex.

Definition 1.3: Sunflower graph S_t is the resultant graph obtained from flower graph of wheel W_t by adding t-1 pendent edges to the central vertex (Harary, 1972).

MATERIALS AND METHODS

Extened medium domination number for some special type graphs

Extended medium domination number for windmill graph

Theorem 2.1: For any windmill graph:

$$\binom{k}{2}(k^2-4k+5)$$

Proof: Construct the wind mill graph by joining j copies of K_j at a shared vertex. The vertices of K_j's are (1, 2, …, k), (k+1, …, 2k), …, (((j-1) k+1, …., jk). Merge the vertices 1, k+1, …, (j-1) k+1 and name that vertex as 1. Extended medium domination number for a complete graph is:

$$\binom{k}{2}(k^2-4k+5)$$

We have m copies of K_k. edom(u, v) = $(k-1)^{k-1}$ for all u, v; u, v ≠ ik+1 where i = 0 to j-1. Therefore:

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$$ETDV(G) = j\binom{k}{2}(k^2\text{-}4k\text{+}5)\text{+}\binom{j}{2}(k\text{-}1)^k$$

$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{j\binom{k}{2}(k^2\text{-}4k\text{+}5)\text{+}\binom{j}{2}(k\text{-}1)k}{\binom{(j(k\text{-}1)\text{+}1)}{2}}$$

Extended medium domination number for sunflower graph

Theorem 2.2: For any sunflower graph:

$$S_t,\ EMD(G) = \left(\frac{31t^2\text{-}55t\text{+}48}{2\binom{3(t\text{-}1)\text{+}1}{2}}\right)$$

Proof: Consider the sunflower graph S_t name the center vertex as a_1, name the vertices of the cycle are (b_1, b_2, ..., b_{t-1}), name the pendent vertex which is adjacent to b_i is c_{2i-1} therefore, the pendent vertices are (c_1, c_2, ..., $c_{2(t-1)}$). edom(a_1, b_i) = 8 for i = 1 to t-1. Therefore:

$$\sum_{i=1}^{t-1} edom\ (a_i, b_i) = 8(t\text{-}1)$$

edom(a_1, c_i) = 4 for i(odd) = 1-2 (t-1); Therefore:

$$\sum_{i(odd)=1}^{2(t-1)} edom(a_1, c_i) = 4(t\text{-}1)$$

edom(a_1, c_i) = 4 for i(even) = 1-2 (t-1); Therefore:

$$\sum_{i(even)=1}^{2(t-1)} edom(a_1, c_i) = (t\text{-}1)$$

edom(b_i, b_{i+1}) = 6 for i = 1-t-1; Therefore:

$$\sum_{i=1}^{t-1}(b_i, b_{i+1}) = 6(t\text{-}1)$$

edom(b_i, b_{i+2}) = 7 for i = 1 to t-1; Therefore:

$$\sum_{i=1}^{t-1}(b_i, b_{i+2}) = 7(t\text{-}1)$$

edom(b_i, b_{i+3}) = 8 for i = 1 to t-1; Therefore:

$$\sum_{i=1}^{t-1}(b_i, b_{i+3}) = 8(t\text{-}1)$$

edom(b_1, b_j) = 7 for j = 5 to t-4; Therefore:

$$\sum_{j=5}^{t-4}(b_1, b_j) = 7(t\text{-}8)$$

edom(b_2, b_j) = 7 for j = 6 to t-3; Therefore:

$$\sum_{j=6}^{t-3}(b_2, b_j) = 7(t\text{-}8)$$

edom(b_3, b_j) = 7 for j = 7 to t-2; Therefore:

$$\sum_{j=7}^{t-2}(b_3, b_j) = 7(t\text{-}8)$$

edom(b_4, b_j) = 7 for j = 8 to t-1; Therefore:

$$\sum_{j=8}^{t-1}(b_4, b_j) = 7(t\text{-}8)$$

edom(b_5, b_j) = 7 for j = 9 to t-1; Therefore:

$$\sum_{j=9}^{t-1}(b_5, b_j) = 7(t\text{-}9)$$

edom(b_6, b_j) = 7 for j = 10 to t-1; Therefore:

$$\sum_{j=10}^{t-1}(b_6, b_j) = 7(t\text{-}10)$$

edom(b_{t-5}, b_{t-1}) = 7; edom(b_i, c_{i-1}) = 4 for i = 1 to t-1; Therefore:

$$\sum_{j=1}^{t-1}(b_i, c_{i-1}) = 4(t\text{-}1)$$

edom(b_i, c_j) = 6 for i = 1 to t-1; j = 1 to 2 (t-1); j ≠ 2i-1; j is odd only; Therefore, Σ edom(b_i, c_j) = 6 (t-1) (t-2) for i = 1 to t-1; j = 1-2 (t-1); j ≠ 2i-1; j is odd only. edom(b_i, c_j) = 2 for i = 1 to t-1; j = 1-2 (t-1); j is even only; Therefore Σ edom(b_i, c_j) = 2 $(t\text{-}1)^2$ for i = 1 to t-1; j = 1 to 2(t-1); j is even only. edom(c_i, c_j) = 1 for i, j-even; Therefore:

$$\sum_{i,j-even} edom\,(c_i, c_j) = (t\text{-}2)+(t\text{-}1)+, ..., +1$$

$edom(c_i, c_j) = 1$ for i, j-even; Therefore:

$$\sum_{i,j-even} edom\left(c_i, c_j\right) = (t\text{-}2)+(t\text{-}1)+, ..., +1$$

$edom(c_i, c_j) = 2$ for i-odd, j-even; Therefore:

$$\sum_{i-odd, j-even} edom\left(c_i, c_j\right) = 2(t\text{-}1)^2$$

$edom(c_i, c_j) = 4$ for i, j-odd, i = 1-2(t-1)-1, j = i+2; Therefore, Σ $edom(c_i, c_j) = 4$ (t-1) for i, j-odd, i = 1-2(t-1)-1, j = i+2; $edom(c_i, c_j) = 3$ for i, j-odd, i = 1-2 (t-1)-5, j = 5 to 2 (t-1)-1, $j \neq i+2$; Therefore, Σ $edom(c_i, c_j) = 3$ [(t-4)+(t-4)+(t-6)+(t-7)+(t-8)+, .., +1]:

$$ETDV(G) = 38(t\text{-}1)+21(t\text{-}8)+$$
$$(6t^2\text{-}18t+12)+2(t\text{-}1)^2+7\left(\frac{(t\text{-}8)(t\text{-}7)}{2}\right)+$$
$$\left(\frac{(t\text{-}2)(t\text{-}1)}{2}\right)+2(t\text{-}1)2+4t\text{-}4+6$$
$$(t\text{-}4)+3\left(\frac{(t\text{-}6)(t\text{-}5)}{2}\right)$$
$$EMD(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{31t^2\text{-}55t+48}{2\binom{3(t\text{-}1)+1}{2}}$$

RESULTS AND DISCUSSION

Extended medium domination number for jahangir graph if m = 1

Theorem 3.1: If $G = J_{1, n}$, then:

$$EMD(G) = \frac{3n^2+6}{\binom{n+1}{2}}$$

Proof: Consider the Jahangir graph $J_{1,n}$ and name the vertices of the outer cycle are (1, 2, …, n) and the center vertex name as a. edom(i, i+1) = 4 for i = 1 to n; Therefore:

$$\sum_{i=1}^{n} edom(i, i+1) = 4n \text{ for } i = 1\text{-}n$$

edom(1, x) = 6 for x = 3 to n; Therefore:

$$\sum_{x=3}^{n} edom(1, x) = 6n(n\text{-}2) \text{ for } x = 3 \text{ to } n$$

edom(2, x) = 6 for x = 4 to n; Therefore:

$$\sum_{x=4}^{n} edom(2, x) = 6n(n\text{-}3) \text{ for } x = 4 \text{ to } n$$

edom(n-1, n) = 6. edom(a, x) = 5 for x = 1 to n:

$$\sum_{x=1}^{n} edom(a, x) = 5n \text{ for } x = 1 \text{ to } n$$

$$ETDV(G) = 9n+6\left(1+2+, ..., +(n\text{-}2)\right) =$$
$$9n+6\left[\frac{(n\text{-}2)(n\text{-}1)}{2}\right] = 9n+3\left(n^2\text{-}3n+2\right) =$$
$$9n+3n^2\text{-}9n+6 = 3n^2+6$$

$$ETDV(G) = \frac{ETDV(G)}{\binom{p}{2}} = \frac{3n^2+6}{\binom{n+1}{2}}$$

Extended medium domination number for Jahangir graph if m = 2

Theorem 3.2: If $G = J_{2, n}$ then:

$$EMD(G) = \frac{5n^2+17n}{2\binom{2n+1}{2}}$$

Proof: Consider the Jahangir graph J2, n and name the vertices are (1, 2, …, 2n) for outer cycle and the center vertex as a. $edom(C_{2n}) = 6n$; edom(i, i+1) = 1 for i = 1 to 2n; Therefore:

$$\sum_{i=1}^{2n} edom(i, i+1) = 2n \text{ for } i = 1\text{-}2n$$

edom(x, y) = 1 for x, y odd number; Therefore, Σ edom(x, y) = (n-1)+(n-2)+, …, 2+1 for x, y odd number; edom(x, y) = 0 for x, y even number; Therefore , Σ edom(x, y) = 0 for x, y even number, edom(1, x) = 2 for x even number; Therefore, Σ edom(1, x) = 2 (n-2) for x even number, edom(3, x) = 2 for x even number; Therefore, Σ edom(3, x) = 2(n-2) for x even number, edom(5, x) = 2 for x even number; Therefore, Σ edom(5,x) = 2 (n-3) for x even number, edom(2n-5, 2n-2) = 2; Therefore, Σ edom (2n-5, 2n-2) = 2(2), edom(2n-3, 2n) = 2, edom(2, x) = 2 for x odd number; Therefore, Σ edom(2, x) = 2 (n-2) for x odd number, edom(4, x) = 2 for x odd number; Therefore, Σ edom(4, x) = 2 (n-3) for x odd number, edom(2n-4, 2n-1) = 2, edom(a, x) = 3 for x odd number:

$$\sum_{x=1}^{2n} \text{edom}\ (a, x) = 3n\ \text{for}\times\text{odd number}$$

edom(a, x) = 2 for x even number:

$$\sum_{x=1}^{2n} \text{edom}\left(a, x\right) = 2n\ \text{for}\times\text{even number}$$

$$\text{ETDV}(G) = 13n+\left[1+2+, ..., +(n\text{-}1)\right]+2\left[1+2+,...,(n\text{-}2)\right]+2n\text{-}4+2\left[1+2+, ..., +(n\text{-}2)\right] = 15n\text{-}4+4\left[1+2+, ..., +(n\text{-}2)\right]+\left[1+2+, ..., +(n\text{-}1)\right] = 15n\text{-}4+2n^2\text{-}6n+4+\frac{n^2\text{-}n}{2} = \frac{5n^2+17n}{2}$$

$$\text{EMD}(G) = \frac{\text{ETDV}(G)}{\binom{p}{2}} = \frac{5n^2+17n}{2\binom{2n+1}{2}}$$

Extended medium domination number for Jahangir graph in general

Theorem 3.3: If $G = J_{m,n}$, then:

$$\text{EMD}(G) = \frac{n(6m+5(n+1))}{\binom{mn+1}{2}}$$

Proof: Consider the Jahangir graph $J_{m,\,n}$ and name the outer cycle vertices are (1, 2, ..., mn) and name the center vertex as a. Join the middle vertex a to some vertices of the outer cycle say (1, m+1, 2m+1, ..., ..., ..., ..., (n-1) m+1). edom(C_{mn}) = 3mn. edom(a, 1) = edom(a, 2) = edom(a, 3) = edom(a, mn) = edom(a, mn-1) = 1. Similarly we get 5 for (n-1) set of 5 vertices say (xm+1, xm+2, xm+3, xm, xm-1) for 1 = x = n. Therefore, Σedom (a, x) = 5n for all x. edom (xm+1, ym+1) = edom(xm+1, ym) = edom(xm+1, ym+2) = edom (xm+2, ym+1) = edom(xmn, ym+1) = 1 for 0 = x = n-2, 1 = y = n-1, x < y; Therefore, we get 5[(n-1)+(n-2)+, ..., +1]:

$$\text{ETDV}(G) = \sum \text{edom}(u, v)\ \text{for u, v} \in V(G) = 3mn+5n+5\left(\frac{n(n\text{-}1)}{2}\right) = 6mn+10n+5n^2\text{-}5n = 5n^2+6mn+5n = n(6m+5n+5) = n(6m+5(n+1))$$

$$\text{EMD}(G) = \frac{\text{ETDV}(G)}{\binom{p}{2}} = \frac{n(6m+5(n+1))}{\binom{mn+1}{2}}$$

CONCLUSION

Jahangir, sunflower and windmill graph plays an vital role in special type graphs. We found extended medium domination number for so many special type graphs. By this way in this study, we investigated extended medium domination number of Jahangir graph, Sun flower graph and windmill graph. This will be very useful in computer communication network.

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REFERENCES

Buckley, F. and F. Harary, 1990. Distance in Graphs. Addison-Wesley, Boston, Massachusetts, USA., ISBN:9780201095913, Pages: 335.

Harary, F., 1972. Graph Theory. Addison-Wesley Publishing Company, USA.

Mahadevan, G., V. Vijayalakshmi and C. Sivagnanam, 2015. Extended medium domination number of a graph. Intl. J. Appl. Eng. Res., 10: 355-360.

Ramachandran, M. and N. Parvathi, 2015. The medium domination number of a Jahangir graph Jm, N. Indian J. Sci. Technol., 8: 400-406.

Ramya, N., K. Rangarajan and R. Sattanathan, 2012. On prime labeling of some classes of graphs. Intl. J. Comput. Appl., 44: 1-3.

Vargor, D. and P. Dundar, 2011. The medium domination number of a graph. Intl. J. Pure Appl. Math., 70: 297-306.