

Position Control of Vehicles with Multi-Contour Adaptation

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Abstract: The study presents multi-contour position vehicle control system ensuring astaticism and adaptive tuning of parameters. Positioning of the controlled object in a pre-defined point is considered. In the first contour the control action is calculated to provide the desired dynamics of the closed-loop system with astaticism of the second order. The second control contour performs tuning of the parameters according to the known algorithms of searchless adaptation using the reference models. An algorithm is proposed resulting in pre-set values of the characteristic equation of the tuned system. The first and the second contours of the control system compensate the external disturbances and adapt to parametric disturbances. The third contour performs tuning of the reference model that makes it possible to change the requirements to the closed-loop system depending on whether the system satisfies the limitations on control actions. When the limitations on the amplitude of control actions are reached, the system increases the time constants of the reference control. Stability of the closed-loop system was analyzed using the Lyapunov functions method. The proposed control algorithms are demonstrated using an example of a vehicle described by equations of solid body kinematics and dynamics. In the equations of dynamics the resisting forces are accounted. The disturbances are presented in a form of linear functions of time. The presented modeling results confirm the correctness of the theoretical conclusions. The proposed algorithms can be used as well for controlling the trajectory of the vehicle.

Key words: Position control, adaptive control, vehicle, reference model, multi-contour adaptation, body

INTRODUCTION

The foundations of vehicle's adaptive control were laid in the works (Parks, 1966; Zemlyakov and Rutkovskii, 1966; Donalson and Leondes, 1963; Zemlyakov, 1966) presenting the methods of synthesis of self-tuning systems. A survey of the main results obtained in the class of searchless adaptive systems is presented in the works (Voronov and Rutkovsky, 1984; Tao, 2014). At the present time this approach is being developed both theoretically and in practical applications (Rutkovsky *et al.*, 2011; Kharisov *et al.*, 2014). In the class of searchless systems one can notice the same type active development of a scientific direction connected to estimation and compensation of unmeasured disturbances (Tomita *et al.*, 1998; Nikiforov, 2004). Basing on the analysis of works on searchless adaptive systems, it can be mentioned that at the present time the most theoretical development was achieved in methods of adaptation of linear and stationary systems. So, it seems to be promising to apply the methods of adaptation of linear systems to the objects described by nonlinear equations.

This research presents a position-path control system with multi-contour adaptation. Lately, the method of position-path control (Pshikhopov and Medvedev, 2001) was successfully used in the control systems for

various vehicles (Pshikhopov *et al.*, 2014). In the research Pshikhopov *et al.* (2015) a method of synthesis of adaptive position-path control systems provides the quality of astaticism for the closed-loop system and adaptation using the reference model. This method was applied for control of autonomous underwater vehicle. By Pshikhopov *et al.* (2015) it was shown that characteristic equation of the closed-loop system consists of multiplication of 3 polynomials ensuring independent tuning of the basic controller, adaptation algorithm and astaticism contour. This research presents the development of approach presented in works (Pshikhopov *et al.*, 2015).

MATERIALS AND METHODS

Position controller with a reference model: Here, we consider a vehicle model based on equations of kinematics and dynamics of a solid body (Pshikhopov *et al.*, 2014, 2015).

$$\dot{y} = R(y)x, \quad \dot{x} = M^{-1}(F_u + F_d) \quad (1)$$

Where:

y = Vector of linear and angular coordinates of vehicle positions in external system of

- coordinates
- x = Vector of vehicle's linear and angular velocities in the bound system of coordinates
- $R(y)$ = Kinematics matrix
- M = Matrix of inertial parameters
- F_u = Vector of controlling forces and torques
- F_d = Vector of other forces and torques acting on the vehicle, vector F_d can include components depending on state variables x and y as well as external disturbances.

In addition to the Model (1) a nominal model of the following form is introduced:

$$\dot{y}_m = R(y_m)x_m, \quad \dot{x}_m = M^{-1}(F_{um} + F_{dm}) \quad (2)$$

Where:

- Y_m = Vector of linear and angular coordinates of nominal model positions in external system of coordinates
- x_m = Vector of linear and angular velocities of the nominal model in the bound system of coordinates
- $R(y_m)$ = Kinematics matrix of the nominal model
- F_{um} = Vector of controlling forces and torque of the nominal model
- F_{dm} = Nominal vector of other forces and torques. Matrix $R(Y_m)$ and vector F_{dm} match the structure of the matrix $R(y)$ and vector F_d , respectively

Let's synthesize control for the nominal Model (1). Let the requirements to the steady state model of the nominal Model (2) be represented in the form of the following equation

$$e_m = A_1 y_m + A_2 = 0 \text{ or } y_m = -A_1^{-1} A_2 \quad (3)$$

where, A_1 and A_2 matrix and vector of constant coefficients. Vector A_2 consists of elements reflecting the requirements put on positioning point and matrix A_1 is often selected to be diagonal. Let's take the first and second derivatives of the Eq. 3 accounting for the Eq. 2:

$$\dot{e}_m = A_1 \dot{y}_m = A_1 R(y_m)x_m \quad (4)$$

$$\ddot{e}_m = A_1 \dot{R}(y_m)x_m + A_1 R(y_m)M^{-1}(F_{um} + F_{dm}) \quad (5)$$

Let's require of the vector (Eq. 3) to satisfy the reference differential Eq:

$$\ddot{e}_m + T_2 \dot{e}_m + T_1 e_m = 0 \quad (6)$$

where, T_1 and T_2 positively definite diagonal matrices of constant coefficients, determining requirements to the behavior of the nominal model. Let's substitute the Eq. 3-5 in the Eq. 6 and solve it with respect to vector of controlling forces and torques F_{um} :

$$F_{um} = -F_{dm} + (A_1 R(y_m)M^{-1})^{-1} \times \{-A_1 \dot{R}(y_m)x_m - T_2 \dot{e}_m - T_1 e_m\} \quad (7)$$

In Eq. 7 it is assumed that the matrix $R(y_m)$ is nonsingular. For the solid body this requirement comes down to the pitch angle not to be equal to 90°. It is also assumed that elements of the matrix $R(y_m)$ are calculated basing of measuring the vectors x_m and y_m according to the following Eq:

$$\dot{R}_{ij}(y_m) = \sum_k \frac{\partial R_{ij}(y_m)}{\partial y_m^k} \dot{y}_m^k = \sum_k \frac{\partial R_{ij}(y_m)}{\partial y_m^k} R_k(y_m)x_m \quad (8)$$

Where:

- $R_{ij}(y_m)$ = Matrix elements $R(y_m)$
- $R_{ij}(y_m)$ = Matrix elements $R(y_m)$
- y_m^k = Vector elements y_m
- y_m^k = Vector elements y_m
- $R_k(y_m)$ -kth = Row of the matrix $R(y_m)$; $I, j, k = 1, 6$

Equations 2-4 and 7 form the reference model of the vehicle that has the following form:

$$\begin{cases} \dot{y}_m = R(y_m)x_m \\ \dot{x}_m = -(A_1 R(y_m))^{-1} (A_1 \dot{R}(y_m)x_m + T_2 \dot{e}_m + T_1 e_m) \end{cases} \quad (9)$$

Let's demonstrate stability of the equilibrium point of the reference Model (9). Assuming the derivatives in left-hand sides of the system (Eq. 9) to be equal to zero we obtain the steady state mode:

$$\begin{cases} 0 = R(y_m)x_m, \\ 0 = -(A_1 R(y_m))^{-1} (A_1 \dot{R}(y_m)x_m + T_2 \dot{e}_m + T_1 e_m) \end{cases}$$

Since, $\dot{y}_m = 0, \dot{y}_m^k = 0$ Then accounting for (Eq. 3, 4 and 8) the last system is transformed to the following form:

$$\begin{cases} 0 = R(y_m)x_m, \\ 0 = -(A_1 R(y_m))^{-1} (T_2 (A_1 R(y_m)x_m) + T_1 (A_1 y_m + A_2)) \end{cases}$$

Noting that matrix $R(y_m)$ is nonsingular from the last system we get the equations of the state variable of reference model in the steady state:

$$x_m^* = 0, y_m^* = -A_1^{-1}A_2 \tag{10}$$

Let's introduce the following function:

$$V_m = 0.5e_m^T T_1^T e_m + 0.5\dot{e}_m^T \dot{e}_m = 0.5(A_1 y_m + A_2)^T T_1^T (A_1 y_m + A_2) + 0.5(A_1 R(y_m) x_m)^T A_1 R(y_m) x_m \tag{11}$$

Now let's prove that function (Eq. 11) is a Lyapunov function for the reference model. Time derivative of the function (Eq. 11) along the system trajectories (Eq. 3-9) is:

$$\begin{aligned} \dot{V}_m &= e_m^T T_1^T \dot{e}_m + \dot{e}_m^T \dot{e}_m = e_m^T T_1^T \dot{e}_m + \dot{e}_m^T (-T_2 \dot{e}_m - T_1 e_m) \\ &= e_m^T T_1^T \dot{e}_m - \dot{e}_m^T T_2 \dot{e}_m - \dot{e}_m^T T_1 e_m = -\dot{e}_m^T T_2 \dot{e}_m - \\ &(A_1 R(y_m) x_m)^T T_2 A_1 R(y_m) x_m \end{aligned} \tag{12}$$

Nonsingularity of the matrix $R(y_m)$ results in function V_m being negative semidefinite. In order to show that V_m is negative definite, let's show that it zeroes only at the point (Eq. 10). Let's consider 3 cases:

Let $\dot{e}_m = 0$. Then accounting for Eq. 4 we get:

$$A_1 R(y_m) x_m = 0$$

Differentiating the left and the right sides of this equation yields:

$$A_1 \dot{R}(y_m) x_m + A_1 R(y_m) \dot{x}_m = 0$$

Let's rewrite the last equation as follows:

$$\dot{x}_m = (A_1 R(y_m))^{-1} (-A_1 \dot{R}(y_m) x_m)$$

Comparing the right sides of the last equation and second equation of our system (Eq. 9) yields:

$$\begin{aligned} (A_1 R(y_m))^{-1} (-A_1 \dot{R}(y_m) x_m) &= \\ (A_1 R(y_m))^{-1} (-A_1 \dot{R}(y_m) x_m - T_2 \dot{e}_m - T_1 e_m) &- \\ -T_2 \dot{e}_m - T_1 e_m &= 0 \end{aligned}$$

Since $\dot{e}_m = 0$, from the last expression we obtain:

$$e_m = 0 \text{ or } y_m = -A_1^{-1}A_2$$

So, in case function (Eq. 12) zeroes only at point (Eq. 10). Let $y_m = -A_1^{-1}A_2$ ($e_m = 0$). Suppose, $A_1 R(y_m) x_m \neq 0$ ($\dot{e}_m \neq 0$). Then y_m should change (e_m should become not equal to zero) which contradicts the supposition. The obtained contradiction proves that $y_m = -A_1^{-1}A_2$ ($e_m = 0$). So, in case the function (Eq. 12) zeroes only at point (Eq. 10) as well. c) Let $y_m = -A_1^{-1}A_2$ and $A_1 R(y_m) x_m = 0$. In this case from (Eq. 12) it follows that $V_m = 0$.

So, the function V_m (Eq. 12) zeroes only at the point (Eq. 10) at all the other points it remains negative, so, the function (Eq. 11) is a Lyapunov function or the reference Model (9). Now let's synthesize the vehicle control (Eq. 1). The following is the control system's error:

$$e = A_1 y - A_1 y_m + B_1 z_1 + B_2 z_2 \tag{13}$$

$$\dot{z}_1 = z_2, \dot{z}_2 = A_1 y - A_1 y_m \tag{14}$$

where, z_1, z_2 vectors of additional variables introduced to ensure astaticism $B_i, i = 1, 2$ -matrices of arbitrary coefficients.

Assume that the system error e (Eq. 13) satisfies the expression (Eq. 6). Calculating the first and the second time derivatives of the expression (Eq. 13) accounting for the equations (Eq. 1, 9, 13 and 14) yields:

$$\dot{e} = A_1 R(y) x - A_1 R(y_m) x_m + B_1 z_2 + B_2 (A_1 y - A_1 y_m) \tag{15}$$

$$\begin{aligned} \ddot{e} &= A_1 \dot{R}(y) x + A_1 R(y) \dot{x} - A_1 \dot{R}(y_m) x_m - A_1 R(y_m) \dot{x}_m + \\ &B_1 (A_1 y - A_1 y_m) + B_2 (A_1 R(y) x - A_1 R(y_m) x_m) \end{aligned} \tag{16}$$

Substituting e_m with e in (Eq. 6) and accounting for (Eq. 16) we obtain the following algebraic equation:

$$\begin{aligned} A_1 \dot{R}(y) x + A_1 R(y) \dot{x} - A_1 \dot{R}(y_m) x_m - \\ A_1 R(y_m) \dot{x}_m + B_1 (A_1 y - A_1 y_m) + \\ B_2 (A_1 R(y) x - A_1 R(y_m) x_m) + T_2 \dot{e} + T_1 e = 0 \end{aligned}$$

Then we rewrite the second equation:

$$\begin{aligned} A_1 R(y) \dot{x} &= -A_1 \dot{R}(y) x + A_1 \dot{R}(y_m) x_m + \\ A_1 R(y_m) \dot{x}_m &- B_1 (A_1 y - A_1 y_m) - \\ B_2 (A_1 R(y) x - A_1 R(y_m) x_m) &- T_2 \dot{e} - T_1 e \end{aligned}$$

And substitute the Eq. 9, 13 and 15 into the second equation:

$$\begin{aligned} A_1R(y)\dot{x} &= -A_1\dot{R}(y)x + A_1\dot{R}(y_m)x_m - \\ & (A_1\dot{R}(y_m)x_m + T_2A_1R(y_m)x_m + T_1(A_1y_m + A_2)) - \\ & B_1(A_1y - A_1y_m) - B_2(A_1R(y)x - A_1R(y_m)x_m) - \\ & T_2(A_1R(y)x - A_1R(y_m)x_m + B_1z_2 + B_2(A_1y - A_1y_m)) \\ & T_1(A_1y - A_1y_m + B_1z_1 + B_2z_2). \end{aligned}$$

Let's expand and simplify the last expression:

$$\begin{aligned} A_1R(y)\dot{x} &= -(B_1A_1 + T_2B_2A_1 + T_1A_1)y - \\ & (A_1\dot{R}(y) + B_2A_1R(y) + T_2A_1R(y))x + \\ & (B_1A_1 + T_2B_2A_1)y_m + B_2A_1R(y_m)x_m - \\ & T_1B_1z_1 - (T_2B_1 + T_1B_2)z_2 - T_1A_2 \end{aligned}$$

Substituting the second equation of the system (Eq. 2) into the second expression yields the following expression for the controlling forces and torques:

$$\begin{aligned} F_u &= -F_d + (A_1R(y)M^{-1})^{-1} \{ -(B_1A_1 + T_2B_2A_1 + T_1A_1)y - \\ & (A_1\dot{R}(y) + B_2A_1R(y) + T_2A_1R(y))x + \\ & (B_1A_1 + T_2B_2A_1)y_m + B_2A_1R(y_m)x_m - \\ & T_1B_1z_1 - (T_2B_1 + T_1B_2)z_2 - T_1A_2 \} \end{aligned} \quad (17)$$

Equation 1, 9, 14 and 17 are the equations of the closed-loop system. Let's bring them together for convenience

$$\begin{cases} \dot{y} = R(y)x, \\ \dot{x} = (A_1R(y))^{-1} \{ -(B_1A_1 + T_2B_2A_1 + T_1A_1)y - \\ (A_1\dot{R}(y) + B_2A_1R(y) + T_2A_1R(y))x + \\ (B_1A_1 + T_2B_2A_1)y_m + B_2A_1R(y_m)x_m - \\ T_1B_1z_1 - (T_2B_1 + T_1B_2)z_2 - T_1A_2 \}, \\ \dot{y}_m = R(y_m)x_m, \\ \dot{x}_m = -(A_1R(y_m))^{-1} (A_1\dot{R}(y_m)x_m + \\ T_2A_1R(y_m)x_m + T_1(A_1y_m + A_2)), \\ \dot{z}_1 = z_2, \\ \dot{z}_2 = A_1y - A_1y_m. \end{cases} \quad (18)$$

The structure of the closed-loop system is presented in Fig. 1. It matches the structure of a position-path control system with astaticism of the second order provided by introduction of two integrators.

Assuming that the derivatives in the left side of the system (Eq. 18) are equal to zero, let's consider the steady state mode. Note that $R(y) = R(y_m)$ in the steady state:

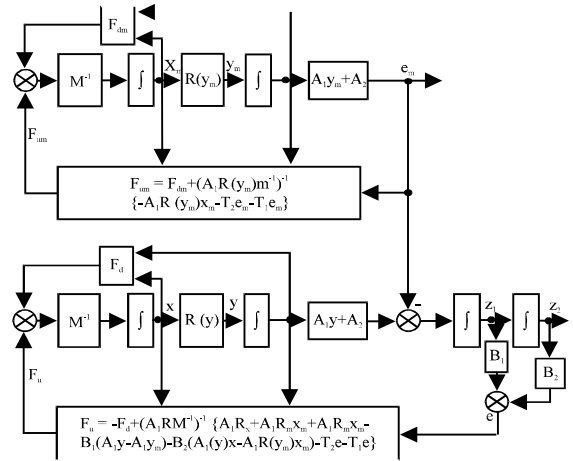


Fig.1: Structure of Astatic Position Control System with a Reference Model

$$\begin{cases} 0 = R(y)x, \\ 0 = (A_1R(y))^{-1} \{ -(B_1A_1 + T_2B_2A_1 + T_1A_1)y - \\ (A_1\dot{R}(y) + B_2A_1R(y) + T_2A_1R(y))x + \\ + (B_1A_1 + T_2B_2A_1)y_m + B_2A_1R(y_m)x_m - \\ - T_1B_1z_1 - (T_2B_1 + T_1B_2)z_2 - T_1A_2 \}, \\ 0 = R(y_m)x_m, \\ 0 = -(A_1R(y_m))^{-1} (A_1\dot{R}(y_m)x_m + \\ + T_2A_1R(y_m)x_m + T_1(A_1y_m + A_2)), \\ 0 = z_2, \\ 0 = A_1y - A_1y_m. \end{cases} \quad (19)$$

From the last equation it follows that:

$$y = y_m \rightarrow R(y) = R(y_m) = R \quad (20)$$

Since, in the steady state the derivatives are equal to zero, we obtain:

$$\dot{y} = \dot{y}_m \rightarrow Rx = Rx_m \rightarrow x = x_m \quad (21)$$

Substituting (Eq. 20 and 21) into the first five equations of the system (Eq. 19), we find:

$$\begin{cases} 0 = Rx, \\ 0 = -T_1A_1y - T_2A_1Rx - T_1B_1z_1 - \\ (T_2B_1 + T_1B_2)z_2 - T_1A_2, \\ 0 = Rx, \\ 0 = -T_2A_1Rx - T_1A_1y - T_1A_2, \\ 0 = z_2 \end{cases} \quad (22)$$

Expressing T_1A_2 from the fourth equation of (Eq. 22) and substituting it together with the 5th equation into the second equation of (Eq. 22) yields:

$$0 = -T_1B_1z_1$$

So, the equilibrium point of the closed-loop control system is defined by the following expressions:

$$y^* = y_m^* = -A_1^{-1}A_2, x^* = x_m^* = 0, z_1^* = z_2^* = 0 \quad (23)$$

Asymptotic stability of the equilibrium point (Eq. 23) is analyzed using a Lyapunov function that has the following form:

$$\begin{aligned} V = & 0.5e_m^T T_1^T e_m + 0.5\dot{e}_m^T \dot{e}_m + 0.5e^T T_1^T e + 0.5\dot{e}^T \dot{e} = \\ & 0.5(A_1 y_m + A_2)^T T_1^T (A_1 y_m + A_2) + \\ & 0.5(A_1 R(y_m) x_m)^T A_1 R(y_m) x_m + \\ & 0.5(A_1 y - A_1 y_m + B_1 z_1 + B_2 z_2)^T T_1^T \\ & (A_1 y - A_1 y_m + B_1 z_1 + B_2 z_2) + \\ & 0.5(A_1 R(y) x - A_1 R(y_m) x_m + B_1 z_2 + B_2 (A_1 y - A_1 y_m))^T \\ & (A_1 R(y) x - A_1 R(y_m) x_m + B_1 z_2 + B_2 (A_1 y - A_1 y_m)) \end{aligned} \quad (24)$$

Structure of the function (Eq. 25) lets us claim that it is negative semi-definite. In order to prove that is negative definite, we should prove that it becomes equal to zero at the point (Eq. 23) only. The first item of the expression (Eq. 25) matches the expression (Eq. 12). So, let's consider the second item. There are 3 cases.

Assume $e = 0$ then:

$$A_1 y - A_1 y_m + B_1 z_1 + B_2 z_2 = 0$$

Differentiating the last expression we get:

$$\begin{aligned} A_1 R(y) x - A_1 R(y_m) x_m + \\ B_1 z_2 + B_2 (A_1 y - A_1 y_m) = \dot{e} = 0 \end{aligned}$$

Now, assume $\dot{e} = 0$, then:

$$A_1 R(y) x - A_1 R(y_m) x_m + B_1 z_2 + B_2 (A_1 y - A_1 y_m) = 0$$

Differentiating the last expression with respect to time yields:

$$\begin{aligned} A_1 \dot{R}x + A_1 R\dot{x} - A_1 \dot{R}x_m - A_1 R_m \dot{x}_m + \\ B_1 (A_1 \dot{y} - A_1 \dot{y}_m) + B_2 (A_1 R(y) x - A_1 R(y_m) x_m) = 0 \end{aligned}$$

Substituting the derivative \dot{x} from the system (Eq. 18) into the last equation and simplifying, we get:

$$-T_2 \dot{e} - T_1 e = 0$$

Assuming $\dot{e} = 0$ from the last expression we see that $e = 0$. Let $e = 0$ and $\dot{e} = 0$. In this case from (Eq. 25) we see that, the second item of V is equal to zero.

For automatic tuning of the matrices B_1 and B_2 the algorithms basing on the known results (Zemlyakov and Rutkovskii, 1966; Zemlyakov, 1966) were used. We were using the following procedure. As it was shown by (Pshikhopov *et al.*, 2015) in the linear case the adaptive system is described by the characteristic equation of the following form

$$D(s) = (Is^2 + T_2s + T_1)(Is^2 + B_2s + B_1)(Is^2 + T_2s + T_1) \quad (25)$$

where, I -a unity matrix:

Assume that matrices B_1 and B_2 are selected so that the roots of the second item of the characteristic equation (Eq. 26) are real and are connected by a certain relation:

$$\begin{aligned} Is^n + B_2s + B_1 = (Is + S_1)(Is + aS_1) = \\ Is^2 + (1+a)S_1s + aS_1^2 \end{aligned} \quad (26)$$

Where:

- S_1 = Matrix of adjustable parameters
- a = Positive constant coefficient

In this case adaptation of matrices B_1 and B_2 can be performed using the following expression:

$$\dot{s}_1 = -?_1 z_1^T A_1 (y - y_m), B_1 = a s_1^2, B_2 = (1+a) s_1 \quad (27)$$

RESULTS AND DISCUSSION

Numerical modeling results: Let's consider a vehicle described by the following matrices of kinematics and inertial parameters:

$$R = \begin{bmatrix} A & 0 \\ 0 & A_7 \end{bmatrix}$$

$$A = \begin{bmatrix} \cos? \cos J & -\cos? \sin J \cos? + \cos? \sin J \sin? + \\ & + \sin? \sin? & + \sin? \cos? \\ \sin J & \cos J \cos? & -\cos J \sin? \\ -\sin? \cos J & \cos? \sin? + \cos? \cos? - \\ & + \sin? \sin J \cos? & -\sin? \sin J \sin? \end{bmatrix}$$

$$A_\gamma = \begin{bmatrix} 0 & \frac{\cos\theta}{\cos\theta} & \frac{\sin\theta}{\cos\theta} \\ 0 & \sin\theta & \cos\theta \\ 1 & -\text{tg}\theta\cos\theta & \text{tg}\theta\sin\theta \end{bmatrix}$$

$$M = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30 & 0 & 0 \\ 0 & 0 & 0 & 0 & 200 & 0 \\ 0 & 0 & 0 & 0 & 0 & 200 \end{bmatrix}$$

The vector of dynamic forces and torques is defined by the following expressions:

$$F_{dm} = F_d^0 = 0, 5s^2 V^2 \begin{bmatrix} -c_x \\ c_y \\ c_z \\ m_x l \\ m_y l \\ m_z l \end{bmatrix}$$

Where:

- c_x, c_y, c_z = Coefficient of environmental resistance forces
- m_x, m_y, m_z = resistance forces
- ρ = 1000 kg/m³-water density
- V = Object's linear speed M/c
- s = 0.18 M² l = 3 M
- $c_x = 0,06 - 0,003V - 0,142a - 0,05\beta$
- $c_y = -0,0009 + 1,07a + 0,31a|a| - 0,0077|\beta| - 0,398\beta^2$
- $c_z = -1,207\beta - 0,563\beta|\beta|$
- $m_x = 0,098\beta + 0,162a\beta - 0,056a\beta|\beta|$
- $m_y = 0,071\beta + 0,042\beta|\beta|$
- $m_z = 0,00058 + 0,031a + 0,086a|a|$
- Ψ, J, γ Euler angles; α and β -pitch and yaw angles

The modeling results for the closed-loop system are presented in Fig. 2. Controller parameters are $T_1 = 0.25I, T_2 = I, A_1 = I, A_2 = -[10 \ 10 \ 10 \ 0 \ 0 \ 0]^T, I-6 \times 6$ unity matrix. Adaptation contour parameters: $\gamma_1 = 1, S_1(0) = \text{diag}(3); \alpha = 1$.

Figure 2a presents variables y_{m2} and y_2 . Figure 2b presents changing of the tuning parameters according to the expressions (Eq. 22). Changing of the initial value S_1 reduces the control system's error. An increase of S_1 and therefore, coefficients T_1 and T_2 , leads to an increase in amplitude of control actions that can be unacceptable due to limitations put on control. Unmeasured parametric and external disturbances:

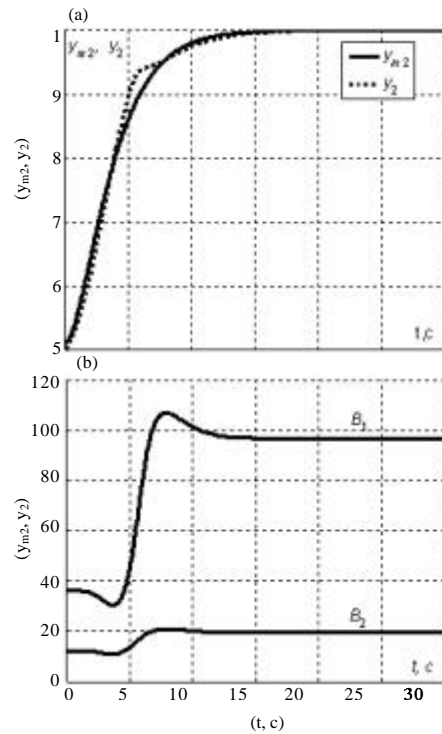


Fig. 2: Adaptive control system modeling results

$$F_d = 0, 3F_d^0 + [5+0,2t \quad -3+0,5t \quad 1+0,3t \quad 0 \quad 0 \quad 0]^T$$

In order to account for limitations, we propose to introduce another adaptation contour changing the parameters of the reference model T_1, T_2 , depending on the amplitude of controlling forces and torques. Particularly, we propose add the following algorithms to the considered adaptive system.

$$\begin{aligned} &\text{if } \text{abs}(F_{ui}) > u_i^{\text{max}} \\ &\text{then } t_i = t_i^0 - \frac{t_i^0 - t_i^{\text{min}}}{u_i^{\text{sup}}} (\text{abs}(F_{ui}) - u_i^{\text{max}}) \quad (28) \\ &\text{if } t_i < t_i^{\text{min}} \text{ then } t_i = t_i^{\text{min}} \end{aligned}$$

where, u_i^{max} -maximal allowed control actions; t_i - parameter determining the of eigenvalues of matrices T_1, T_2 ; t_i^0 -values of parameter t_i for controls within the limitations; t_i^{min} -minimal value of parameter t_i ; u_i^{sup} -calculated value of control, when parameter t_i reaches the value of t_i^{min} . Figure 3 explains the parameters introduced in Eq. 23.

Figure 4 presents the modeling results for the considered adaptive control system using algorithm (Eq. 23) to tune the parameters of the reference control (Eq. 6). Figure 4a presents variables y_{m2} and y_2 , and Fig. 4b

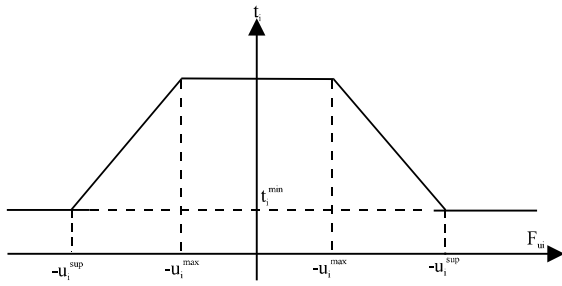


Fig. 3: Changing of parameters of matrices

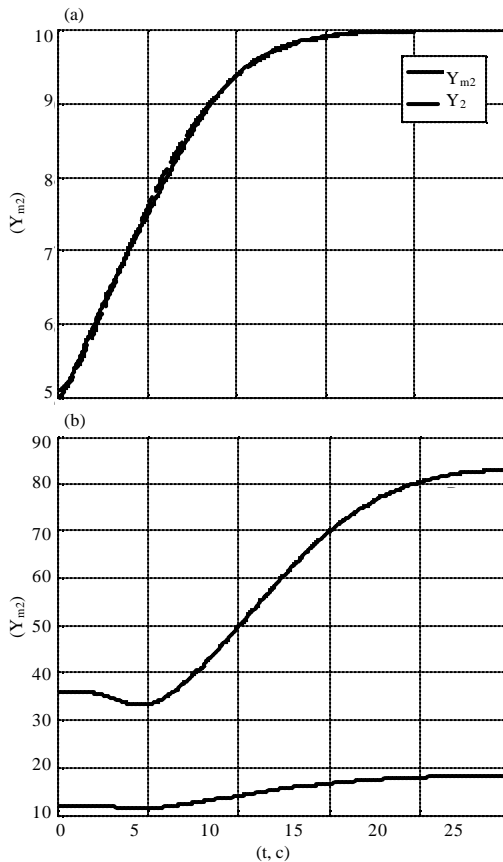


Fig. 4: Modeling of the adaptive system with a reference control

coefficients of the matrices B_1 and B_2 . Comparing Fig. 2a and 4a we can see that transients in the system increased their duration by approx. 20%, yet the accuracy of following the reference signal increased, significantly.

CONCLUSION

The proposed procedure of adaptive control synthesis made it possible to synthesize position-path vehicle control algorithms. Three contours were

introduced into the system. The first one forms a basic control algorithm ensuring astaticism of the closed-loop system. The second contour ensures adaptation of controller's parameters according to the pre-set reference signal. Finally, the third one ensures fine-tuning of the reference object itself depending on whether the control action limitations are satisfied or not. This research develops a method of adaptation in a position-path control system for a vehicle, a method that was originally presented by Pshikhopov *et al.* (2015). Unlike the approach presented by Pshikhopov *et al.* (2015), this study performs an analysis of a closed-loop nonlinear system. Besides, we presented a new adaptation algorithm (Eq. 22) for the coefficients of the matrices B_1 and B_2 . Additionally, an adaptation algorithm for the reference system is proposed.

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