

## Performance Analysis of Inference System According to Partition of Input Space by Means of Gaussian Function

Dong Yoon Lee

Department of Electrical and Electronic Engineering, Joongbu University,  
10279 Goyang, South Korea

**Abstract:** To do modeling of a inference system is necessary to analyze the properties of input-output of inference systems by means of the partition of entire input spaces and the reasoning methods and modeling presents the premise part identification and the consequent part identification. In this study, identification in the premise part separates input space by using Min-Max method and HCM clustering algorithm constructing input output data into the hard clusters and the consequent part is presented by a constant or polynomial functions in the form of simplified inference and quadratic inference and the identification of coefficients expresses the standard least square method. Membership function of the first half is separating input space into relation space partition and respective space partition by using Gaussian function and by using gas furnace process data which is widely used in intelligence science, we evaluate the performance. The generation of inference rules has the problem that the number of inference rules exponentially increases but we divide the input space into the scatter form by using HCM clustering algorithm.

**Key words:** Min-max method, HCM clustering algorithm, simplified inference, quadratic inference  
Gaussian function, polynomial functions

### INTRODUCTION

The membership function is determined as a Gaussian function to identify the fuzzy model in the frontal structure identification and the Min-Max method and the HCM clustering algorithm are used to identify the first-half parameters. The HCM clustering algorithm measures the proximity of the data based on distance by constructing each input space divided by cluster center to form and classify information as a cluster (Pedrycz, 1984). In the second half identification, the inference method is represented by simple inference and quadratic inference and the latter half parameter identification uses the standard least squares method. In order to analyze the characteristics of input/output space and performance, we use process data with gas used by Box and Jenkins widely used in the field of intelligence (Lee, 2014; Box and Jenkins, 1976).

**Fuzzy identification:** In the modeling, the first half identification has an important influence on the selection of membership function, spatial partitioning of input variables and performance of reasoning system. In this paper, we use the Gaussian form as the first half membership function (Park and Lee, 2011). Represent the following Eq. 1. Gaussian membership function:

$$f(x, \sigma, c) = e^{-\frac{(x-c)^2}{2\sigma^2}} \quad (1)$$

The second half of the reasoning system uses simplified reasoning and quadratic reasoning method. The second half is simplified reasoning with only a single constant term and the second half is quadratic inference with the polynomial form of a quadratic function. The standard least squares method is used for identification of the latter part.

The output is the carbon dioxide concentration and the input is the gas flow rate and 296 pairs of time series data with 1 output 1 input gas are used as the output  $y(t)$  for the experiment. With time delay  $[y(t-3), y(t-2), y(t-1), u(t-3), u(t-2), u(t-1)]$  as input  $y(t-1)$  and  $u(t-3)$  which represent the best performance among the inputs are used as inputs. Then, a two-dimensional system is modeled by constructing a one-output two-input system (Park *et al.*, 2015; Park and Lee, 2012). Table 1-4 are divided into mutual space division and individual space

Table 1: Performance index of gaussian function (simplified inference)-relation

Method	No. of MFs	PI	E PI
Min-Max	4×4	0.0919	0.3566
	5×5	0.0351	0.3096
	6×6	0.0203	0.2465
Clustering	4×4	0.1443	0.7867
	5×5	0.1217	0.5655
	6×6	0.0775	0.5801

Table 2: Performance index of gaussian function (simplified inference)-respective

Method	No. of MFs	PI	E PI
Min-Max	4×4	0.1428	0.5873
	5×5	0.1052	0.5569
	6×6	0.0519	0.4543
Clustering	4×4	0.1941	0.7192
	5×5	0.1920	0.6900
	6×6	0.1517	0.5034

Table 3: Performance index of gaussian function (quadratic inference)-relation

Method	No. of MFs	PI	E PI
Min-Max	4×4	0.0113	0.6392
	5×5	0.0096	0.6275
	6×6	0.0090	0.5089
Clustering	4×4	0.0115	0.3369
	5×5	0.0086	0.3190
	6×6	0.0055	0.2579

Table 4: Performance index of gaussian function (quadratic inference)-respective

Method	No. of MFs	PI	E PI
Min-Max	4×4	0.0134	0.3559
	5×5	0.0133	0.3210
	6×6	0.0129	0.3051
Clustering	4×4	0.0150	0.2987
	5×5	0.0126	0.3016
	6×6	0.0116	0.2910

division according to input space division. We show the performance index by the learning data and test data obtained by the Min-max method, the input space division of Gaussian function by HCM clustering algorithm and the inference method when there are 4-6 Gaussian functions for each input. (Table 1) shows the case where the second half structure is simplified reasoning by mutual space division and (Table 2) is the case where the second half structure is simplified reasoning by individual space division. (Table 3) shows the case where the second half structure is the quadratic form due to the mutual space division.

Table 4 shows the case where the second half structure is the quadratic form due to the individual space division (Nagalakshmi and Uthra, 2016; Kumar *et al.*, 2015). Here, the number of Membership Functions (MFs) represents the number of membership functions by each input and the performance index for 145 training data is shown as PI (Performance Index) and the performance index for 145 test data for model testing is shown as E\_PI (Evaluation\_Performance Index). In Table 1-4 as the number of membership functions increases, the performance becomes better.

Comparing the simplified reasoning with the quadratic reasoning method, the quadratic reasoning method has better overall performance. Fig. 1-2 are a mutual space division according to input space division and Fig. 3 and

4 show the membership function by the Min-Max method and the HCM clustering algorithm and the membership function by simplified inference method when there are six Gaussian functions for each input. Fig. 1-8 represent the Gaussian membership function of variable x1 in u (t-3) and variable x2 in y (t-1) in the first row, first column and second column. The Min-Max method of uniformly dividing the input/output data by the number of membership functions is to divide the space in the portion corresponding to the minimum or maximum of the data.

The HCM clustering algorithm which classifies input data into clusters based on the characteristics of the data, and classifies the data as adjacent to each other with reference to each other, shows that the input/output space is unevenly distributed at the center of the cluster.

As shown in the result of the membership function, the characteristics are reflected even when the space is divided in consideration of the characteristics of the input/output data in the output of the inference system. Figure 5 and 6 are mutual space divisions according to input space division and Fig. 7 and 8 show the membership function by the quadratic inference when the number of Gaussian functions is 6 for each input and the input space division of the Gaussian function by the HCM clustering algorithm, respectively, as the individual space division. The second row, column 1, shows the first half fitting strength of the two inputs u (t-3) and y (t-1) as a membership function defined by the Min-Max method and HCM. The y-axis and x-axis represent the input space and the z-axis represents the degree of belonging. In the second row and the second column, the normalized firing strength is obtained by normalizing the first half fitting to [0, 1] because the two input variable values of the variables x1 and x2 are different from each other. It is the same before and after normalization as shown in the first column. In the third row and the first column, the simplified reasoning method is represented by a constant, and the quadratic reasoning method is represented by a polynomial expression. The third row and the second column are model output pictures with the z-axis range normalized to [0 100]. Column 1 of row 4 is the unqualified input/output space which is the data before the normalization which shows the training data in the z-axis range [45 65]. The 4 rows and 2 columns also, show evaluation data as input/output space with [45 65] in the z-axis range. Except for Fig. 3 and 4 all figures indicate that the data is diverging from an area in which data does not exist.

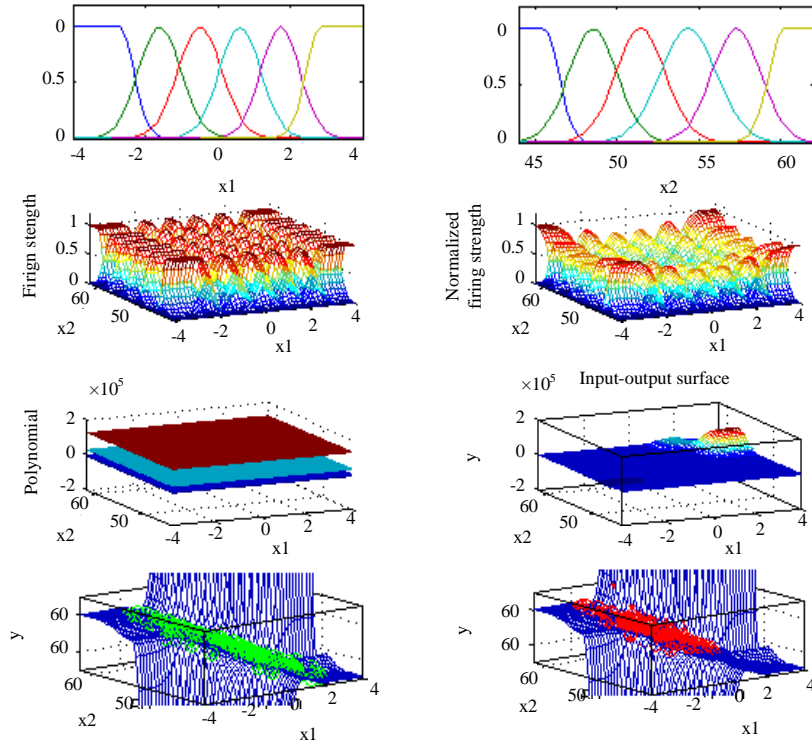


Fig. 1: Membership function characteristics according to relation input partition (Min-max method-simplified inference)

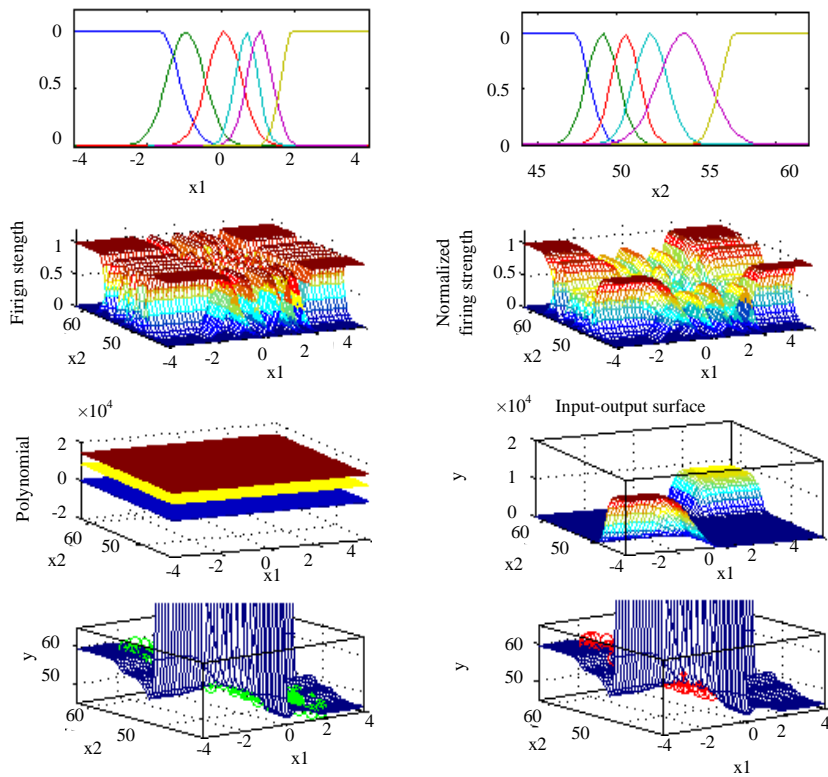


Fig. 2: Membership function characteristics according to relation input partition (HCM-simplified inference)

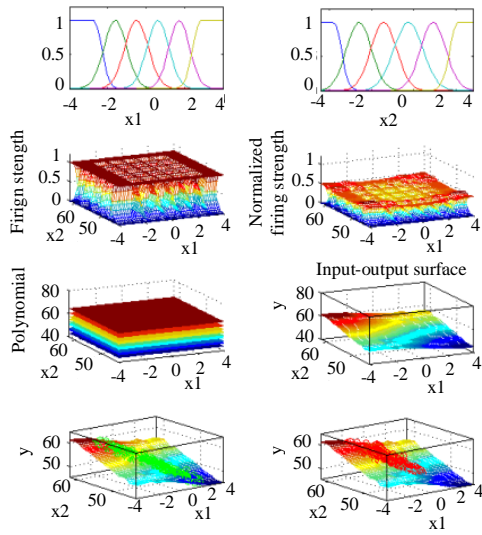


Fig. 3: Membership function characteristics according to respective input partition (Min-max method-simplified inference)

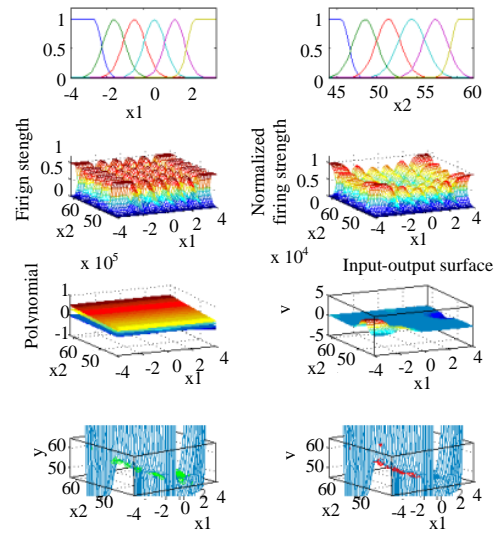


Fig. 5: Membership function characteristics according to relation input partition (Min-max-quadratic inference)

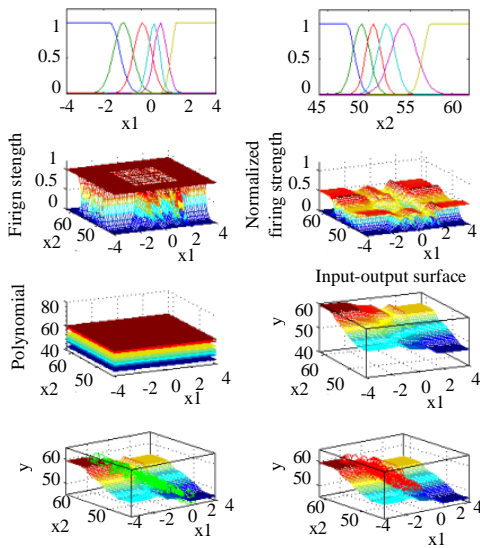


Fig. 4: Membership function characteristics according to respective input partition (HCM-simplified inference)

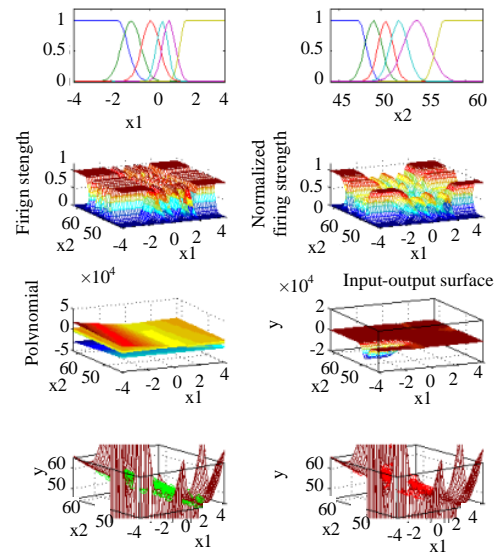


Fig. 6: Membership function characteristics according to relation input partition (HCM-quadratic inference)

The minimum-maximum value indicates that the input/output space is divided uniformly with the data and the HCM can see that the input/output space is unevenly distributed at the center of the cluster. Also, in the part where space overlaps, the structure of the second half is shown according to the combination of Gaussian function, simplified inference

and quadratic inference. It can be seen that the space output form depends on the posterior structure in the space where the space does not overlap. In the latter part of the rule, simple reasoning is expressed as a single constant and quadratic expression is expressed as a quadratic function.

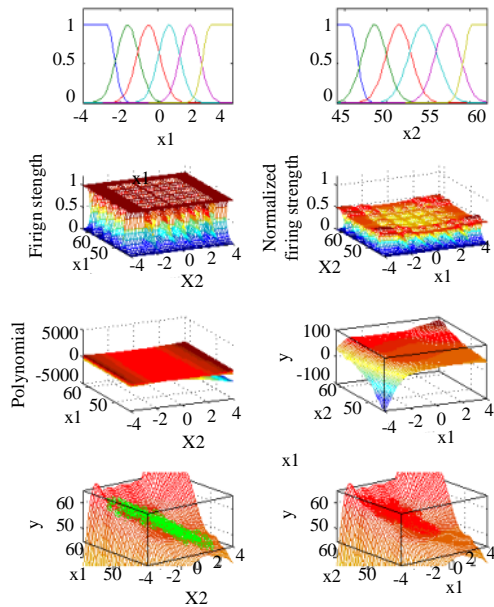


Fig. 7: Membership function characteristics according to respective input partition (min-max-quadratic inference)

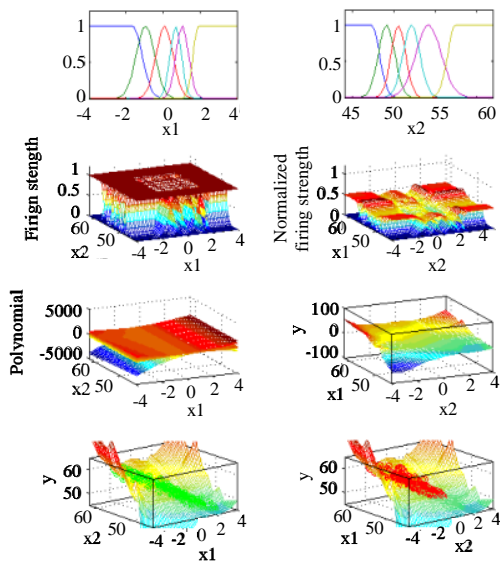


Fig. 8: Membership Function Characteristics according to Respective input Partition (HCM-Quadratic Inference)

**CONCLUSION**

In this study, to analyze the performance of the inference system, the input space is divided by the mutual space division and the individual space division. In the first half of the paper, HCM clustering algorithm and

Min-max method are developed to construct a reasoning system for nonlinear process. Gaussian function was used for the first half structure and the divided subspaces were formed by using 4-6 membership functions. In the second half identification, the inference method is performed by simplified reasoning and quadratic reasoning, and the latter half structure is expressed by the standard least squares method. Experimental results show that when the input space is divided, the characteristics of the inference system are reflected in the output. That is, it can be seen that the input-output space is divided by the minimum-maximum value of the input/output data and the HCM clustering algorithm. Also, it can be seen that each local space according to the fuzzy rule of input/output space is formed according to the division position and shape of the input space and the structure of the latter half. It can be seen that as the number of rules increases, the error value becomes smaller and the performance is further improved. Comparing simplified inference and quadratic inference methods, quadratic inference method shows better overall performance. The reasoning model based on the individual space division shows similar or somewhat error results to the inference model based on the mutual space division. However, by forming fuzzy rules by fuzzy division of each input space, the model can be designed. And today in the Internet of Things (IoT) era, all objects are connected to the Internet and a lot of data is acquired from sensors, so that, system analysis, prediction and inference can be done by data.

**ACKNOWLEDGEMENT**

This study was supported by Joongbu University Research and Development Fund in 2018.

**REFERENCES**

Box, G.E.P and G.M. Jenkins, 1976. Time Series Analysis, Forecasting and Control. Holden-Day, San Francisco.  
 Kumar, B.V., G.R. Karpagam and N.V. Rekha, 2015. Performance analysis of deterministic centroid initialization method for partitional algorithms in image block clustering. Indian J. Sci. Technol., 8: 63-73.  
 Lee, D.Y., 2014. Characteristics of gas furnace process by means of partition of input spaces in trapezoid-type function. J. Digital Convergence, 12: 277-283.  
 Nagalakshmi, T. and G. Uthra, 2016. A fuzzy approach in finding an optimal solution of a fuzzy reliability problem. Indian J. Sci. Technol., 9: 1-5.

- Park, K.J. and D.Y. Lee, 2011. Characteristics of fuzzy inference systems by means of partition of input spaces in nonlinear process. *J. Korea Contents Assoc.*, 11: 48-55.
- Park, K.J. and D.Y. Lee, 2012. Nonlinear characteristics of non-fuzzy inference systems based on HCM clustering algorithm. *J. Korea Academia Ind. Cooperation Soc.*, 13: 5379-5388.
- Park, K.J., J.P. Lee and D.Y. Lee, 2015. Design of scatter partition-based fuzzy neural networks using FCM clustering and genetic optimization for pattern recognition. *Intl. Inf. Inst. Tokyo Inf.*, 18: 2091-2097.
- Pedrycz, W., 1984. An identification algorithm in fuzzy relational systems. *Fuzzy Sets Syst.*, 13: 153-167.