Journal of Engineering and Applied Sciences 13 (Special Issue 12): 9431-9437, 2018
ISSN: 1816-949X


# Fuzzy Logic Application to Coaxial and Non-Coaxial Components of Shear Strength of Soils

[1]J. Rajaraman, [1]K. Thiruvenkatasamy and [2]S. Narasimha Rao
[1]Department of Harbour and Ocean Engineering,
AMET University, 135 East Coast Road, Kanathur, 603 112 Chennai, India
[2]Dredging Corporation of India (Govt. of India), Chennai, India

**Abstract:** Fuzziness is explored as an alternative to randomness for describing uncertainty. The new sets as points geometric view of fuzzy sets is developed. This view identifies a fuzzy set with a point in a unit hyper cube and a non-fuzzy set with a vertex of the cube. In fuzzy logic a crisp set is a set in which all members match the class concept and the class boundaries are sharp. The degree to which an individual observation z is a member of the set is expressed by the membership function F which can take the value of 0 or 1 for Boolean sets. In fuzzy logic the membership function is a number in the range of 0.1 with 0 representing non-membership of the set and 1 representing full membership of the set. In fuzzy sets, the grade of membership is reflected in terms of a scale that can fluctuate continuously between 0 and 1. Normally, we can visualize the membership function by having x axis with the element of the fuzzy set and y axis with the value of the membership function. There are better ways of visualization. "Kosko cube" is the starting point. Kosko uses the term" fit" values to designate the value of the membership function. This is to contrast the term "bit" which is used for binary values the f stands for "fuzzy". To achieve better results the fuzzy power set $F(2^x)$ is examined. Thus, the $2^x$ is the set of all sub-sets of x and x is our universe of discourse. The set of all sub-sets equals the unit hyper cube $I^n = [0.1]^n$. A two dimensional version of Kosko cube is considered. In Geology/sediments/soil samples usually consists of clay and sand in ideal conditions. If p is percentage of clay then (1-p) is % of sand. Clay properties relate to pure shear or cohesion or coaxial shear component of the sample. Similarly the sand % represents simple shear or friction or non-coaxial shear component. Fuzzy logic suits this problem when % are expressed in the range 0, 0.1, 0.2, etc., up to 1.0. The membership functions in the combination of coaxial and non-coaxial shear component is represented in kosko cube. Taking Skempton's experimental data (clay+sand) points distributed in quadrants 1-4 inside the cube as points in are interpreted through simple geological processes.

**Key words:** Fuzzy logic, coaxial and non-coaxial shear, Kosko cube, membership function, experimental data, geological processes

## INTRODUCTION

Zadeh writes, "The notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many a respects the framework used in the case of ordinary sets but it is more general than the latter and potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing (Kosko, 1990). Essentially, such a framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables.

**The geometry of fuzzy sets:sets as points (Kosko cube):** Consider the set of two elements $X = \{x_1 x_2\}$. The non-fuzzy power set $2^X$ contains four sets: $2^X = \{ \varnothing, X, \{x_1\}, \{x_2\}\}$. These four sets correspond, respectively to four bit vectors (0.0), (1.1), (1.0) and (0.1). This 1s and 0 s indicate the presence or absence of the ith element $x_i$ in the sub-set. More abstractly, each sub-set A is uniquely defined by one of the two valued membership functions $m_A$: $X \rightarrow \{0.1\}$.

Now, consider the fuzzy sub-sets of X. The fuzzy sub-set A = (1/3, 3/4) can be viewed as one of the continuum many continuous valued membership functions $m_A$: $X \rightarrow \{0.1\}$. Indeed this is the classical Zadeh sets as functions definition of fuzzy sets. In this

**Corresponding Author:** J. Rajaraman, Department of Harbour and Ocean Engineering, AMET University, 135 East Coast Road, Kanathur, 603 112 Chennai, India

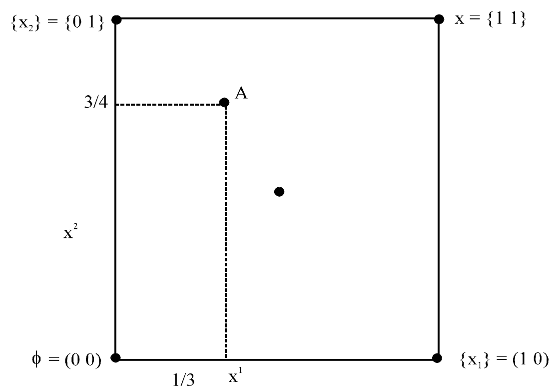


Fig. 1: Kosko cube with particular membership values

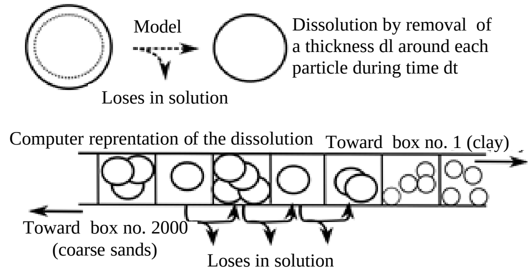


Fig. 2: Model of dissolution of the skeleton grains

example, element $x_1$ belongs to or fits in, sub-set A a little bit-to degree 1/3. Element $x_2$ has more membership than not at 3/4. Analogous to the bit vector representation of finite (countable) sets, we say that A is represented by the fit vector (1/3, 3/4). The element $m_A(x_i)$ is the ith fit or fuzzy unit value. The set as points view then geometrically represents the fuzzy sub-set A as a point in $1^2$, the unit square as in Fig. 1. The midpoint of $1^n$ maximally fuzzy. All its membership values are 1/2. The midpoint is very unique.

**A conceptual model of dissolution of the skeleton grains:** Here, dissolution is the ensemble of phenomenon that chemically attack the particles of various sizes, whether through solubilization or by hydrolysis (Legros, 2012). In this model, the particles are sorted by size in a table. The particles of diameter 200 μm will be in box No. 200. Since, dissolution being gradual, it consists of translocating the particles from box to box without ever skipping a box. This process comes to a halt when the thickness taken off is dl (Fig. 2).

In this a collection of particles containing sands, silts and clays at the same time, the point representing which is located at the centre of the textural triangle as shown in Fig. 3 as point X. In reality, the smallest particles have a greater surface area/mass ratio. They are therefore, more

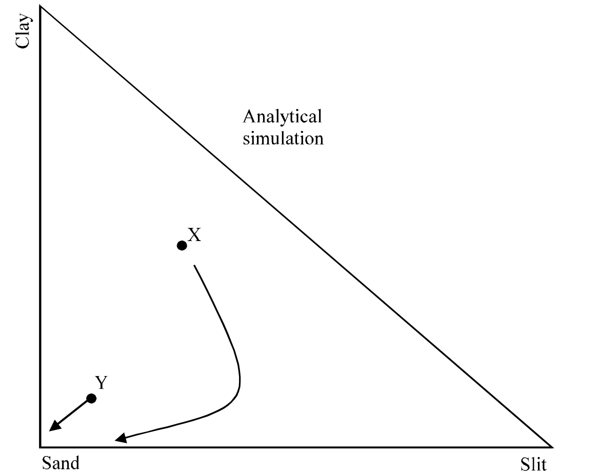


Fig. 3: Result of a simulated dissolution in a textural triangle

susceptible to dissolution and disappear in greater proportion by weight. The result is a trajectory towards the sand corner, at the cost of a very pronounced loss in total mass as shown in the Fig. 3.

Dissolution at first affects mainly the particles of clay size, especially, the fine clays, leading to an evolution towards the base of the triangle. Then the silts are attacked in their turn whereby an inflexion is seen in the trajectory towards the sand corner.

If we start from a collection of already sandy particles (point Y) there is no trajectory observable by simulation (Pluijm, 2003). The point representing the collection remains almost in place in the triangle. In other words, it is impossible to create a clayey or even silty material starting from a sand composed of particles of size 2000 μm. When the diameter will be reduced to 1000 μm this sand will continue to be counted in the coarse sands (particle size class 2000-200 μm). The size class will not change in the system of measurement. However, it would have lost 87.5% of its mass. The chemical reduction of sands does not change the particle size class.

## MATERIALS AND METHODS

**Isotropic stress and deviatoric stress:** The amount of water existing in the soil mass will significantly influence the engineering behavior of soil. Karl Terzaghi has said in effect that there would be no need for soil mechanics if not for water. This is because the presence of water affects the state of stress within a soil mass (Jensen, 1993). The water content also has bearing on potential volume change, progressive failure, densification, shear strength and settlement. The mechanism of soil water interaction is complex and its behavior is not only

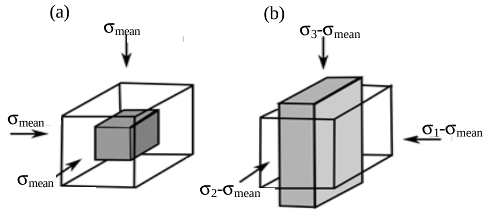


Fig. 4: The mean (hydrostatic) and deviatoric components of the stress: a) Mean stress causes volume change and b) Deviatoric stress causes shape change

dependent on soil types but is also, related to the current and past environmental conditions and stress histories isotropic stress and deviatoric stress Isotropic stress acts equally in all directions, it results in a volume change of the body. Deviatoric stress, on the other hand, changes the shape of a body as shown in Fig. 4.

**The concept of coaxial and non-coaxial components of shear strength:** In a homogeneously strained, two dimensional body there will be at least two material lines that do not rotate relative to each other, meaning that their angle remains the same before and after strain (Skempton, 1964). A material line connects features such as an array of grains that are recognizable throughout a body's strain history. The circle deforms and changes into an ellipse.

In homogeneous strain, two orientations of material lines remain perpendicular before and after strain. These two material lines form the axes of an ellipse that is called strain ellipse. The principal incremental strain axes rotate to the finite strain axes, a scenario that is called non-coaxial strain accumulation. The case in which the same material lines remain the principal strain axes at each increment is called coaxial strain accumulation. The coaxial component of shear strength is called pure shear and the non-coaxial component of shear strength is called simple shear. The combination of simple shear (a special case of non-coaxial strain) and pure shear (coaxial strain) is called general shear or general non-coaxial strain. Two types of general shear are possible. It is explained in Fig. 7. The Fig. 5 explains simple shear and pure shear.

In Fig. 5 the rigid spheres slide past one another to accommodate the shape change without distortion of the individual marbles. In Fig. 4b the shape change is achieved by changes in the shape of individual clay balls to ellipsoids arequite different.

In Figure 6 homogeneous strain describes the transformation of a square to a rectangle or a circle to an ellipse. Two material lines that remain perpendicular before and after strain are the principal axes of the strain ellipse [solid lines]. The dashed lines are material lines that do not remain perpendicular after strain, they rotate toward the long axis of the strain ellipse.

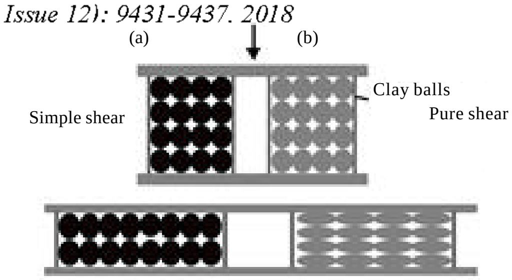


Fig. 5: Simple shear and pure shear

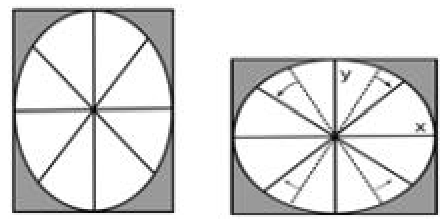


Fig. 6: Homogeneous strain

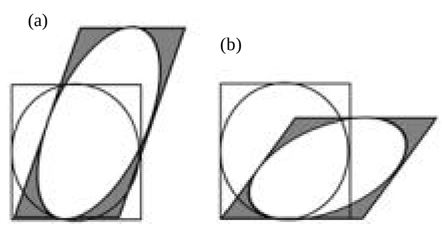


Fig. 7: Types of general shear

In Figure 7 a combination of simple shear [a special case of non-coaxial strain] and pure shear [coaxial strain] is called general shear or general non-coaxial strain. Two types of general shear are transtension Fig. 7a and transpression Fig. 7b, reflecting extension and shortening components.

**The properties of sediments derived from secondary rocks and manifestation of coaxial and non-coaxial components of shear strength:** The properties of sediments derived from secondary rocks are worth mentioning in this context:

Rock is aggregate of minerals. Chemical composition is a direct function of mineralogy and mineral composition varies with grain size. The major-element chemical composition of shales and mudstones is related also to grain size. Rain size and shape, control coaxial and non-coaxial strains of the sediments. Angular grains increase the angle of internal-friction of the soil.

Because the chemical composition of siliciclastic sedimentary rocks is closely related to the mineral composition of these rocks, the chemical composition varies as a function of grain size along with variations in mineralogy. For example, that $SiO_2$ abundance decreases progressively from fine sands to fine clays whereas the $Al_2O_3$ content systematically increases. uartz arenites composed of 90-95% siliceous grains quartz, chert, quartzose rock fragments).

Fine grained siliciclastic sedimentary rocks, composed mainly of particles smaller than approximately 62 μ make up approximately 50% of all sedimentary rocks in stratigraphic record. Quartz tends to be more abundant in coarse grained mudstones and shales whereas clay minerals are more abundant in fine grain mudstones and shales.

Quartz arenites are more poorly sorted and may contain high percentages of sub-angular to angular grains. Some quartz arenites exhibit textural inversions such as a combination of poor sorting and high rounding, a lack of correlation between roundness and size such as small round grains and larger angular grains or mixtures of rounded and angular grains within the same size fraction. These textural inversions probably result from mixing of grains from different sources, erosion of older sandstones or environmental variables such as wind transport of rounded grains into a quiet-water environment.

Angular grains may result also from development of secondary overgrowths. Now, the problem has to do with the inherent relationship of parent rock grain size and size of rock fragments. Only fine size parent rocks yield substantial quantities of rock fragments of sand size. Therefore, coarse grained parent rocks are poorly represented by rock fragments in sandstones.

Collectively, the changes brought about in the composition of sediment by weathering and erosion, transport, reworking at the depositional site can be significant. Provenance analysis requires that we cannot use the absence of particular constituents as a guide to provenance interpretation, we can use only the presence. The fact that feldspars and heavy minerals may be absent or scarce in sandstone, for example, does not mean that they were necessarily absent or scarce in the source rocks. Feldspars would have been converted chemically to clays.

The ultimate products of weathering following the above properties of sediments end up in sand and clay. The coaxial and non-coaxial components of shear strength are the hidden signature to sediments in the presence of water.

## RESULTS AND DISCUSSION

**The complex function-permeability:** Permeability is a complex function of particle size, sorting, shape, packing and orientation of sediments. These variable factors can be expressed in terms of heterogeneity factor. For a formation with a mixture of clay and sand the following equations with this heterogeneity factors $C_{v1}$ and $C_{v2}$. This variable factor $C_V$ is believed to decrease with decreasing particle size and decreasing sorting. This factor $C_V$ is affected by particle orientation. It is also, affected by the orientation parallel to bedding plane or perpendicular to the orientation. To make it a simple factor for the purpose of calculation the heterogeneity of clay is taken as $C_{v1}$ and for sand as $C_{v2}$. The general eqn for $C_V$ total is ($C_V$ = Coefficient of variation or Heterogeneity):

$$C_{v_{total}} = \sqrt{pc_{v_1}^2 + (1\text{-}p)pc_{v_2}^2}$$

P = 1 (Taking element No: 1 as clay)
Element 2 sand (1-p) = o:

$$C_{v_{total}} = \sqrt{1c_{v_1}^2 + (1\text{-}1)c_{v_2}^2}$$

$$C_{v_{total}} = \sqrt{c_{v_1}^2}_{=Cv1(\text{for Clay})}$$

Similarly, for p = 0 for clay:

$$C_{v_{total}} = \sqrt{0c_{v_1}^2 + (1\text{-}0)c_{v_2}^2}$$

$$C_{v_{total}} = \sqrt{c_{v_2}^2} = c_{v_2}\ (\text{for sand})$$

The common shear strength eqn is:

$$\tau = [C+\sigma \tan \phi\ ]$$

Where:

σ = Shear strength
C = Cohesion
σ = Normal stress
σ = The angle of internal friction of the soil

$$\tau = [C+\sigma \tan \phi\ ] \cos \alpha \ x$$

$$\sqrt{pc_{v_1}^2 + (1\text{-}p)c_{v_2}^2}$$

$$\cos\alpha = \sqrt{pc_{v_1}^2 + (1\text{-}p)c_{v_2}^2}$$

$$\tau = (c+\sigma\tan\phi)\sqrt{pc_{v_1}^2 + (1\text{-}p)c_{v_2}^2}$$

When, α = 90°, cos α = 0 for pure clay p = 1,
Sand (1-p) = 0, φ = 0:

$$\tau = (c+\sigma \tan(0))\sqrt{c_{v_1}^2 + 0c_{v_2}^2}, \text{for} \cos(90°) = 0$$

$$\tau = C(C_{v1}) \text{for pure clay}, C_{v1} = 1, \tau = C$$

For pure sand p = 1. $\tau = (C+\sigma \tan \phi)$:

$$\tau = (C+\sigma \tan\phi)\sqrt{pc_{v_1}^2 + (1\text{-}p)C_{v_2}^2} \quad \text{for} \cos 0 = 1$$

For pure sand p = 0:

$$\tau = (C+\sigma \tan\phi)\sqrt{0C_{v_1}^2 + (1\text{-}0)c_{v_2}^2}$$

$$\tau = (C+\sigma \tan\phi)^{(\sqrt{0+C_{v_2}^2})}$$

$$\tau = (C+\sigma \tan\phi)C_{v_2}$$

For clay C = 0:

$$\tau = (C_{v_2})\sigma \tan\phi) \text{and} C_{v_2} = 1$$

Heterogeneity:

$$C_{v_1} \text{ or } C_{v_2} \text{ -1}$$

$$\tau = \sigma \tan\phi$$

**The relation between coaxial and non-coaxial strain and skempton points:** For interpretation the data (after Skempton, 1964) indicating the variations of angle of internal friction (φ) with percentage of clay content is shown in a family of nine points, distributed over the first three quadrants as shown in Fig. 8 and 9.

This Fig. 9 shows the sharing of coaxial and non-coaxial strain or strength by different soil samples. No point lies in quadrant 4 which is high cohesion and high friction zone but in nature high cohesion and high friction cannot exist together in a soil sediment system when sharing the same volume or space between clay and sand (0.0, 1.0 or 1.0, 0.0).

These flow lines represent pure shear Fig. 8a, general shear Fig. 8b, simple shear Fig. 8c and rigid body rotation Fig. 8d. The cosine of the angle α is the kinematic vorticity number, $W_k$ for these strain histories, $W_k = 0$, $0<W_k<1$, $W_k = 1$ and $W_k = 8$, respectively. Avoiding the math, a convenient graphical way to understand this parameter is shown in Fig. 8. When tracking the movement of individual points within a deforming body relative to a reference line, we obtain a displacement field (or flow lines) that enables us to quantify the internal

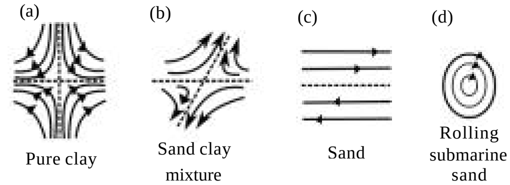


Fig. 8: Particle paths or flow lines during progressive strain accumulation

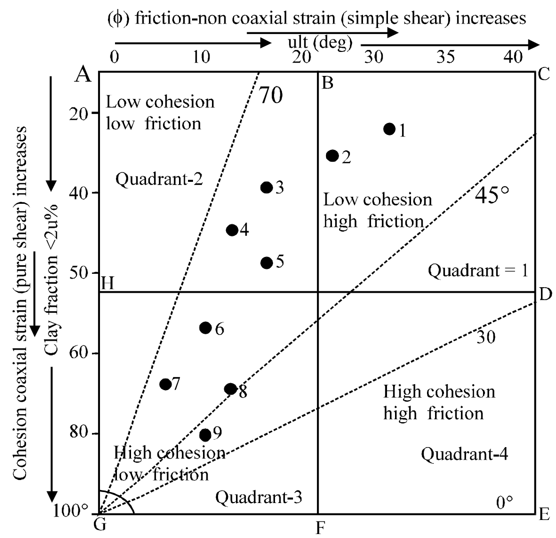


Fig. 9: Variation of $\varphi_{ult}$ with percentage of clay content. After Skempton (1964) all Skempton points lie in quadrants 1-3

vorticity. The angular relationship between the asymptotes and the reference line defines $W_k$. $W_k = \cos \alpha$.

For pure shear $W_k = 0$ Fig. 8a for general shear $0<W_k<1$ Fig. 8b and for simple shear $W_k = 1$ Fig. 8c. Rigid-body rotation or spin can also be described by the kinematic vorticity number (in this case, $W_k = \infty$ Fig. 8d).When, $\alpha = 0°$, $\cos \alpha = 1$, represents simple shear. When, $\alpha = 9°$ $\cos \alpha = 0$, represents pure shear.

The component describing the rotation of the material lines with respect to the principal strain axis is called the internal vorticity which is a measure of the degree of non-coaxiality.

If there is zero internal vorticity, the strain history is coaxial which is sometimes called pure shear. The non-coaxial strain history describes the case in which the distance perpendicular to the shear plane remains constant, this is also known as simple shear.

In Fig. 9, if ∝ = 0, the slope line coincides with x axis, GFE, cos∝ = cos0 = 1.0. If ∝ = 90°, the slope line becomes vertical and coincides with y axis, GHA, cos∝ = cos90° = 0.0.

In Table 1 and 2 the activity increases from quartz to clay minerals or from frictional soil to cohesive soil or simple shear to pure shear combinations as Skempton points is shown in Table 1.

**Fuzzy logic (membership functions) application to coaxial and non-coaxial components of shear strength of soils:** In marine environment the sea beats the waves on the sandy shores whose particles are subjected to intense dissolution which does not however, prevent them from remaining sandy. Now, transferring the triangle in correct orientation and ignoring gravity to each of the quadrants (1-4) into the Kosco's cube the following integrated figure is obtained.

In this integrated Fig. 10 and also in Table 2 the Boolean corner crisp points and the fuzzy paths and the midpoint M with maximum fuzzy are shown. Since, marine conditions are considered only dissolution is considered for the fuzzy logic. The movement from $X_1$ in the direction of the arrow indicates the movement of clay. It is applicable to $X_2$-$X_4$. The small movements of sand are indicated from $Y_1$-$Y_4$. In Fig. 10 the importance is given to the maximum fuzzy point M and all other Boolean crisp corner points of the cube. The sedimentation processes sometime undergo the following processes: sand deposit gaining clay particles in the voids, already existing sandy clay deposits may undergo loss of clay particles, clay deposits may gain sand in the voids in some cases clayey sand may undergo loss of sand. In all these cases the membership function related to geological process will be different. For clarity only dissolution is discussed.

**The suitability of fuzzy logic (membership fuctions) and interpretation of geo-technical parameters**

**The maximum fuzzy, Mid-point M:** The mid-point represents maximum fuzzy with (membership function 0.5, 0.5) and also the percentage of clay 50% and sand 50%intrinsically related to coaxial shear (pure shear) of clay and non-coaxial shear (simple shear) in a unique logic. This mid-point is a reality in fuzzy logic.

But this mid-point is forbidden to classical logic (Boolean logic) and set theory. The arbitrary insistence on bivalence or sharp crisp sets is the main reason for the restriction. At the middle point of the cube the classical theory breaks down completely because both sand and clay exist in reality with membership functions (0.5, 0.5).

Hence, conclusion number 1. is if laws of mathematics applied to reality they are not certain. This is the reason for avoiding mid-point. Fuzzy logic is for reality (in this case geology) and therefore, it admits mid-point under all conditions. In other words when crisp sets in the four corners of the cube contracts to a mid-point, the classical (Boolean logic fails).

**The minimum fuzzy or Boolean or crisp sets:** When the mid-point expands and propagates to include all the four corners of crisp sets then also, the classical logic fails. For

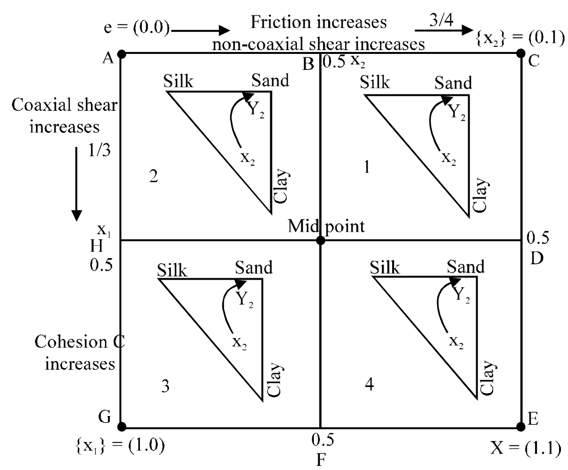


Fig. 10: Fuzzy logic (membership functions) and Shear strength of soils

Table 1: Activity number, clays and skempton points

| Quadrant | No. of skempton points | Type of quadrant |
|---|---|---|
| 1 | 02. | Low cohesion, high friction |
| 2 | 03. | Low cohesion, low friction |
| 3 | 04. | High cohesion, low friction |
| 4 | nil | High cohesion, high friction |

Table 2: Connecting fuzzy logic, fuzzy paths, cohesion and frictional components of shear strength of soils

| | | Fuzzy path | Geo-tech | |
|---|---|---|---|---|
| Quadrant | Fuzzy area | Away from | Towards destination | Classification |
| 1 | BCDM | Mid-point (maximum fuzzy) | Cohesion (0.0), friction (1.0) | Low cohesion, high friction crisp point |
| 2 | BMH | Mid-point (maximum fuzzy) | Cohesion (0.0), friction (0.5) | Low cohesion, low friction |
| 3 | HMFG | Crisp corner cohesion (1.0) friction (0.0) | Mid-point Cohesion (0.5) friction (0.5) | High cohesion, low friction |
| 4 | MDEF | Cohesion (1.0), friction (1.0) | Cohesion (0.5), friction (0.5) | High cohesion,high friction, cannot exist in natural soils |

classical logic at the mid-point nothing is distinguishable. At the vertices everything is distinguishable. At the corners of the cube (vertices) point E representing high friction and high cohesion (1.0, 1.0) cannot exist simultaneously in nature in soils in a given sample.

There is residual friction even for very pure clay. Similarly, a feeble cohesion for fine sands. The corner points (0.0) (0.1), (1.0) can exist in soil processes and samples in ideal cases. But the corner point (1, 1) can not exist in any conditions. Therefore, conclusion number 2 is if the laws of mathematics are certain (sharp or crisp sets), they do not refer to reality.

## CONCLUSION

If laws of mathematics applied to reality they are not certain. This is the reason for avoiding mid-point. Fuzzy logic is for reality (in this case geology) and therefore, it admits mid-point under all conditions. In other words when crisp sets in the four corners of the cube contracts to a mid-point, the classical Boolean logic fails. If the laws of mathematics are certain (sharp or crisp sets) they do not refer to reality.

The Boolean logic says that the shape of the earth is a perfect sphere but as the reality demands, geologists admit the reality and say that the shape of the earth is a spheroid which is ellipsoid of revolution. Seeing always believes.

## REFERENCES

Jensen, J.L., 1993. Statistics for Petroleum Engineers and Geoscientists. Prentice Hall, Upper Saddle River, New Jersey, USA., ISBN:9780131318557, Pages: 390.

Kosko, B., 1990. Fuzziness vs probability. Intl. J. Gen. Syst., 17: 211-240.

Legros, J.P., 2012. Major Soil Groups of the World. CRC Press, Boca Raton, Florida, USA., ISBN:978-1-57808-783-9, Pages: 455.

Pluijm, V.D.B.A., 2003. Earth Structure. W.W.Norton & Company Inc., New York, USA.,.

Skempton, A.W., 1964. Long-term stability of clay slopes. Geotech., 14: 77-102.