Journal of Engineering and Applied Sciences 14 (Special Issue 8): 10226-10233, 2019
ISSN: 1816-949X

A New Family [0, 1] Truncated Frechet Power Distribution

Mahdi W. Namma Nasrallah
Department of Statisticals, Administration and Economics College,
Karbala University, Karbala, Iraq

Abstract: In this study, a new family of continuous distributions based on [0, 1] truncated Frechet distribution is introduced. Special cases of [0, 1] truncated Frechet Power distribution ([0, 1] TFP_{ower}) is argued. The cumulative distribution function, the rth moment and central moment the mean, the variance, the skewness, the kurtosis, the mode, the median, the characteristic function, the reliability function and the hazard rate function are obtained for the distributions under consideration. It is well known that an item fails when a stress to which it is subjected exceeds the corresponding strength. In this sense, strength can be viewed as "resistance to failure". Good design practice is such that the strength is almost greater than the predicted stress. The safety factor can be defined in terms of strength and stress as strength/stress. Consequently, the [0, 1] TFP_{ower} strength-stress will the Shannon entropy and Kullback-Leibler and Renyi entropy will be derived also.

Key words: [0, 1] TFP_{ower}, relative Entropy, Reniy entropy, shannon entropy, stress-strength model, reliability

INTRODUCTION

In statistical analysis, a lot of distributions are used to represent set(sec) data. Recently new distributions are derived to extend some of well-known families of distributions such that the new distributions are more flexible than the others to model real data. The research work is proposed a distribution for wider applicability in other fields. The generalization which is motivated by the work of Eugene *et al*. (2002) will be the guide for this study, Eugene *et al*. In defined the beta G distribution from a quite arbitrary cumulative distribution function (cdf), G(x) by:

$$F(x)=\left(1/\beta(a,b)\right)\int_0^{G(x)} w^{a-1}(1-w)^{b-1}\,dw \quad (1)$$

where, a>0 and b>0 are two additional parameters whose role is to introduce skewness and to vary tail weight and $\beta(a,b)=\int_0^1 w^{a-1}(1-w)^{b-1}\,dw$ is the beta function. The class of distributions of Eq. 1 has increased the attention after the work of Eugene *et al*. (2002) and Jones (2004). Application of $X = G^{-1}(V)$ to the random Variable V following a beta distribution with parameters a and b, V~B (a, b) say, yields X with cdf. Eugene *et al*. (2002) defined the Beta Normal (BN) distribution by taking G(x) to be the cdf of the normal distribution and derived some of its first moments. General expressions for the moments of the BN distribution were derived by Gupta and Nadarajah (2005). An extensive review of scientific literature on this subject is available in Abid and Abdulrazak (2017), they write as:

$$\begin{aligned}F(x)&=\int_0^{G(x)}\frac{ab}{e^{-a}}t^{-(b+1)}e^{-at^{-b}}\,dt\\&=\frac{1}{e^{-a}}e^{-at^{-b}}\Big]_0^{G(x)}\frac{1}{e^{-a}}e^{-aG(x)^{-b}}\end{aligned} \quad (2)$$

With pdf:

$$\begin{aligned}f(x)&=\frac{\partial}{\partial x}F(x)=\frac{\partial}{\partial x}\frac{e^{-a(G(x))^{-b}}}{e^{-2}}\\&=\frac{ab}{e^{-a}}e^{-a(G(x))^{-b}}\left(G(x)\right)^{-(b+1)}g(x)\end{aligned} \quad (3)$$

With G(x) being a baseline distribution, we define in Eq. 2 and 3 above, a generalized class of distributions. The researcherwill name it the [0,1] truncated Frechet -G distribution.

MATERIALS AND METHODS

[0, 1] truncated Frechet-power distribution: Assum that $f(x; p, \theta) = \theta t^{\theta-1}/p^{\theta}$, 0<x<p are pdf and $F(x) = (t/p)^{\theta}$ cdf of power random variable, respectively, then by applying Eq. 2 and 3 above, we get the cdf and pdf of [0,1] TFP_{ower} random variable as follows:

$$F(x)=\frac{1}{e^{-a}}e^{-a\left(1-\left(\frac{x}{p}\right)^{\theta}\right)^{-b}},\ x>0 \quad (4)$$

$$f(x)=\frac{ab\theta p^{-\theta}}{e^{-a}}x^{(\theta-1)}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)},\ 0<x<p \quad (5)$$

Prove that f (x) is probability density function:

$$\int_0^p f(x)dx = \int_0^p \frac{ab\theta p^{-\theta}}{e^{-a}} x^{(\theta-1)} e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}} \left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)} dx$$

Let:

$$y = a\left(\frac{x}{p}\right)^{-\theta b} \Rightarrow x = p\left(\frac{y}{a}\right)^{\frac{-1}{b\theta}} \Rightarrow dx = \frac{-p}{ab\theta}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}-1} dy$$

Then:

$$\int_0^p f(x)dx = \frac{1}{e^{-a}}\int_0^\infty ab\theta p^{-\theta}\left(p\left(\frac{y}{p}\right)^{\frac{-1}{b\theta}}\right)^{-(b+1)} e^{-y}\frac{p}{ab\theta}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}-1} dy$$

$$= \frac{1}{e^{-a}}\int_0^\infty e^{-y}dy = \frac{1}{e^{-a}}\left(-e^{-y}\right)_a^\infty = 1$$

Properties [0, 1] truncated Frechet-power distribution Reliability and hazard rate functions:

$$R(x) = 1\text{-}F(x) = 1\text{-}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}$$

$$\lambda(x) = \frac{f(x)}{R(x)} = \left[\frac{\frac{ab\theta p^{-\theta}}{e^{-a}} x^{(\theta-1)} e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}} \left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)}}{1\text{-}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}}\right] \quad (6)$$

The rth raw uncentral moment:

$$E\left(x^r\right) = \int_0^p x^r \frac{ab\theta p^{-\theta}}{e^{-a}} x^{(\theta-1)} e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}} \left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)} dx$$

Let:

$$y = a\left(\frac{x}{p}\right)^{-b\theta} \Rightarrow x = p\left(\frac{y}{a}\right)^{\frac{-1}{b\theta}} \Rightarrow dx = \frac{-p}{ab\theta}\left(\frac{y}{a}\right)^{\frac{1}{b\theta}-1} dy$$

Then,

$$E\left(x^r\right) = \frac{1}{e^{-a}}\int_a^\infty abp^{-\theta}\left(p\left(\frac{y}{a}\right)^{\frac{-1}{b\theta}}\right)^{r-(b+1)} e^{-y}\left(\left(\frac{y}{a}\right)^{\frac{-1}{b}}\right)^{-(b+1)} \frac{p}{a\theta b}\left(\frac{y}{a}\right)^{\frac{-1}{b}\theta\text{-}1} dx$$

$$= \frac{1}{e^{-a}}\int_a^\infty p^r\left(\frac{y}{a}\right)^{\frac{-r}{b\theta}+\frac{1}{b}+\frac{-1}{b\theta}}\left(\frac{y}{a}\right)^{1+\frac{1}{b}} e^{-y}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}} dy$$

$$= \frac{p^r a^{\frac{r}{\theta b}}}{e^{-a}}\int_a^\infty y^{\frac{-r}{\theta b}} e^{-y}dy \quad (7)$$

$$E\left(x^r\right) = \frac{p^r a^{\frac{r}{\theta b}}}{e^{-a}}\Gamma\left(1\text{-}\frac{r}{\theta b}, a\right)$$

The rth raw central moment:

$$E(x\text{-}\mu)^r = \int_0^p (x\text{-}\mu)^r \frac{ab\theta p^{-\theta}}{e^{-a}} x^{(\theta-1)} e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}} \left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)} dx$$

Let:

$$y = a\left(\frac{x}{p}\right)^{-b\theta} \Rightarrow x = p\left(\frac{y}{a}\right)^{\frac{-1}{b\theta}} \Rightarrow dx = \frac{-p}{ab\theta}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}-1} dy$$

Then:

$$= \frac{abp^{-\theta}}{e^{-a}}\int_a^\infty\left(p\left(\frac{y}{a}\right)^{\frac{-1}{b\theta}}\text{-}\mu\right)^r\left(p\left(\frac{y}{a}\right)^{\frac{-1}{b\theta}}\right)^{(\theta+1)}$$

$$e^{-y}\left(\left(\frac{y}{a}\right)^{\frac{-1}{b}}\right)^{-(b+1)} \frac{p}{a\theta b}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}-1} dy$$

$$= \frac{1}{e^{-a}}\int_a^\infty\left(pa^{\frac{1}{\theta b}}\right)^j(-\mu)^{r-j} e^{-y}dy \quad (8)$$

$$= \frac{1}{e^{-a}}\sum_{j=0}^\infty C_j^r\left(pa^{\frac{1}{\theta b}}\right)^j(-\mu)^{r-j}\int_a^\infty y^{\frac{J}{\theta b}} e^{-y}dy$$

$$E(x\text{-}\mu)^r = \frac{1}{e^{-a}}\sum_{j=0}^\infty C_j^r\left(pa^{\frac{1}{\theta b}}\right)^j(-\mu)^{r-j}\Gamma\left(1\text{-}\frac{j}{\theta b}, a\right)$$

The characteristic function:

$$Q_x(t) = E\left(e^{ixt}\right) = E\left(\sum_{r=0}^\infty\frac{(ixt)^r}{r!}\right) = \sum_{r=0}^\infty\frac{(it)^r}{r!}E\left(x^r\right)$$

$$= e^a\sum_{r=0}^\infty\frac{(itp)^r a^{\frac{1}{\theta b}}}{r!}\Gamma\left(1\text{-}\frac{r}{\theta b}, a\right) \quad (9)$$

Mean and variance of the of [0,1] TFp random variable:

$$\mu = E(x) = \frac{pa^{\frac{1}{\theta b}}}{e^{-a}}\Gamma\left(1\text{-}\frac{1}{\theta b}, a\right) \quad (10)$$

$$\sigma^2 = \frac{p^2 a^{\frac{2}{\theta b}}}{e^{-a}}\left(\Gamma\left(1\text{-}\frac{2}{\theta b}, a\right) - \frac{\Gamma^2\left(1\text{-}\frac{1}{\theta b}, a\right)}{e^{-a}}\right) \quad (11)$$

Mode M_o and the Median M_e: The mode can be derived as:

$$f'(x) = \frac{-ab\theta^2 p^{-\theta}}{e^{-a}}(b+1)x^{(\theta-1)} e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left(\frac{x}{p}\right)^{\theta-1}\left[\left(\frac{x}{p}\right)^{\theta}\right]^{-(b+2)}$$

$$+\frac{-a^2b^2\theta^2p^{-\theta}}{pe^{-a}}x^{\theta-1}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left(\frac{x}{p}\right)^{\theta-1}\left[\left(\frac{x}{p}\right)^{\theta}\right]^{-2(b+1)} + \quad (12)$$

$$\frac{ab\theta p^{-\theta}}{e^{-a}}(\theta\text{-}1)x^{(\theta-2)}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left[\left(\frac{x}{p}\right)^{\theta}\right]^{-(b+1)} = 0$$

$$\Rightarrow \theta(b+1)\text{-}(\theta\text{-}1) = ab\theta\left(\frac{x}{p}\right)^{-\theta b} \quad x = M_o = P\left(\frac{\theta ab}{\theta b+1}\right)^{\frac{1}{\theta b}}$$

The last equation has no closed form therefor using numerical solution to slove equation to get the mode of x. Then the median of x is:

$$F(x)=\frac{1}{e^{-a}}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}=\frac{1}{2}$$
$$\Rightarrow x=M_e=p\left[1+\frac{\text{In}(2)}{a}\right]^{\frac{-1}{\theta b}} \tag{13}$$

Coefficient of Skeweness of [0, 1)TFP$_{ower}$:

$$C.S=\frac{E(x-\mu)^3}{\sigma^3}$$

$$C.S=\frac{\left[\begin{array}{c}\left(pa^{\frac{1}{\theta b}}\right)^3\Gamma\left(1-\frac{3}{\theta b},a\right)\\ +3\left(\left(\frac{pa^{\frac{1}{\theta b}}}{e^{-a}}\right)\Gamma\left(1-\frac{1}{\theta b},a\right)\right)^2\left(pa^{\frac{1}{\theta b}}\right)\Gamma\left(1-\frac{1}{\theta b},a\right)\\ -3\left(\frac{pa^{\frac{1}{\theta b}}}{e^{-a}}\right)\Gamma\left(1-\frac{1}{\theta b},a\right)\left(pa^{\frac{1}{\theta b}}\right)^2\Gamma\left(1-\frac{2}{\theta b},a\right)\\ -\left(\left(\frac{pa^{\frac{1}{\theta b}}}{e^{-a}}\right)\right)\Gamma\left(1-\frac{1}{\theta b},a\right)^3\end{array}\right]}{\left[\frac{p^2a^{\frac{2}{\theta b}}}{e^{-a}}\left(\Gamma\left(1-\frac{2}{\theta b},a\right)-\frac{\Gamma^2\left(1-\frac{1}{\theta b},a\right)}{e^{-a}}\right)\right]^{\frac{3}{2}}} \tag{14}$$

Coefficient of kurtosis of [0, 1) TFP$_{ower}$:

$$C.k=\frac{E(x-\mu)^4}{\sigma^4}-3$$

$$C.k=\frac{\left[\begin{array}{c}\left(\left(\frac{pa^{\frac{1}{\theta b}}}{e^{-a}}\right)\Gamma\left(1-\frac{1}{\theta b},a\right)\right)^4\\ +4\left(\left(\frac{pa^{\frac{1}{\theta b}}}{e^{-a}}\right)\Gamma\left(1-\frac{1}{\theta b},a\right)\right)^3\left(pa^{\frac{1}{\theta b}}\right)\Gamma\left(1-\frac{1}{\theta b},a\right)\\ +6\left(\left(\frac{pa^{\frac{1}{\theta b}}}{e^{-a}}\right)\Gamma\left(1-\frac{1}{\theta b},a\right)\right)^2\left(pa^{\frac{1}{\theta b}}\right)^2\Gamma\left(1-\frac{2}{\theta b},a\right)\\ -4\left(\left(\frac{pa^{\frac{1}{\theta b}}}{e^{-a}}\right)\Gamma\left(1-\frac{1}{\theta b},a\right)\right)^1\left(pa^{\frac{1}{\theta b}}\right)^3\Gamma\left(1-\frac{3}{\theta b},a\right)\\ \left(pa^{\frac{1}{\theta b}}\right)^4\Gamma\left(1-\frac{4}{\theta b},a\right)\end{array}\right]}{\left[\frac{p^2a^{\frac{2}{\theta b}}}{e^{-a}}\left(\Gamma\left(1-\frac{2}{\theta b},a\right)-\frac{\Gamma^2\left(1-\frac{1}{\theta b},a\right)}{e^{-a}}\right)\right]^2}-3 \tag{15}$$

Quantile function x_q of [0, 1] TFP$_{ower}$:

$$q=p\left(x\le x_q\right)=F\left(x_q\right)0<q<1,\quad x_q>0$$
$$q=\frac{1}{e^{-a}}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\Rightarrow qe^{-a}=e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\Rightarrow -a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}= \tag{16}$$
$$\text{In}\left(qe^{-a}\right)\ x_q=F^{-1}(q)=p\left[1-\frac{\text{In}(q)}{a}\right]^{\frac{-1}{\theta b}}$$

So by using the inverse transform method, we can generate [0, 1] TFP$_{ower}$ random variable as follows:

$$U=e^{a-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}$$
$$\text{Ln }U=a\left[1-\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}\right]$$
$$\frac{\text{Ln }U}{a}=\left[1-\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}\right]\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}= \tag{17}$$
$$1-\frac{\text{Ln }U}{a}\left(\frac{x}{p}\right)=\left(1-\frac{\text{Ln }U}{a}\right)^{\frac{-1}{\theta b}}$$
$$x=P\left[1-\frac{\ln(u)}{a}\right]^{\frac{-1}{\theta b}}$$

where, u is a random number distributed in the unit interval [0,1].

RESULTS AND DISCUSSION

Shannon entropies: An entropy of a random variable X is a measure of variation of the uncertainty. The Shannon entropy of [0,1] TFP$_{ower}$ (a, b, θ, P) random variable X can be found as follows:

$$SE=E\left(-\ln\left(f(x)\right)\right)=\int_0^p\frac{ab\theta p^{-\theta}}{e^{-a}}x^{(\theta-1)}$$
$$e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)}\left[\begin{array}{c}-\text{Ln}\left(\frac{ab\theta P^{-\theta}}{e^{-a}}\right)+(\theta+1)\ln(x)+\\ a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}+(b+1)\text{Ln}\left(\frac{x}{p}\right)\end{array}\right]dx \tag{18}$$
$$=-\ln\left(\frac{ab\theta p^{-\theta}}{e^{-a}}\right)+(\theta+1)E\left(\text{Ln}(x)\right)+a\,p^{\theta b}E\left(\frac{1}{X^{\theta b}}\right)+$$
$$(b+1)E\left[\text{Ln}\left(\frac{x}{p}\right)^{\theta}\right]$$

Let:

$$I_1 = (b+1)E(Ln(x)) = \frac{ab\theta p^{-\theta}}{e^{-a}}(b+1)\int_0^p \ln(x)x^{-(b+1)} e^{-a\theta^b x^{-b}}\left(\left(\frac{x}{p}\right)^\theta\right)^{-(b+1)} dx$$

Let:

$$y = x^{-\theta b} \Rightarrow x = y^{\frac{-1}{\theta b}} \Rightarrow dx = \frac{-1}{\theta b} y^{\frac{-1}{\theta b}-1} dy$$

Then:

$$I_1 = \frac{ab\theta^b}{e^{-a}}(b+1)\int_{p^{-\theta b}}^{\infty} \ln\left(y^{\frac{-1}{\theta b}}\right)\left(y^{\frac{-1}{\theta b}}\right)^{\theta-1} e^{-ap^{\theta b}y}\left[\left(y^{\frac{-1}{\theta b}}\right)^\theta\right]^{-(b+1)} \frac{1}{\theta b} y^{\frac{-1}{\theta b}-1} dy = \frac{ap^{\theta b}(\theta+1)}{e^{-a}}\int_{p^{-\theta b}}^{\infty} \ln(y) e^{-ap^{\theta b}y} dy = \frac{-(b+1)ap^{\theta b}}{\theta b e^{-a}}\left[\int_0^\infty \ln(y)e^{-ap^{\theta b}y}dy - \int_0^{p^{-\theta b}} \ln(y)e^{-ap^{\theta b}y}dy\right]$$

Since:

$$\int_0^\infty x^{s-1}e^{-mx}\ln(x)dx = m^{-s}\Gamma(s)\{\Psi(s)-\ln(m)\},\ \Psi(1) = \Gamma(1) = -\gamma$$

where, $\gamma = 0.5772$ is an Euler constant. Then:

$$\int_0^\infty \ln(y)e^{-ap^{\theta b}y}dy = \frac{1}{ap^{\theta b}}\{-\gamma-\ln(ap^{\theta b})\}$$

For:

$$I_{11} = \int_0^{\infty-b} \ln(y)e^{-ap^{\theta b}y}dy$$

Let:

$$u = p^{\theta b}y \Rightarrow y = p^{-\theta b}u \Rightarrow dy = p^{-\theta b}du$$

Then:

$$I_{11} = \int_0^1 \ln(\theta^{-b}u)e^{-au}p^{-\theta b}dz = p^{-\theta b}\ln(p^{-\theta b})\int_0^1 e^{-au}du + p^{-\theta b}\int_0^1 \ln(u)e^{-au}du = \frac{\theta b}{a}p^{-\theta b}\ln(p)e^{-au}\Big]_0^1 + p^{-\theta b}\int_0^1 \ln(z)\sum_{m=0}^{\infty}\frac{(-au)^m}{m!}du$$

Since:

$$e^{-az} = \sum_{m=0}^{\infty}\frac{(-az)^m}{m!} I_{11} = \frac{\theta bp^{-\theta b}}{a}\ln(p)[e^{-a}-1] + p^{-\theta b}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!}\int_0^1 \ln(u)u^m du$$

Since:

$$\int x^m \ln(x)dx = x^{m+1}\left\{\frac{\ln(x)}{m+1} - \frac{1}{(m+1)^2}\right\}$$

Then:

$$I_{11} = \frac{\theta bp^{-\theta b}}{a}\ln(p)[e^{-a}-1] + p^{-\theta b}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2}$$

$$I_1 = \frac{\theta-1}{e^{-a}}\{+\gamma+\ln(ap^{\theta b})\} + \frac{(\theta-1)}{e^{-a}}\ln(p)(e^{-a}-1) - \frac{a(b+1)}{e^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2} I_2 = ap^{\theta b}E\left(\frac{1}{x^{\theta b}}\right) = \frac{a^2b\theta p^{\theta(b-1)}}{e^{-a}}\int_0^p\left(\frac{1}{x^{\theta b}}\right)x^{\theta-1}e^{-a\left(\left(\frac{x}{p}\right)^\theta\right)^{-b}}\left(\left(\frac{x}{p}\right)^\theta\right)^{-(b+1)} dx$$

Let:

$$y = a\left(\left(\frac{x}{p}\right)^\theta\right)^{-b} \Rightarrow x = p\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}} \Rightarrow dx = \frac{-p}{ab\theta}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}-1} dy$$

Then:

$$I_2 = \frac{a^2b\theta p^{\theta(b-1)}}{e^{-a}}\int_a^\infty\left(p\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}}\right)^{-\theta b+\theta-1} e^{-y}\left(\left(\frac{y}{a}\right)^{\frac{-1}{b}}\right)-(b+1) \frac{-p}{ab\theta}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}-1} dy = \frac{1}{e^{-a}}\int_a^\infty y e^{-y} dy\ I_2 = \frac{1}{e^{-a}}\Gamma(2,a)$$

Let:

$$I_3 = (b+1)E\left(Ln\left(\frac{x}{p}\right)^\theta \int_0^p Ln\left(\frac{x}{p}\right)^\theta \frac{ab\theta p^{-\theta}}{e^{-a}}x^{(\theta-1)}e^{-a\left(\left(\frac{x}{p}\right)^\theta\right)^{-b}}\left(\left(\frac{x}{p}\right)^\theta\right)^{-(b+1)} dx\right) \frac{(b+1)ab\theta p^{-\theta}}{e^{-a}}\int_0^p Ln\left(\frac{x}{p}\right)^\theta x^{(\theta-1)}e^{-a\left(\left(\frac{x}{p}\right)^\theta\right)^{-b}}\left(\left(\frac{x}{p}\right)^\theta\right)^{-(b+1)} dx$$

$$y = \left(\frac{x}{p}\right)^\theta \Rightarrow y^{\frac{1}{\theta}} = \frac{x}{p} \Rightarrow x = py^{\frac{1}{\theta}} \Rightarrow dx = \frac{p}{\theta}y^{\frac{1}{\theta}-1}dy$$

$$\frac{(b+1)ab\theta p^{-\theta}}{e^{-a}}\int_0^1 Ln(y)\left[py^{\frac{1}{\theta}}\right]^{(\theta-1)} e^{-ay^{-b}}(y)^{-(b+1)}\frac{p}{\theta}y^{\frac{1}{\theta}-1}dy$$

$$\frac{(b+1)ab\theta}{e^{-a}}\int_0^1 Ln(y)(y)^{-(b+1)}e^{-ay^{-b}}dy$$

Let:

$$U = y^{-b} \Rightarrow y = u^{-\frac{1}{b}} \Rightarrow dy = -\frac{1}{b}u^{-\frac{1}{b}-1}$$

Then:

$$\frac{(b+1)ab\theta}{e^{-a}}\int_0^1 Ln\left(u^{\frac{-1}{b}}\right)\left[u^{-\frac{1}{b}}\right]^{-(b+1)} e^{-au}\text{-}\frac{1}{b}u^{-\frac{1}{b}-1}dy\frac{(b+1)a}{be^{-a}}\int_0^1 Ln(u)e^{-au}dy$$

Since:

$$e^{-au} = \sum_{m=0}^{\infty}\frac{(-a)^m}{m!}u^m\frac{(b+1)a}{be^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!}\int_0^1 Ln(u)u^m dy$$

$$I_3 = \frac{(b+1)a}{be^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2}$$

Then the Shannon entropy is:

$$SE = -\ln\left(\frac{ab\theta P^{-\theta}}{e^{-a}}\right)+\frac{\theta\text{-}1}{e^{-a}}\left\{+\gamma+\ln\left(a p^{\theta b}\right)\right\}+\frac{(\theta\text{-}1)}{e^{-a}}\ln(p)\left(e^{-a}\text{-}1\right)- \frac{a(b+1)}{e^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2}+\frac{1}{e^{-a}}\Gamma(2,a)+\frac{(b+1)a}{be^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2} \quad (19)$$

Kullback–Leibler divergence: The Kullback–Leibler divergence or the relative entropy is a measure of the difference between two probability distributions F and F*. It is not symmetric in F and F*. In applications, normally represents the "true" distribution of data, observations or a precisely calculated theoretical distribution while F* normally represents a theory, model, description or approximation of F. Precisely, the Kullback–Leibler divergence of F* from denoted $D_{KL}(F||F^*)$ is a measure of the information gained when one revises ones beliefs from the prior probability distribution F^* to the posterior probability distribution F. More exactly, it is the quantity of information that is lost when F^* is used to approximate F, defined practically as the expected extra number of bits needed to code samples from F using a code optimized for F* rather than the code optimized for F. Hence, the relative entropy D_{KL}(F F^*) for a random variable [0,1] TFP_{ower} (a, b, θ, P) can be found as follows, Since:

$$\frac{f(x)}{f^*(x)} = \left(\frac{\frac{ab\theta p^{-\theta}}{e^{-a}}x^{\theta-1}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)}}{\frac{a_1 b_1\theta_1 p_1^{-\theta_1}}{e^{-a_1}}x^{\theta_1-1}e^{-a_1\left(\left(\frac{x}{p_1}\right)^{\theta_1}\right)^{-b_1}}\left(\left(\frac{x}{p_1}\right)^{\theta_1}\right)^{-(b_1+1)}}\right)$$

Then:

$$Dkl\left(F\|F^*\right) = \int_0^p f(x)\ln\left(\frac{f(x)}{f^*(x)}\right)dx\int_0^p\frac{ab\theta p^{-\theta}}{e^{-a}}x^{(\theta-1)}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}} \quad (19)$$

$$\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)}\ln\left(\frac{\frac{ab\theta p^{-\theta}}{e^{-a}}x^{\theta-1}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)}}{\frac{a_1 b_1\theta_1 p_1^{-\theta_1}}{e^{-a_1}}x^{\theta_1-1}e^{-a_1\left(\left(\frac{x}{p_1}\right)^{\theta_1}\right)^{-b_1}}\left(\left(\frac{x}{p_1}\right)^{\theta_1}\right)^{-(b_1+1)}}\right)dx = \quad (20)$$

$$\ln\left(\frac{ab\theta p^{-\theta}e^{-a_1}}{a_1 b_1\theta_1 p_1^{-\theta_1}e^{-a}}\right)+(\theta\text{-}\theta_1)E(\ln(x))-a p^{\theta b}E\left(\frac{1}{x^{\theta b}}\right)+ a_1 p_1^{\theta_1 b_1}E\left(\frac{1}{x^{\theta_1 b_1}}\right)-(b+1)E\left(Ln\left(\frac{x}{p}\right)^{\theta}\right)+(b_1+1)E\left(Ln\left(\frac{x}{p_1}\right)^{\theta_1}\right)$$

Since:

$$I_1 = (\theta\text{-}\theta_1)E(\ln(x)) = (\theta\text{-}\theta_1)\int_0^p \ln(x)\frac{ab\theta p^{-\theta}}{e^{-a}}x^{\theta-1}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}} \left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)}dx\ I_1 = \frac{\theta\text{-}1}{e^{-a}}\left\{\gamma+\ln\left(a p^{\theta b}\right)\right\}+\frac{(\theta\text{-}1)}{e^{-a}}\ln(p)\left(e^{-a}\text{-}1\right)- \frac{a(b+1)}{e^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2}$$

And:

$$I_2 = ap^{\theta b}E\left(\frac{1}{\chi^{\theta b}}\right)-a p^{\theta b}\int_0^p\frac{1}{x^{\theta b}}\frac{ab\theta p^{-\theta}}{e^{-a}}x^{\theta-1}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)}dx$$

Let:

$$y = a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b} \Rightarrow x = p\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}} \Rightarrow dx = \frac{-p}{ab\theta}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}-1}dy$$

Then:

$$= -a p^{\theta b}\frac{ab\theta p^{-\theta}}{e^{-a}}\int_a^{\infty}\left[p\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}}\right]^{-\theta b+\theta+1}e^{-y}\left(\left(\frac{y}{a}\right)^{\frac{-1}{b}}\right)^{-(b+1)}\frac{-p}{ab\theta}\left(\left(\frac{y}{a}\right)^{\theta}\right)^{\frac{-1}{\theta b}-1}dy$$

$$= \frac{1}{e^{-a}}\int_0^{\infty}ye^{-y}dy = \frac{1}{e^{-a}}\Gamma(2,a)I_2 = e^a\,\Gamma(2,a)$$

And:

$$I_3 = a_1 p_1^{\theta_1 b_1}E\left(\frac{1}{x^{\theta_1 b_1}}\right) = a_1 p_1^{\theta_1 b_1}\int_0^p\frac{1}{x^{\theta_1 b_1}}\frac{ab\theta p^{-\theta}}{e^{-a}} x^{\theta-1}e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)}dx$$

Let:

$$y = a\left(\frac{x}{p}\right)^{-\theta b} \Rightarrow x = p\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}} \Rightarrow dx = \frac{-p}{a\theta b}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}-1} dy$$

Then:

$$I_3 = \frac{-a_1 p_1^{\theta_1 b_1} p^{\theta_1 b_1}}{e^{-a} a^{\frac{\theta_1 b_1}{\theta b}}}\int_a^{\infty} y^{\frac{\theta_1 b_1}{\theta b}} e^{-y} dy \; I_3 = \frac{-a_1 p_1^{\theta_1 b_1} p^{\theta_1 b_1 - \theta}}{e^{-a} a^{\frac{\theta_1 b_1}{\theta b}}}\Gamma\left(\frac{\theta_1 b_1}{\theta b}+1, a\right)$$

$$I_4 = (b+1)E\left(Ln\left(\frac{x}{p}\right)^{\theta}\right) = \int_0^p Ln\left(\frac{x}{p}\right)^{\theta} \frac{ab\theta p^{-\theta}}{e^{-a}} x^{\theta-1} e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}$$

$$\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)} dx \; y = \left(\frac{x}{p}\right)^{\theta} \Rightarrow y^{\frac{1}{\theta}} = \frac{x}{p} \Rightarrow x = py^{\frac{1}{\theta}} \Rightarrow dx = \frac{p}{\theta} y^{\frac{1}{\theta}-1} dy =$$

$$\frac{(b+1)ab\theta p^{-\theta}}{e^{-a}}\int_0^1 Ln(y)\left[py^{\frac{1}{\theta}}\right]^{(\theta-1)} e^{-ay^{-b}} (y)^{-(b+1)} \frac{p}{\theta} y^{\frac{1}{\theta}-1} dy$$

Then:

$$I_4 = \frac{(b+1)a}{be^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2}$$

$$I_5 = (b_1+1)E\left(Ln\left(\frac{x}{p_1}\right)^{\theta_1}\right) = (b_1+1)\int_0^p Ln\left(\left(\frac{x}{p_i}\right)^{\theta_1}\right)\frac{ab\theta p^{-\theta}}{e^{-a}} x^{\theta-1}$$

$$e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}}\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)} dx$$

$$y = \left(\frac{x}{p}\right)^{\theta} \Rightarrow y^{\frac{1}{\theta}} = \frac{x}{p} \Rightarrow x = py^{\frac{1}{\theta}} \Rightarrow dx = \frac{p}{\theta} y^{\frac{1}{\theta}-1} dy$$

Then:

$$(b_1+1)\frac{ab\theta p^{-\theta}}{e^{-a}}\int_0^1 Ln\left(\frac{py^{\frac{1}{\theta}}}{p_1}\right)^{\theta_1}\left(py^{\frac{1}{\theta}}\right)^{\theta-1} e^{-a(y)^{-b}} (y)^{-(b+1)} \frac{p}{\theta} y^{\frac{1}{\theta}-1} dy =$$

$$(b_1+1)\frac{ab}{e^{-a}}\int_0^1 Ln\left[\left(\frac{p}{p_1}\right)^{\theta_1} (y)^{\frac{\theta_1}{\theta}}\right] e^{-a(y)^{-b}} (y)^{-(b+1)} dy$$

Let:

$$U = y^{-b} \Rightarrow y = u^{-\frac{1}{b}} \Rightarrow dy = -\frac{1}{b} u^{-\frac{1}{b}-1}$$

Then:

$$= (b_1+1)\frac{ab}{e^{-a}}\int_0^1 Ln\left[\left(\frac{p}{p_1}\right)^{\theta_1}\left(u^{-\frac{1}{b}}\right)^{\frac{\theta_1}{\theta}}\right] e^{-au}\left(u^{-\frac{1}{b}}\right)^{-(b+1)} \frac{-1}{b} u^{-\frac{1}{b}-1} dy =$$

$$(b_1+1)\frac{-a}{e^{-a}}\int_0^1 Ln\left[\left(\frac{p}{p_1}\right)^{\theta_1}\left(u^{-\frac{1}{b}}\right)^{\frac{\theta_1}{\theta}}\right] e^{-au} dy =$$

$$(b_1+1)\frac{-a}{e^{-a}}\left[\int_0^1 Ln\left[\left(\frac{p}{p_1}\right)^{\theta_1}\right] e^{-au} dy + \int_0^1 Ln\left(u^{-\frac{\theta_1}{\theta b}}\right) e^{-au} dy\right]$$

$$I_{51} = (b_1+1)\frac{-a}{e^{-a}}\int_0^1 Ln\left[\left(\frac{p}{p_1}\right)^{\theta_1}\right] e^{-au} dy = (b_1+1)\frac{-a}{e^{-a}} Ln\left(\frac{p}{p_1}\right)^{\theta_1} + \int_0^1 e^{-au} dy$$

$$I_{51} = (b_1+1)\theta_1 Ln\left(\frac{p_1}{p}\right)$$

$$I_{52} = (b_1+1)\frac{-a}{e^{-a}}\int_0^1 Ln\left(u^{\frac{-\theta_1}{\theta b}}\right) e^{-au} dy = (b_1+1)\frac{a\theta_1}{\theta b e^{-a}}\int_0^1 Ln(u) e^{-au} dy$$

$$I_{52} = (b_1+1)\frac{a\theta_1}{\theta b e^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!}\int_0^1 Ln(u) u^m dy = (b_1+1)\frac{a\theta_1}{\theta b e^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2}$$

$$I_5 = (b_1+1)\theta_1 Ln\left(\frac{p_1}{p}\right) + (b_1+1)\frac{a\theta_1}{\theta b e^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2}$$

The Kullback–Leibler divergence is:

$$\begin{aligned} Dkl(F \| F^*) = &\ln\left(\frac{ab\theta p^{-\theta} e^{-a_1}}{a_1 b_1 \theta_1 p_1^{-\theta_1} e^{-a}}\right) + \frac{\theta-1}{e^{-a}}\{\gamma + \ln(a\, p^{\theta b})\} + \frac{(\theta-1)}{e^{-a}} \\ &\ln(p)(e^{-a}-1) - \frac{a(b+1)}{e^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2} + e^{-a}\Gamma(2, a) - \\ &\frac{p_1^{\theta_1 b_1} p^{\theta_1 b_1}}{e^{-a} a^{\frac{\theta_1 b_1}{\theta b}}}\Gamma\left(\frac{\theta_1 b_1}{\theta b}+1, a\right) + \frac{(b+1)a}{b\, e^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2} + \\ &(b_1+1)\text{-}\theta_1 Ln\left(\frac{p_1}{p}\right) + (b_1+1)\frac{a\theta_1}{\theta b e^{-a}}\sum_{m=0}^{\infty}\frac{(-a)^m}{m!(m+1)^2} \end{aligned} \quad (21)$$

Stress-Strength reliability: Inferences about R = P[Y<X] where X and Y are two independent random variables is very common in the reliability literatures. For example, if X is the strength of a component which is subject to a stress Y, then R is a measure of system performance and arises in the context of mechanical reliability of a system. The system fails if and only if at any time the applied stress is more than its strength. Let Y and X be the stress and the strength random variables, independent of each other, follow, respectively, [0,1] TFP (a, b, θ, P) and [0,1] TFP (a_1, b_1, θ_1, P_1), then:

$$R = P(y<x) = \int_0^p f_x(x) F_y(x) dx = \int_0^p \frac{ab\theta p^{-\theta}}{e^{-a}} x^{\theta-1} e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}} \left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)} \frac{1}{e^{-a_1}} e^{-a_1\left(\left(\frac{x}{p_1}\right)^{\theta_1}\right)^{-b_1}} dx \quad (22)$$

Since:

$$e^{-a_1\left(\left(\frac{x}{p_1}\right)^{\theta_1}\right)^{-b_1}} = \sum_{m=0}^{\infty} \frac{\left(-a_1 p_1^{\theta_1 b_1}\right)^m}{m!} x^{-\theta_1 b_1 m}$$

Then:

$$R = \frac{1}{e^{-a}e^{-a_1}} \sum_{m=0}^{\infty} \frac{\left(-a_1 p_1^{\theta_1 b_1}\right)^m}{m!} \int_0^p ab\theta p^{-\theta} x^{(\theta+1)} e^{-a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}} \left(\left(\frac{x}{p}\right)^{\theta}\right)^{-(b+1)} x^{-\theta_1 b_1 m} dx$$

Let:

$$y = a\left(\frac{x}{p}\right)^{-\theta b} \Rightarrow x = p\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}} \Rightarrow dx = \frac{-p}{ab\theta}\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}-1} dy$$

So:

$$R = \frac{ab\theta p^{-\theta}}{e^{-a}e^{-a_1}} \sum_{m=0}^{\infty} \frac{\left(-a_1 p_1^{\theta_1 b_1}\right)^m}{m!} \int_a^{\infty} \left[p\left(\frac{y}{a}\right)^{\frac{-1}{\theta b}}\right]^{-\theta_1 b_1 m+\theta+1} e^{-y} \left(\left(\frac{x}{a}\right)^{\frac{-1}{b}}\right)^{-(b+1)} \frac{-p}{ab\theta}\left(\left(\frac{y}{a}\right)^{\theta}\right)^{\frac{-1}{\theta b}-1} dy = \frac{p^{-\theta_1 b_1 m}}{e^{-a}e^{-a_1}} \sum_{m=0}^{\infty} \frac{\left(-a_1 p_1^{\theta_1 b_1}\right)^m}{m!} \quad (23)$$

$$\int_a^{\infty} \left(\frac{y}{p}\right)^{\theta_1 b_1 m} e^{-y} dy \; R = \frac{p^{-\theta_1 b_1 m} p_1^{\theta_1 b_1 m}}{a^{\frac{\theta_1 b_1 m}{\theta b}} e^{-a} e^{-a_1}} \sum_{m=0}^{\infty} \frac{(-\alpha)^m}{m!} \Gamma\left(\frac{\theta_1 b_1 m}{\theta b}+1, a\right)$$

Reniy entropy: Nonpara metric method, the researcher worked on a formula called Renyi entropy which is a probability measure to reduce uncertainty in the results of randomized trials or in the results of experiments from different data sizes, so, the formula of the Renyi entropy function will be derived for the proposed distribution according to the following formula:

$$I_R(r) = \frac{1}{1-\zeta} Log \int_0^p \left[f(a, b, \theta, p)\right]^{\zeta} dx \int_0^p \left(\frac{ab\theta p^{-\theta}}{e^{-a}}\right)^{\zeta} x^{\zeta(\theta-1)} e^{-\zeta a\left(\left(\frac{x}{p}\right)^{\theta}\right)^{-b}} \left(\left(\frac{x}{p}\right)^{\theta}\right)^{-\zeta(b+1)} dx \quad (24)$$

Let:

$$y = a\zeta\left(\frac{x}{p}\right)^{-\theta b} \Rightarrow x = p\left(\frac{y}{a\zeta}\right)^{\frac{-1}{\theta b}} \Rightarrow dx = \frac{-p}{\zeta ab\theta}\left(\frac{y}{\zeta a}\right)^{\frac{-1}{\theta b}-1} dy$$

Then:

$$= \left(\frac{ab\theta p^{-\theta}}{e^{-a}}\right)^{\zeta} \int_a^{\infty} \left[p\left(\frac{y}{\zeta a}\right)^{\frac{-1}{\theta b}}\right]^{\zeta(\theta-1)} e^{-y} \left(\left(\frac{y}{\zeta a}\right)^{\frac{-1}{b}}\right)^{-\zeta(b+1)} \frac{-p}{\zeta ab\theta}\left(\frac{y}{\zeta a}\right)^{\frac{-1}{\theta b}-1} dy$$

$$= \left(\frac{ab\theta p^{-\theta}}{e^{-a}}\right)^{\zeta} \int_a^{\infty} (p)^{\zeta\theta-\zeta+1} \left(\frac{y}{\zeta a}\right)^{\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}-1} e^{-y} dy = \left(\frac{ab\theta p^{-\theta}}{e^{-a}}\right)^{\zeta} \frac{1}{(a)^{\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}}}$$

$$\int_a^{\infty} (p)^{\zeta\theta-\zeta+1} (y)^{\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}-1} e^{-y} dy = \frac{(ab\theta)^{\zeta}}{\theta b e^{-\zeta a}} \frac{p^{1-\zeta}}{(a)^{\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}}} \int_a^{\infty} (y)^{\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}-1} e^{-y} dy$$

$$= \frac{(ab\theta)^{\zeta}}{\theta b e^{-\zeta a}} \frac{p^{1-\zeta}}{(a)^{\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}}} \Gamma\left(\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}, a\right)$$

$$I_R(r) = \frac{1}{1-\zeta} Log\left[\frac{(ab\theta)^{\zeta}}{\theta b e^{-\zeta a}} \frac{p^{1-\zeta}}{(a)^{\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}}} \Gamma\left(\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}, a\right)\right]$$

$$I_R(\zeta) = \frac{1}{1-\zeta}\left[\begin{array}{l} \zeta Log(ab\theta) + (1-\zeta) Log(p) - Log(\theta b) + \zeta a - \\ \left(\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}\right) Log(a) + Log\left(\Gamma\left(\frac{\zeta}{\theta b}+\zeta-\frac{1}{\theta b}, a\right)\right) \end{array}\right] \quad (25)$$

After obtaining Eq. 25 we will work to give the initial values of the parameters (a, b, θ, p) and for different groups of $0 \leq \zeta \leq 1$. The research depends on the parameters are estimated at that value (ζ) which achieves the smallest value of R (ζ) probability measure to reduce uncertainty.

CONCLUSION

In statistical analysis, a lot of distributions are used to represent Set (s) data. Recently new distributions are derived to extend some of well-known families of distributions such that the new distributions are more flexible than the others to model real data. The composing of some distributions with each other's in some way has been in the foreword of data modeling.

In this study we presented a new family of continuous distributions based on [0, 1] truncated Frechet distribution. [0, 1] truncated Frechet power [0, 1] TFP_{ower} distributions is discussed as special cases. Properties of [0, 1] TFP_{ower} is derived. We provide forms for characteristic function, rth raw moment and central moment, mean, variance, skewness, kurtosis, mode, median, reliability function, hazard rate function, Shannonentropy function Reney entropy and function. This study deals also with the determination of stress-strength reliability R = P [Y<X] when X (strength) and Y (stress) are two independent [0,1] TFP_{ower} distributions with different parameters.

REFERENCES

Abid, S.H. and R.K. Abdulrazak, 2017. [0,1] Truncated frechet-uniform and exponential distributions. Am. J. Syst. Sci., 5: 13-27.

Eugene, N., C. Lee and F. Famoye, 2002. The Beta-normal distribution and its applications. Commun. Stat. Theory Methods, 31: 497-512.

Gradshteyn, I.S. and I.M. Ryzhik, 2000. Table of Integrals, Series and Products. 6th Edn., Academic Press, San Diego, California, USA., Pages: 1163.

Gupta, A.K. and S. Nadarajah, 2005. On the moments of the beta normal distribution. Commun. Stat. Theor. Methods, 33: 1-13.

Jones, M.C., 2004. Families of distributions arising from distributions of order statistics. Test, 13: 1-43.