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Study of Some Graphs Types via. Soft Graph

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Abstract: In this study, the soft graph for some types of graphs which valid like null graph, complete graph, cycle graph, bipartite graph, star graph and wheel graph are be determined.

Key words: Soft graph, null graph, path graph, complete graph, cycle graph, complete bipartite graph, star graph

INTRODUCTION

We consider a graph G = (V, E) for simplicity G as finite, simple and undirected. Let V (G) and E (G) denote the vertex set and edge set, respectively for a graph G. We refer the reader to (Harary, 1969; Haynes *et al*., 1998; Omran and Rajihy, 2017; Omran and Oda, 2019; Al-Harere and Breesam, 2019) for all other terms and notions which are not provided in this study. The sub graph of induced by the vertices in D is denoted by G (D).

Molodtsov the connotation of soft set theory in (1999) as a general mathematical for dealing with uncertainties which is free from the above difficulties. Soft set theory has a rich potential for applications in several directions, a few of which had been shown by Molodtsov (Al-Harere and Breesam, 2019). Maji, defined and studied many processes on the soft set (Molodtsov, 1999). Thumbakara and George (2014) have been introduced soft graph and investigated some of their properties (Maji *et al*., 2003).

We calculate the soft graph for null graph, complete graph, cycle graph, bipartite graph and star graph.

Preliminaries: In this section, we presented some definitions about graphs, soft set and soft graph. These will be helpful in later section.

Definition 2.1; Molodtsov (1999): Let U be an initial universe set and E a set of parameters or attributes with respect to U. Let P (U) denote the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over, U where F is a mapping given by: $F: A \rightarrow P(U)$ in other words, a soft set (F, A) over U is a parameterized family of subsets of U for $e \in A$, F (e) may be considered as the set of e-elments of the soft sets (F, A). Thus (F, A) is defined as:

$$(F, A) = \{F(e) \in P(U) : e \in E, F(e) = \phi \text{ if } e \notin A$$

Definition 2.2; Harary (1969): A graph G consists of a pair (V (G), E (G)) where V (G) is a nonempty finite set whose element are called points, vertices or nods and E (G) is a set of unordered pairs of distinct elements of (V (G)). The members of E (G) are called lines, curves or edges of the graph G.

The number of vertices in a given graph is called order of the graph denoted by |V(G)|. The number of edges in a given graph is calledsize of the graph denoted by |E(G)|.

Definition 2.3; Harary (1969): If two vertices are joined by an edge then these vertices are called adjacent vertices.

Definition 2.4; Haynes *et al.* (1998): If $E = \varphi$ in a graph G = (V, E) then such graph without any edges is called a null graph and denoted by N_n.

Definition 2.5; Haynes *et al.* (1998): A path of length n-1 denoted by P_n is a sequence of distinct edge $v_1 v_2, v_2 v_3, \ldots, v_{n-1} v_n$.

Defi nition 2.6; Haynes *et al.* (1998): If all vertices in a graph are adjacent to each other, then the graph is called a complete graph. The symbol K_n is used to denote a complete graph with n vertices.

Definition 2.7; Haynes *et al.* (1998): A cycle in a graph is closed path in which the only repetition are the first and last vertex and denoted by C_n.

Definition 2.8; Haynes *et al.* (1998): A complete bipartite graph is a bipartite graph in which each vertex in X is joined to each vertex in Y. The complete graph having bipartition (X, Y) such that |X| = m and |Y| = n is denoted by $K_{m,n}$.

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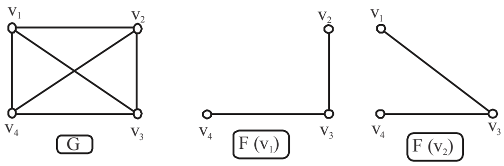

Fig. 1: Soft graph in G

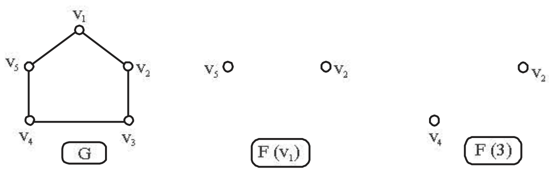

Fig. 2: (F, A) is not soft graph

Definition 2.9; Kitaev and Lozin (2015): A star graph S_n is the complete bipartite graph $K_{1,n}$. The centrum of a star is all-adjacent vertex in it.

Definition 2.10; Harary, (1969): A wheel graph is the graph C_n+k_1 denoted by W_n.

Definition 2.11; Gross *et al.* (2013): A subgraph H of G such that whenever u, $v \in V$ (H) are adjacent in G then they are in H is called an induced subgraph of G and denoted by G [V (H)].

Definition 2.12; Maji *et al.* (2003): A graph is called connected if in it any two vertices are joined by some path; otherwise is called disconnected.

Definition 2.13; Gross *et al.* (2013): The distance between two vertices in a graph is the length of the shortest path between them.

Definition 2.14; Gross *et al.* (2013): The eccentricity of a vertex v in a connected graph is its distance to a vertex farthest from v. The radius of a connected graph is its minimum eccentricity denoted by rad G. The diameter of a connected graph is its maximum eccentricity denoted by diam (G).

Soft graph: Let G = (V, E) be a simple graph, A any nonempty set, let R be an arbitrary relation between elements of A and element of V. That is $R \subseteq A \times V$. A set valued function $F: A \rightarrow P(V)$ can be defined as:

$$F(x) = \{y \in V \mid xRy\}$$

The pair (F, A) is soft set over V.

Definition 3.1; Gross *et al.* (2013): Let (F, A) be a soft set over V. Then (F, A) is said to be a soft graph of G if the subgraph induced by F(x) in G is connected subgraph of G for all $x \in A$. And the set of all soft graph of G is denoted by S.

Example 3.1: Let G(V, E) be simple graph as shown in Fig. 1. Let A = $\{v_1, v_2\}$ and the set valued function F by (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) = 1\}$ then F (v_1) = $\{v_2, v_3, v_4\}$, F (v_2) = $\{v_1, v_3, v_4\}$. Therefore, the induced subgraph by F(v_1), F (v_2) in G is connected subgraph of G for all $x \in A$, hence, (F, A) is a soft graph.

Example 3.2: Let G (V, E) be a simple graph as shown in Fig. 2. Let A = $\{v_1, v_2\}$ and the set valued function F by F (x), = $\{y \in V | xRy \Leftrightarrow x$ is adjacent to y in G$\}$, then F (v_1) = $\{v_2, v_5\}$ F (v_3) = $\{v_2, v_4\}$. Therefore, the induced subgraph by F (v_1), F (v_2) in G is not connected for all $x \in A$, hence, (F, A) is not a soft graph .

Theorem 3.1: Let N_n be a null graph then (F, A) is soft graph if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) = 0\}$.

Proof: Let A = (v_i) for some i, then F (v_i) = (v_i) $\forall i$, therefore, the induced subgraph by F (v_i) is connected. Hence, (F, A) is soft graph.

Theorem 3.2: Let P_n be a path graph then (F, A) is soft graph if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq = k$ where either $k \leq$ rad (G) or k = diam (G)$\}$.

Proof: Let A = $\{v_i\}$ for some, i then there are two cases depend on F (x) as follows.

Case(1): If $k \leq$ rad (G) then there are two cases as follows: (i) If $v_1 \in$ cent (G) then F $(v_i) \cong F = G \cong \forall i$. Thus, the induced subgraph F (v_i) is connected subgraph, hence (F, A) is soft graph; (ii) if $v_i \notin$ cent (G) and A is single set then F $(v_i) \ncong$ G $\forall$ i and the induced subgraph by F (v_i) is connected subgraph, hence, (F, A) is soft graph.

Case (2): If k = diam (G) then, F $(v_i) \cong G \forall i$. Therefore, the induced subgraph by F (v_i) is a connected subgraph, hence (F, A) is a soft graph.

Example 3.3: Let G = P_5 be path graph then there are two cases depend on F (x) as follows:

Case (1): If k = rad (G) then rad (G) = $2 \Rightarrow k = 2$ and F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq 2\}$ (i) if $v_i \in$ cent (G) =A = (v_3) and F (v_3) = $\{v_1, v_2, v_3 v_4, v_5\} \ncong$ G. Therefore, the induced subgraph by F (v_3) is connected subgraph, hence (F, A) is soft graph (Fig. 3a) (ii) if $v_i \notin$ cent (G) then let A= (v_2) and F (v_2) = $\{v_1, v_2, v_3, v_4\} \ncong$ G. Therefore, the induced subgraph by F (v_3) is connected subgraph, hence (F, A) is soft graph (Fig. 3b).

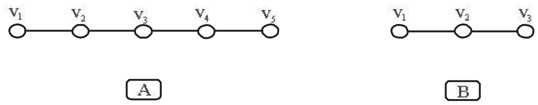

Fig. 3: Soft graph in path graph P_5

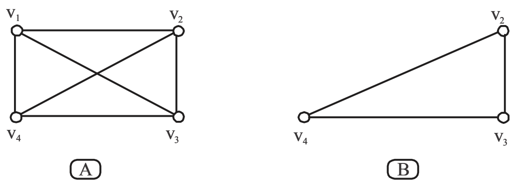

Fig. 4: Soft graph in complete graph K_4

Case (2): If k = diam (G) then diam (G) = 4 ⇒ k = 4 F (x) = {y∈V|xRy ⇔d (x, y)≤4}; (i) if v_i ∈cent (G) A= v_3 and F (v_1) = {v_1, v_2, v_3, v_4, v_5} ≅G. Therefore, the induced subgraph by F (v_1) is a connected subgraph, hence (F, A) is soft a graph Fig. 3a.

Theorem 3.3: Let K_n be complete graph then (F, A) is a soft graph if F (x) = {y∈V|xRy⇔d (x, y)≤ 1}.

Proof: Let A = (v_i) for some i then there are two cases depend on F (x) as follows:

Case (1): If F (x) = {y∈V|xRy⇔d (x, y) = 1},then F (v_i) = {v_j j = 1, ..., n, j ≠ i} ≇ G. Therefore, the induced subgraph by F (v_i) is isomorphic to K_{n-1}, so, it is a connected subgraph, hence (F, A) is a soft graph.

Case (2): If F (x) = {y∈V|xRy⇔d (x, y) = 1}then F (v_i) = {v_i, i = 1, ..., n} ≇G. Therefore, the induced subgraph by F (v_i) is a connected subgraph, hence (F, A) is a soft graph.

Example 3.4: Let G be a complete graph of order n, let A = (v_1) and if F (x) = {y∈V|xRy⇔d (x, y) = 1} F (v_1) = {v_2, v_3, v_4} ≅ K_3. Therefore, the induced by F (v_1) is a connected subgraph, hence, (F, A) is a soft graph. Figure 4 (B) if F (x) = {y∈V|xRy⇔d(x, y)≤1} then (v_1) = {v_1, v_2, v_3, v_4}≅G. Therefore, the induced by F (v_1) is a connected subgraph, hence (F, A) is a soft graph (Fig. 4A).

Theorem 3.4: Let C_n be a cycle graph then (F, A) is soft graph if and only if F (x) = {y∈V|xRy⇔d (x, y) = $|n/2|$}∀n∈N.

Proof: Let A = {v_i} for some i and F (x) = {y∈V|xRy⇔d (x, y) = |n/2|} then there are two case 1 as follows.

Case (1): If n even then, F (v_i) $\{v_{i+n/2}\}$. Therefore, the induced subgraph by F (v_i) is a connected, hence (F, A) is a soft graph.

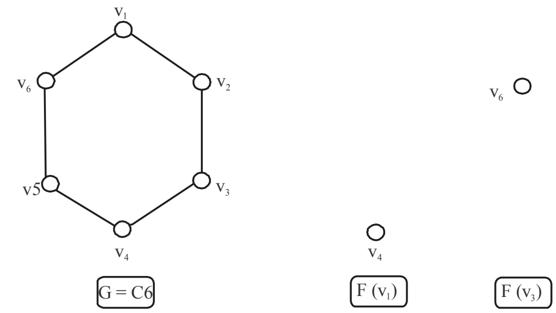

Fig. 5: (F, A) is soft graph in C_6

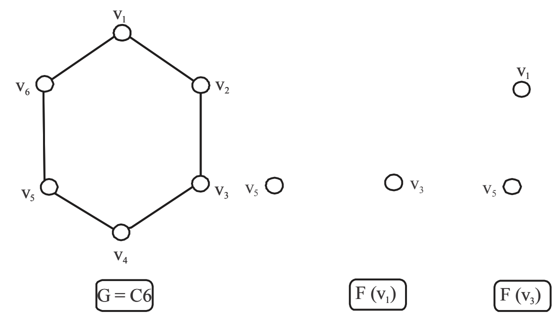

Fig. 6: (F*, A) is not soft graph in C_6

Case (2): If n is odd then F (v_i) = $\left\{v_i+\frac{n}{2}, v_i+\frac{n}{2}+1\right\}$. Therefore, the induced subgraph by F (v_i) is connected, hence, (F, A) is a soft graph.

Let F* (x) ≠ F (x) to proof F* (x) is not soft graph, let A = (v_i) for some i and F* (x) = $\{y\in V|xRy \Leftrightarrow d(x,y)=c \text{ where } c\neq|n/2|\}$.

Therefore, the induced subgraph by F* (v_i) is not a connected subgraph, since, it is contains at least two independent vertices. Figure 5 and 6. Hence, F*(A) is not a soft graph.

Example 3.5: Let G≅C_n be a cycle graph if n is an even number. Let n = 6, let A = {v_1, v_3} then:

$$F(x)=\left\{y\in V\middle|xRy\Leftrightarrow d(x,y)=\left|\frac{n}{2}\right|\right\}\Rightarrow d(x,y)=\frac{6}{2}=3, \text{then } F(v_1)=\{v_4\}, F(v_3)=\{v_6\}$$

Therefore, the induced subgraph by F (v_1) F (v_3) is connected, hence (F, A) is the soft graph.
If:

$$F^*(x)=\left\{y\in V\middle|xRy\Leftrightarrow d(x,y)=c \text{ where } c\neq\left|\frac{n}{2}\right|\right\} \text{ since, } n=6 \text{ so } c\neq 3 \text{let } c=2$$

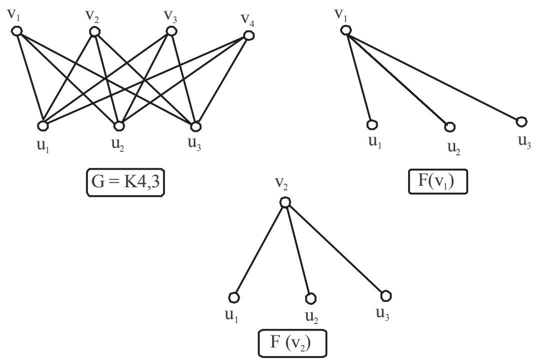

Fig. 7: (F, A) is soft graph in $K_{4,3}$

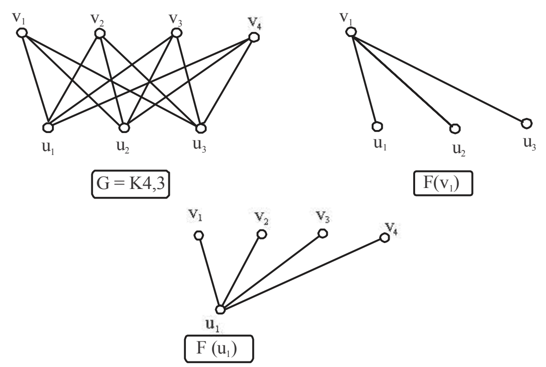

Fig. 8: (F, A) is soft graph in $K_{4,3}$ F(u1)

Then $F^*(v_1) = \{v_3, v_5\}$, $F^*(v_1) = \{v_1, v_1\}$. Therefore, the induced subgraph by F^* (v_1), $F^*(v_3)$ is not connected subgraph, hence, (F, A) is not soft graph.

Theorem 3.5: Let $G = K_{n,m}$ be complete pibartite graph of order nm and n, $m \geq 1$ then is (F, A) soft graph if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq k$; where k = 1, 2$\}$.

Proof: There are two cases depend on the set A as follows:

Case (1): If A = $\{v_i\}$ for some i then there are two casesas follows; if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq 1\}$ then F (v_i) = $\{v_i, u_j, i = 1, ..., m\}$. Therefore, the induced subgraph by F (v_i) is connected subgraph, hence (F, A) is soft graph. if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq 2\}$ then F (v_i) = $\{v_i, u_i, i = 1, ..., n, j = 1, ..., m\} \cong G$. Therefore, the induced subgraph by F (v_i) is connected subgraph, hence, (F, A) is soft graph..

Case(2): If A = $\{v_i, u_j\}$ for some i, j then there are two cases as follows: if F (x) = $\{Y \in V | xRY \Leftrightarrow d(x, y) \leq 1\}$ then F (v_i) = $\{v_i, u_j, j = 1, ..., m\}$ $\forall$i, F (μ_i) = $\{v_i, u_j, j = 1, .., n\}\forall$j. Therefore, the induced subgraph by F (v_i), F (u_i) is connected subgraph, hence, (F, A) is soft graph. if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq 2\}$ then, F (v_i) = $\{v_i, u_j, i = 1, ..., n, j = 1, ..., m\} \cong G$, F ($u_i$) = $\{v_i, u_j, i = 1, ..., n, j = 1, ..., m\} \cong G$. Therefore, the induced subgraph by F (v_i), F (u_i) is connected subgraph, hence, (F, A) is soft graph.

Example3.6: Let G = $K_{4,3}$ be complete bipartite graph Fig. 7.

Case (1): if A = $\{v_1, v_2\}$ if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq 1\}$ then F (v_i) = $\{v_i, u_1, u_2, u_3\}$, F (u_2) = $\{v_2, u_1, u_2, u_3\}$. Therefore, the induced subgraph by F (v_1), F (v_2) is connected subgraph, hence (F, A) is soft graph.

If F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq 2\}$ then F (v_1) = $\{v_1, v_2, v_3, v_4, u_1, u_2, u_3\}$. Therefore, the induced subgraph by F (v_1), (v_2) is connected subgraph, hence (F, A) is soft graph.

Case (2): A = $\{v_1, u_1\}$ if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq 1\}$ then F (v_1) = $\{v_1, u_1, u_2, u_3\}$, F (u_1) = $\{u_1, v_1, v_2, v_3, v_4\}$. Therefore, the induced subgraph by F (v_1), F (u_1) is connected subgraph, hence (F, A) is soft graph (Fig. 8).

If F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq 2\}$ then F (v_1) = $\{v_1, v_2, v_3, v_4, u_1, u_2, u_3\} \cong G$ F (u_1) = $\{v_1, v_2, v_3, v_4, u_1, u_2, u_3\} \cong G$. Therefore, the induced subgraph by F (v_1), F (u_1) is connected subgraph, hence, (F, A) is soft graph.

Corollary 3.1: Let S_n be star graph then (F, A) is soft graph if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) \leq k\}$ where k = 1, 2.

Theorem 3.6: Let W_n be wheel graph then (F, A) is soft graph if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) = k\}$ where k = 1, 2.

Proof: There are two cases depend on the set A as follows:

Case (1): If A = $\{v_i\}$ for some i, then there are two case as follows: if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) = 1\}$ then F (v_i) = $\{v_0, v_j\}$ for some j, j $\neq$ i and v_0 cent (G). Therefore, the induced subgraph by F (v_i) is connected subgraph, hence, (F, A) is soft graph; if F (x) = F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) = 2\}$ then, F (v_i) = F $\{v_j\}$ for some j and i $\neq$j. Therefore, the induced subgraph by is connected subgraph by F (v_i) is conneted subgraph, hence, (F, A) is soft graph.

Case (2): If A = $\{v_0\}$ where $v_0 \in$ cent (G), then F (x) = F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) = 1\}$ and F (v_0) = $\{v_i, i = 1, ..., n\} \neq C_n$. Therefore, the induced subgraph by F (v_i) is connected subgraph, hence, (F, A) is soft graph.

Example 3.7: Let G = W_5 be wheel graph. There are two cases depend on the set of follows:

Case (1): If A = $\{v_1, v_3\}$ then there are two case as follows: if F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) = 1\}$ then F (v_1) =

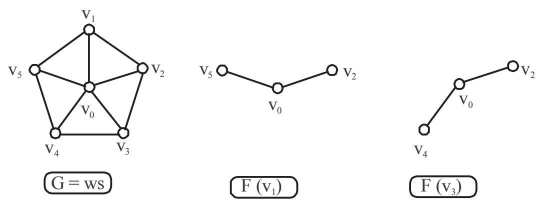

Fig. 9: (F, A) is soft graph in W_5

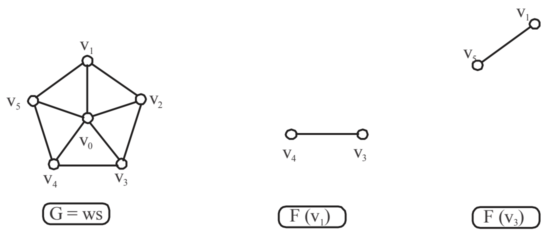

Fig. 10: (F, A) is soft graph in W_5 ($F(v_3)$, (v_5, v_1))

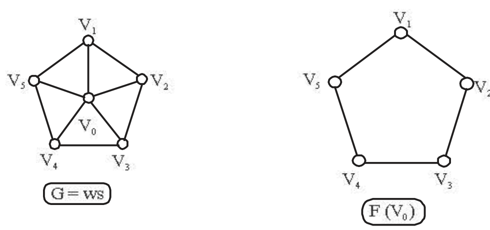

Fig. 11: (F, A) is soft graph in W_5 ($F(v_0)$)

$\{v_0, v_2, v_5\}$, F (v_3) = $\{v_0, v_2, v_4\}$. Therefore, the induced subgraph by F (v_1), F (v_3) is connected subgraph, hence, (F, A) is soft graph (Fig 9).

If = F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) = 2\}$, then F ($v_1$) = $\{v_3, v_4\}$, F (v_3) = $\{v_1, v_5\}$. Therefore, the induced subgraph by F (v_1), F (v_3) is connected subgraph, hence (F, A) is soft graph (Fig. 10).

Case (2): If A = $\{v_0\}$, then F (x) = $\{y \in V | xRy \Leftrightarrow d(x, y) = 1\}$ and F (v_0) = $\{v_1, v_2, v_3, v_4, v_5\} \cong C_5$. Therefore, the induced subgraph by F (v_0) is connected subgraph, hence (F, A) is soft graph (Fig. 11).

CONCLUSION

In this study, we conducted a study the soft graph on the special types of graphs such as null, path, cycle, complete bipartite, star and wheel graph. The theories of each species are proved with illustrative examples.

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