Journal of Engineering and Applied Sciences 14 (Special Issue 8): 10413-10418, 2019
ISSN: 1816-949X

SU (3) Symmetry of Even-Even $^{150,\,152}$Ce Isotopes in the Microscopic Interacting Boson Model (IBM-1)

Ali H. Al-Musawi and Amir A. Mohammed-Ali
Department of Physics, College of Science, University of Kerbala, 56001 Karbala, Iraq

Abstract: Deformed nuclei represent perhaps the largest and most studied class of nuclear-level schemes. The Interacting Boson Model (IBM-1) has been used to calculate the positive parity states of ^{150,152}Ce below 2 MeV. A simple parameterisation has been used that corresponds to a description close to the SU (3) limit of the model. The energy levels and B (E2) values are obtained for the ^{150,152}Ce by using the IBM. The E (γ) transition energy has been plotted as a function of the spin to determine the existence of the back-bending phenomenon. The results show that these nuclei have rotational properties. The moment of inertia, the rotational energy, two-nucleon transfer reactions and the Potential Energy Surfaces (PES) for these isotopes have been calculated.

Key words: IBM, two-nucleon transfer, B (E2) values, potential energy surface, SU (3) limit, moment of inertia

INTRODUCTION

Although, Cerium (Ce) isotopes are located in an interesting and complex region within the periodic table (Turkan and Maras, 2007) previous studies of the cerium nucleus series are relatively few, especially, studies of cerium nuclei with mass numbers 150 and 152. The ^{152}Ce has not been studied previously, so, it has been studied here using the Interacting Boson Model (IBM-1). In nuclei with mass number 150 such as Nd, it is found that these isotopes are nearly perfect rotors (Jinfu *et al*., 2000). By Jinfu *et al*. (2000) studied the positive parity collective states in $^{128-150}$Ce isotopes using the IBM-1. They studied both energy levels and the reduced transition probability for these isotopes.

The ^{150,152}Ce nuclei have Z = 58 and 92 and 94 neutrons, respectively. These nuclei have four proton bosons as particles close to the magic number 50 and they have five and six neutron bosons as particles, respectively. These configurations have been used to interpret the yrast levels (Iachello, 1981). Initially, the ratio R ($4^+_1/2^+_1$) of ^{150,152}Ce is 3.15 and 3.25. These ratios indicate that these nuclei are located in the SU (3) region and have rotational properties. The collective nuclear motion for heavy mass nuclei has been successfully described by Arima and Iachello using the IBM (Armia and Iachello, 1976). The IBM is mainly rooted in the shell model and the collective model (Pfeifer, 1998). In IBM-1, the three symmetry limits are the vibrational limit U (5), the rotational limit SU (3) and the γ-unstable O (6). The IBM-1 makes no distinction between neutrons and protons boson (Iachello, 1981; Iachillo and Arima, 1987). The aim of this study is to interpret energy levels for even-even ceusing the IBM-1 with two-nucleon transfer reactions. The B (E2) values and moment of inertia are calculated and compared with previous available experimental data and the dynamical symmetries of these isotopes investigated.

MATERIALS AND METHODS

Theoretical basis

Microscopic Interacting Boson Model (IBM-1): The IBM-1 of Iachillo and Arima (1987) is widely accepted as a traceable theoretical model to describe the low nuclear collective properties of complex nuclei across an entire closed shell using a suitable Hamiltonian (Casten and Warner, 1988; Al-Dahan, 2017; Hussain *et al*., 2015). The low-lying collective characteristics of even nuclei are described in terms of the interacting s-boson and d-boson, with a total angular momentum of L = 0 and L = 2, respectively (Iachello, 1981; Al-Dahan, 2017; Sharrad *et al*., 2013). The number of bosons depends on the number of active nuclei or hole pairs outside a closed shell (Pfeifer, 1998). The Hamiltonian of the IBM-1 can be expressed as (Iachello, 1981):

$$\begin{aligned} H =\ & \epsilon_s N + \frac{1}{2} u_0 N (N-1) + \epsilon' \left(d^\dagger, \tilde{d}\right) + \\ & \sum_{L=0,2,4} \frac{1}{2} (2L+1)^{\frac{1}{2}} C'_L \left[\left[d^\dagger \times d^\dagger\right]^{(L)} \times \left[\tilde{d} \times \tilde{d}\right]^{(L)} \right]^{(0)} + \\ & \frac{1}{2^{\frac{1}{2}}} \tilde{v}_2 \begin{bmatrix} \left[d^\dagger \times d^\dagger\right]^{(2)} \times \left[\tilde{d} \times \tilde{s}\right]^{(2)} + \left[d^\dagger \times s^\dagger\right]^{(2)} \times \\ \left[d^\dagger \times d^\dagger\right]^{(2)} \left[\tilde{d} \times \tilde{d}\right]^{(2)} \end{bmatrix}^{(0)} + \\ & \frac{1}{2} \tilde{v}_0 \begin{bmatrix} \left[d^\dagger \times d^\dagger\right]^{(0)} \times \left[\tilde{s} \times \tilde{s}\right]^{(0)} + \\ \left[s^\dagger \times s^\dagger\right]^{(0)} \times \left[\tilde{d} \times \tilde{d}\right]^{(0)} \end{bmatrix}^{(0)} \end{aligned} \qquad (1)$$

where, the dot indicates scalar products and the cross indicates tensor products. The above equation is specified

Corresponding Author: Ali H. Al-Musawi, Department of Physics, College of Science, University of Kerbala, 56001 Karbala, Iraq

by nine parameters, two appearing in one-body terms and seven in two-body terms (Iachello, 1981; Arima and Iachello, 1981). The Hamiltonian used to calculate energy levels (Casten and Warner, 1988) is:

$$H = \varepsilon\hat{n}_d + a_2 Q^2 + a_1 L^2 \quad (2)$$

Moment of inertia and rotational energy: The transition energy (Eγ) between levels is E(L)-E(L-2) while the equation of calculating rotational energy (ħ) (Castenholz, 1993; Krane and Halliday, 1987) is:

$$\hbar\omega = \frac{E(L)-E(L-2)}{\sqrt{L(L+1)}-\sqrt{(L-2)(L-1)}}(\text{MeV}) \quad (3)$$

The general form for calculating the moment of inertia (Castenholz, 1993) is:

$$2\vartheta/\hbar^2 = \frac{4L-2}{E\gamma} = \frac{4L-2}{E(L)-E(L-2)}(\text{MeV})^{-1} \quad (4)$$

Potential Energy Surface (PES): The IBM-1 energy surface is created using the expectation value E of the IBM-1with coherent states to describe the intrinsic structure of quantum systems that have multi-nucleon out-closed shells in a clear geometric picture form (Iachillo and Arima, 1987; Casten and Warner, 1988; Sharrad *et al*., 2013). The intrinsic state of the ground-state of the IBM-1can be given (Casten and Warner, 1988) as:

$$\left|N, \beta, \gamma\right\rangle = 1/\sqrt{N!}\left(b_c^{\dagger}\right)^N\left|0\right\rangle \quad (5)$$

where, $|0\rangle$ is a vacuum state that is annihilated by all annihilation operators. The PES as a β–γ function can been written (Casten and Warner, 1988; Sharrad *et al*., 2013) as:

$$E(N, \beta, \gamma) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} + \frac{N(N+1)}{(1+\beta^2)^2}\begin{pmatrix}\alpha_1\beta^4+\alpha_2\beta^3 \\ \cos3\gamma+\alpha_3\beta^2+\alpha_4\end{pmatrix} \quad (6)$$

where, the α_1, α_2, α_3 and α_4 are related to the coefficients C_L, v_2, v_0, u_2 and u_0 of Eq. 1.

Two-nucleon transfer intensities: An important property in the changing interpretation mechanism of nuclear shape is the two-nucleon transfer intensity. Within the framework of the IBM-1, two nucleon transfer intensities can be described. The operator for L = 0 transfer is:

$$T^{\dagger} = \alpha + s^{\dagger}, \quad T^{-} = \alpha\text{-}s$$

($s^{\dagger}$, s) refers to creation and annihilation of an s-boson operator. The expressions of the boson transfer intensity for ground-state to ground-state in limits U (5) and SU (3) are given, respectively (Iachillo and Arima, 1987; Casten and Warner, 1988; Scholten, 1980) as:

$$I(N_v \rightarrow N_v+1) = \alpha_v^2(N_v+1)(\Omega_v\text{-}N_v) \quad (7)$$

$$I(N_v \rightarrow N_v+1) = \alpha_v^2(N_v+1)\left(\frac{2N+3}{3(2N+1)}\right)\left(\Omega_v - N_v - \frac{4}{3}\frac{(N\text{-}1)}{(2N\text{-}1)}N_v\right) \quad (8)$$

Where:

N_v : The number of neutrons boson
N : Indicates the number of the total particles bosons
Ω_v : Represents the neutron-pair degeneracies which is taken equal to (126-82)/2 = 22

RESULTS AND DISCUSSION

In this study, the energy levels, the reduced quadrupole transition probabilities B (E2) and PES for ^{150,152}Ce isotopes have been investigated using IBM-1, also the moment of inertia for the ground-band was calculated.

Energy levels: The EPS term was introduced into the Hamiltonian equation of the pure SU (3) limit to calculate the energy levels of ^{150}Ce and the SU (3) limit was applied to calculating the energy levels of ^{152}Ce. The even-even ^{150,152}Ce isotopes have proton boson particle number 4 and neutron boson particles number 5 and 6, respectively.

The experimental energy ratio $R(4_1^+/2_1^+) = E4_1^+/E2_1^+$ for ^{150,152}Ce is 3.15 and 3.25, respectively, whilst the calculated energy ratio $R(4_1^+/2_1^+) = E4_1^+/E2_1^+$ using IBM-1 for the same isotopes is 3.34 and 3.36, respectively. The energy levels of the γ-band have been predicted and the results of the comparison between the theoretical and experimental energy spectrum were good as shown in Fig. 1. The QQ parameters for ^{150}Ce and ^{152}Ce were found by estimating the energy of the second state with the spin or parity 2^+_2. The parameters used in the PHINT code to calculate energy levels are shown in Table 1.

B (E2) values: An important aspect of the IBM-1 predictions focuses on electric quadrupole transitions (E2) (Mulligan *et al*., 1972. The general E2 operator can be introduced (Casten and Warner, 1988) as:

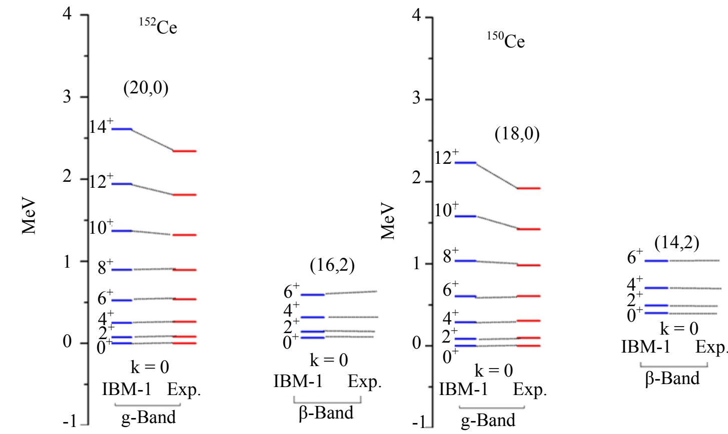

Fig. 1: Comparison between the calculated energy levels with the experimental energy levels (Artna-Cohen, 1996; Basu and Sonzogni, 2013 Martin, 2013) for ^{150,152}Ce

Table 1: The parameters used in the PHINT code for ^{150,152}Ce in (MeV) units exceptthe boson number (N)

Isotopes	N	ε	PAIR	ELL	QQ	OCT	HEX	CHI
^{150}Ce	9	0.09	0	0.02	−0.0174	0	0	−2.9
^{152}Ce	10	0	0	0.024	−0.0024	0	0	−2.9

Table 2: The parameters used in the FBEM code for ^{150,152}Ce

Isotope	E2SD (e_B)	E2DD (e_B)
^{150}Ce	0.13	−0.37
^{152}Ce	0.155	−0.458

Table 3: Comparison of the calculated and experimental (Artna-Cohen, 1996; Basu and Sonzogni, 2013 Martin, 2013) with B(E2) values for ^{150,152}Ce isotopes

	B(E2) value in (e^2b^2) units				
	^{150}Ce			^{152}Ce*	
$J_i \to J_f$	IBM-1 cal.	EXP.	Cal. of Jinfu *et al.* (2000)	IBM-1 cal.	EXP.
$2_1^+ \to 0_1^+$	0.605	0.615	0.568	1.10	1.19
$4_1^+ \to 2_1^+$	0.844	0.47	-	1.54	-
$6_1^+ \to 4_1^+$	0.886	-	-	1.626	-
$8_1^+ \to 6_1^+$	0.861	-	-	1.603	-
$10_1^+ \to 8_1^+$	0.795	-	-	1.508	-
$12_1^+ \to 10_1^+$	0.694	-	-	1.36	-
$14_1^+ \to 12_1^+$	-	-	-	1.17	-
$2_2^+ \to 0_2^+$	0.431	-	-	0.0572	-
$4_2^+ \to 2_2^+$	0.597	-	-	0.0144	-
$6_2^+ \to 4_2^+$	0.620	-	-	0.08	-
$2_2^+ \to 0_1^+$	0.001-	-	-	0	-
$4_2^+ \to 2_1^+$	0.0007	-	-	0	-
$6_2^+ \to 4_1^+$	0.001	-	-	0	-

*There are no calculated B(E2) values for^{152} Ce isotope in reference (Jinfu *et al*., 2000)

$$\hat{T}(E2) = \alpha_2\hat{Q} = e_B\hat{Q} = e_B\left(\left[d^\dagger\tilde{s}+s^\dagger\tilde{d}\right]^{(2)} + \chi\left[d^\dagger\times\tilde{d}\right]^{(2)}\right) \quad (9)$$

Where:

e_B : The effective charge of the boson
Q : The quadrupole operator
χ : Denotes that deformation magnitude is equal to $\pm\frac{\sqrt{7}}{2}$ or $(\tilde{s}, \tilde{d})$

($s^\dagger$, s) are annihilation and creation operators s- and d-bosons, respectively (Casten and Zamfir, 2001). Table 2 shows the parameters used to calculate the B (E2) values in the FBEM programme dependence on experimental $B(E;2_1^+ \to 0_1^+)$ transition. The calculated B (E2) values were compared with available experimental data as shown in Table 3.

In the pure SU (3) limit, the reduced electric quadrupole transitions are allowable between interbands while the transitions between the various bands are forbidden such as the (β→g) band, so, the observed B (E2) values of the ^{152}Ce isotope in Table 3 show that this isotope has a perfect rotor. In the U (5) limit, the transitions between various bands are allowable in addition to the transitions between interbands which achieves the selection rule (Basu and Sonzogni, 2013). The ($2_2^+ \to 0_1^+$), ($4_2^+ \to 2_1^+$) and ($6_2^+ \to 4_1^+$) transitions of ^{150}Ce with very small values (0.001, 0.0007, 0.0003) (e^2b^2) lead to small breaking in the pure SU (3) limit.

Moment of inertia and the back-bending effect: The moment of inertia and the rotational energy for the ground-band of even-even Ce isotopes have been calculated using Eq. 3 and 4 as shown in Table 4. The relationship between transition energy and spin has been plotted as shown in Fig. 2 .

Neither the ^{150}Ce isotope nor the ^{152}Ce isotope show the back-bending effect which means the isotope's properties run unchanged (Al-Dahan, 2017). The back-bending usually does not occur at a low spin in the ground-band, so, the moment of inertia increases directly proportional to the rotational energy as show in

Table 4: Comparison of the transition energy, the rotational energy and moment of inertia of the experimental (Basu and Sonzogni, 2013) and the theoretical for the ^{150}Ce isotope

$J_f \to J_i$	E_γ(MeV) Exp.	$2\vartheta/\hbar^2$ (MeV)$^{-1}$ Exp.	$\hbar\omega$ (MeV)Exp.	E_γ (MeV) Cal.	$2\vartheta/\hbar^2$ (MeV Cal.	$\hbar\omega$ (MeV) Cal.
$2_1^+ \to 0_1^+$	0.097	61.58	0.039	0.086	69.7	0.035
$4_1^+ \to 2_1^+$	0.208	67.30	0.102	0.202	69.3	0.099
$6_1^+ \to 4_1^+$	0.301	73.08	0.149	0.316	69.6	0.157
$8_1^+ \to 6_1^+$	0.376	79.78	0.187	0.430	69.7	0.214
$10_1^+ \to 8_1^+$	0.440	86.36	0.219	0.540	69.58	0.271
$12_1^+ \to 10_1^+$	0.495	92.74	0.247	0.658	69.9	0.328

Table 5: Comparison of the transition energy, the rotational energy and moment of inertia of the experimental (Artna-Cohen, 1996; Martin, 2013) and the theoretical for the ^{152}Ce isotope

$J_f \to J_i$	E_γ(MeV) Exp.	$2\vartheta/\hbar^2$(MeV)$^{-1}$ Exp.	$\hbar\omega$ (MeV)Exp.	E_γ (MeV) Cal.	$2\vartheta/\hbar^2$ (MeV) Cal.	$\hbar\omega$ (MeV) Cal.
$2_1^+ \to 0_1^+$	0.081	73.8	0.033	0.074	81	0.03
$4_1^+ \to 2_1^+$	0.182	76.5	0.09	0.175	80	0.0865
$6_1^+ \to 4_1^+$	0.274	80.2	0.136	0.274	80.2	0.136
$8_1^+ \to 6_1^+$	0.355	84.3	0.177	0.373	80.4	0.186
$10_1^+ \to 8_1^+$	0.425	89.2	0.212	0.472	80.3	0.236
$12_1^+ \to 10_1^+$	0.487	94.2	0.243	0.573	80.27	0.286
$14_1^+ \to 12_1^+$	0.538	100.1	0.269	0.667	80.8	0.333

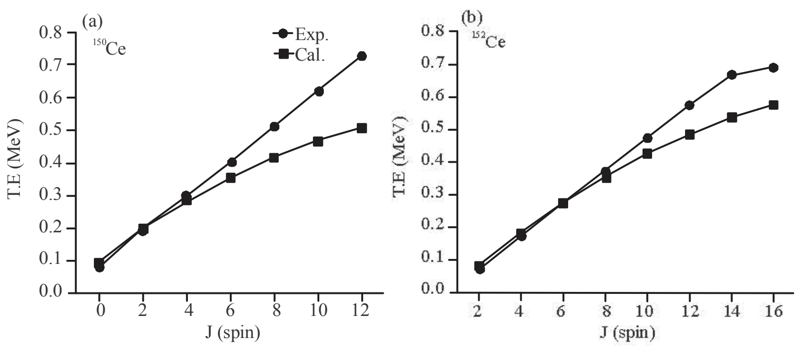

Fig. 2(a, b): The realtionship between transition energy and spin

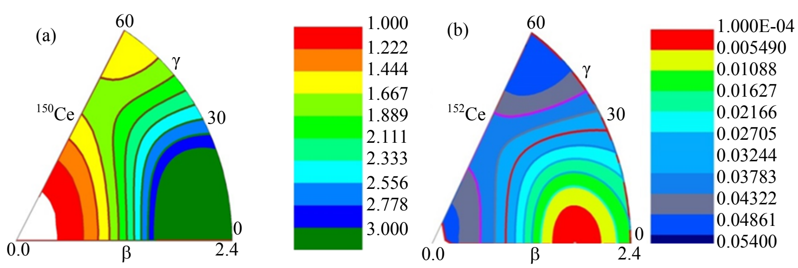

Fig. 3(a, b): The PES as function (β–γ) parameters for ^{150,152}Ce

Table 4 and 5. Energy levels begin to decline as the nuclei turn to the SU (3) limit which gradually decreases their rotational energy.

Potential Energy Surface (PES): In this study, the geometrical properties of the ^{150,152}Ce nuclei have been studied using the IBM-1 (Martin, 2013).

In the SU(3) limit, the energy surface is at its minimum at $\beta = \sqrt{2}$ while the $\gamma = 60$ applies to oblate deformation and the $\gamma = 0$ to prolate deformation (Casten and Warner, 1988). The plane of (β–γ) in Fig. 3 shows the phase shape. This figure illustrates nearly perfect rotational characteristics for the ^{152}Ce isotope with small breaking in the SU (3) limit for the ^{150}Ce isotope. This breaking indicates the transition characteristics from U (5) to SU (3) (Artna-Cohen, 1996).

Two-nucleon transfer reaction intensities: Further to the studied nuclear properties in this research, the two-nucleon transfer reaction intensity for ground-state ($0_1^+ \to 0_1^+$) to ground-state as (t, p) and (p, t) reactions have been investigated. The intensities per α^2 as a function of neutron numbers were plotted for ^{150,152}Ce isotopes as shown in Fig. 4. There are no experimental data for cross-sections of ^{150,152}Ce (Sahan *et al*., 2017; Mulligan *et al*., 1972) so, the calculated results of the

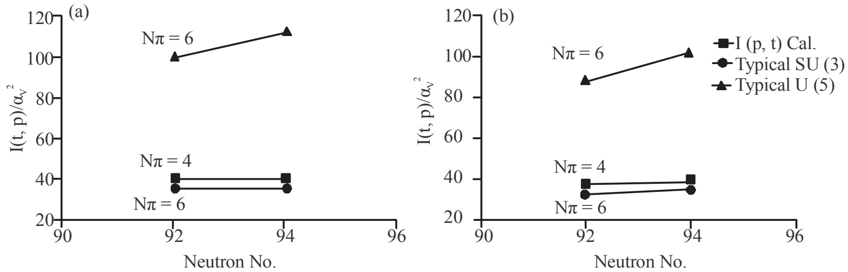

Fig. 4 (a, b): Comparison between the calculated and typical results (Scholten, 1980) of Interacting Boson Model-1 for ground-state to ground-state two neutron transfer reaction intensities of $^{150,152}Ce$ (a) Represent (t, p) reaction while and (b) Represent (p, t) reaction

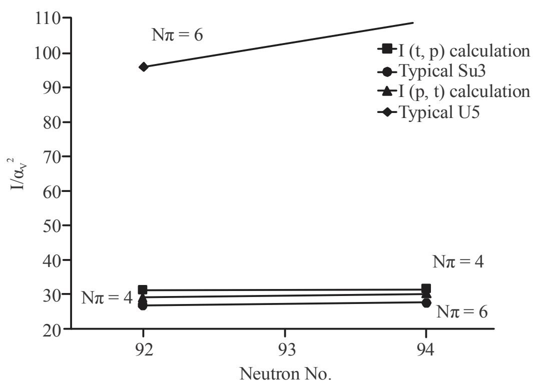

Fig. 5: Comparison between the calculated and typical results (Scholten, 1980) of the IBM-1 for ground-state to ground-state two-neutron transfer reaction intensities of $^{150,152}Ce$

Interacting Boson Model have been compared with typical results of the IBM-1. The results showed good agreement. The drop in ground-state to ground-state transition occurs when nuclei transition from U (5)-SU (3) (Scholten, 1980). The observed results show the characteristics of the SU (3) limit (Iachillo and Arima, 1987).

The observed results from Fig. 5 show the difference between the (t, p) and (p, t) reaction intensities which represents, the changing magnitude value of the ground-state energy levels when adding or removing two nucleon for these two isotopes.

CONCLUSION

The IBM-1has been used to study the nuclear properties of $^{150,152}Ce$ isotopes. The IBM-1 calculations of the energy levels, the B (E2) values and of the two-nucleon transfers reactions show that these isotopes are close to having a perfect rotational shape. Nuclei no longer have the back-bending effect. It can also be concluded from the PES calculations that the prolate shape is deeper than, the oblate shape in these isotopes (Hussain *et al.*, 2015).

The results of the IBM-1 calculations have been compared with available experimental data. These comparisions were in good agreement, so, the IBM-1 is a convenient model to describe the low-lying collective states of a medium nucleus.

REFERENCES

Al-Dahan, N., 2017. Descriptive study of the even-even actinide nuclei $^{230-234}Th$ isotopes using IBM-1. Chin. Phys. C., Vol. 41, No. 6.

Arima, A. and F. Iachello, 1981. The interacting Boson model. Annu. Rev. Nucl. Particle Sci., 31: 75-105.

Armia, A. and F. Iachello, 1976. Interacting boson model of collective states 1: The Vibrational limit. Ann. Phys., 99: 253-317.

Artna-Cohen, A., 1996. Nuclear data sheets for A= 152. Nucl. Data Sheets, Vol. 79, No. 1. 10.1006/ndsh.1996.0012

Basu, S.K. and A.A. Sonzogni, 2013. Nuclear data sheets for A = 150. Nucl. Data Sheets, 114: 435-660.

Casten, R.F. and D.D. Warner, 1988. The interacting boson approximation. Rev. Mod. Phys., 60: 389-469.

Casten, R.F. and N.V. Zamfir, 2001. Empirical realization of a critical point description in atomic nuclei. Phys. Rev. Lett., Vol. 87, No. 5. 10.1103/PhysRevLett.87.052503

Castenholz, A., 1993. Algebraic Approaches to Nuclear Structure. Vol. 6, CRC Press, Boca Raton, Florida, USA., ISBN: 9783718605378, Pages: 535.

Hussain, A.M.A., F.H. Al-Khudair and A.R.H. Subber, 2015. Search for mixed-symmetry state in even-even $^{130-138}Ce$ isotopes within the interacting boson model-2. Turk. J. Phys., 39: 137-148.

Iachello, F., 1981. An Introduction to the Interacting Boson Model. In: Nuclear Structure, Abrahams, K., K. Allaart and A.E.L. Dieperink (Eds.). Springer, Berlin, Germany, ISBN: 978-1-4684-3952-6, pp: 53-89.

Iachillo, F. and A. Arima, 1987. The Interacting Boson Approximation. Cambridge University Press, Cambridge, England, UK., ISBN:052130282X,.

Jinfu, Z., P. Jaeyon and L. Fengying, 2000. Low-lying spectra and E2 transition rates in even-even Ce isotopes in the interacting Boson model. High Energy Phys. Nucl. Phys., 24: 1066-1072.

Krane, K.S. and D. Halliday, 1987. Introductory Nuclear Physics. John Wiley, New York, pp: 845.

Martin, M.J., 2013. Nuclear data sheets for A = 152. Nucl. Data Sheets, 114: 1497-1847.

Mulligan, T.J., E.R. Flynn, O. Hansen, R.F. Casten and R.K. Sheline, 1972. (t,p) and (p,t) reactions on even Ce isotopes. Phys. Rev. C., Vol. 6, No. 5.

Pfeifer, W., 1998. An Introduction into the Interacting Boson Model of the Atomic Nucleus. ETH Zurich, Zurich, Switzerland, ISBN: 9783728125200, Pages: 184.

Sahan, M., H. Sahan and E. Tel, 2017. The Study of (n, 2n) Reaction Cross Sections for Ce and Nd Isotopes up to 20 MeV. Phys. Atomic Nuclei, 80: 1050-1060.

Scholten, O., 1980. The interacting boson approximation model and applications. MS.c. Thesis, Trippenhuis, Amsterdam, Netherlands.

Sharrad, F.I., H.Y. Abdullah, N. Al-Dahan, N.M. Umran, A.A. Okhunov and H.A. Kassim, 2013. Low-lying states of ^{184}W and ^{184}Os nuclei. Chin. Phys. C., Vol. 37, No. 3. 10.1088/1674-1137/37/3/034101

Turkan, N. and I. Maras, 2007. Microscopic interacting boson model calculations for even-even $^{128-138}$Ce nuclei. Pramana, 68: 769-778.