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Comparing Different Estimators of Reliability Function for Stress-Strength Models with Applications

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Abstract: One of the most practical application of reliability as a function of time is a well stress-strength model were this model have several applications like Physics, engineering and components, so here, we introduce the Stress-Strength (S-S) reliability model for system contains one component denoted by [$R = p(y<x)$] where (y) is stress random variable and (x) is strength random variable were:

$$R = p(y < x) = \iint_{y<x} f(x)f(y)dydx$$

The studied model introduced represents reliability function for stress-strength model, assuming the components of stress and strength are independent and identically distributed as Exponentiated Weibull Distribution (EWD). The model of S-S derived and the reliability of it also found. Then estimating by maximum likelihood and least square methods. The comparison done through simulation using different sets of sample size (n, m) also different sets of initial values of (β_1, β_2, θ), all the results of comparison explained by tables.

Key words: Stress-strength model, reliability function, maximum likelihood method, least square method, MSE, sets

INTRODUCTION

There are many models for stress-strength reliability but here we explain the S-S system (Ng, 2006; Panahi and Asadi, 2010), i.e., we work on estimating [$R = p(y<x)$] where (y) is stress random variable and (x) is strength random variable and they are independent random variables follow exponentiated Weibull distribution (Hassan and Basheikh, 2012; Salehi and Ahmadi, 2015; Mokhlis, 2006; Singh *et al*., 2015).

In statistical approach most (stress-strength) models assumed the component of stress and strength are independent and identically distributed, many researchers discuss the reliability of stress-strength model (Al-Zahrani and Basloom, 2016; Amin, 2017; Hussian, 2014; Karam and Sabea, 2017; Salem, 2013) and other like Church and Harris (1970), Bhattacharyya and Johnson (1974) developed model for stress-strength system for (k) components system. Many other researchers studied the model like Pandey and Uddin (1991).

Theoretical aspects: Estimating reliability for stress-strength model (with one component), here we have (stress-strength) model were the stress is (y) and strength is (x), two independent random variables taking (x) to be random variable follow Exponentiated Weibull Distribution (EWD), (β_1, θ) and (y) is independent random variable EWD (β_2, θ), i.e.:

$$f(x) = \beta_1 \theta x^{\theta-1}(1-e^{-x^{\theta}})^{\beta_1-1} e^{-x^{\theta}} \quad (1)$$

$$f(y) = \beta_2 \theta y^{\theta-1}(1-e^{-y^{\theta}})^{\beta_2-1} e^{-y^{\theta}} \quad (2)$$

Where:
θ = Scale
β = The shape

while the cumulative distribution function is:

$$F(x, \beta, \theta) = (1-e^{-x^{\theta}})^{\beta}\ x, \beta, \theta>0 \quad (3)$$

The reliability function is:

$$R(x, \beta, \theta) = 1-(1-e^{-x^{\theta}})^{\beta}\ x, \beta, \theta>0 \quad (4)$$

Reliability is the probability that the unit or equipment or machines still work over time (t) when it is necessary to consider the effects of environmental conditions while evaluating reliability, we need here to estimate reliability of (stress-strength) model were this model have applications in Physics and Engineering, similar to strength failure and system break down, it have many applications, especially on engineering devices which have more than one component reliability function for stress-strength system (Khan and Jan, 2014; Kizilaslan and Nadar, 2015; Rao, 2012a, b) which defined above, the estimated reliability is:

$$\hat{R} = \frac{\hat{\beta}_1}{\hat{\beta}_1+\hat{\beta}_2}$$

MATERIALS AND METHODS

Estimation methods

Maximum likelihood method: The estimation of (β_1, β_2) through this method as: Let (x_1, x_2, ..., x_n) be a random variable from PDF in Eq. 1 and let (y_1, y_2, ..., y_m) be a random variable from PDF in Eq. 1, then:

$$L = \prod_{i=1}^{n} f(x_i, \beta_1, \theta)\prod_{j=1}^{m} f(y_j, \beta_2, \theta) = \left[\beta_1^n\theta^n\prod_{i=1}^{n}x_i^{\theta-1}\prod_{i=1}^{n}\left(1-e^{-x_i^\theta}\right)^{\beta_1-1}e^{-\sum_{i=1}^{n}x_i^\theta}\right]\times \left[\beta_2^m\theta^m\prod_{j=1}^{m}y_j^{\theta-1}\prod_{j=1}^{m}\left(1-e^{-y_j^\theta}\right)^{\beta_2-1}e^{-\sum_{j=1}^{m}y_j^\theta}\right] \quad (6)$$

$$\log L = n\log\beta_1+m\log\beta_2+n\log\theta+m\log\theta+(\theta-1)\sum_{i=1}^{n}\log x_i+(\beta_1-1)\sum_{i=1}^{n}\log\left(1-e^{-x_i^\theta}\right)+(\theta-1)-\sum_{j=1}^{m}\log y_i+(\beta_2-1)\sum_{j=1}^{m}\log\left(1-e^{-y_i^\theta}\right) \quad (7)$$

$$\log L = n\log\beta_1+m\log\beta_2+(n+m)\log\theta+\sum_{i=1}^{n}\log x_i^{\theta-1}+\sum_{j=1}^{m}\log y_j^{\theta-1}-\sum_{i=1}^{n}x_i^\theta-\sum_{j=1}^{m}y_j^\theta+(\beta_1-1)\sum_{i=1}^{n}\log\left(1-e^{-x_i^\theta}\right)+(\beta_2-1)\sum_{j=1}^{m}\log\left(1-e^{-y_j^\theta}\right)$$

$$\log L = n\log\beta_1+m\log\beta_2+(n+m)\log\theta+(\theta-1)\left(\sum_{i=1}^{n}\log x_i+\sum_{j=1}^{m}y_i\right)-\sum_{i=1}^{n}x_i^\theta-\sum_{j=1}^{m}y_j^\theta+(\beta_1-1)\sum_{i=1}^{n}\log\left(1-e^{-x_i^\theta}\right)+(\beta_2-1)\sum_{j=1}^{m}\log\left(1-e^{-y_j^\theta}\right) \quad (8)$$

$$\frac{\partial\log L}{\partial\beta_1} = \frac{n}{\beta_1}+\sum_{i=1}^{n}\log\left(1-e^{-x_i^\theta}\right) \quad (9)$$

$$\frac{\partial\log L}{\partial\beta_2} = \frac{m}{\beta_2}+\sum_{j=1}^{m}\log\left(1-e^{-y_i^\theta}\right) \quad (10)$$

From these two Eq. 9 and 10, we obtain MLE's for two shape parameters β_1, β_2:

$$\hat{\beta}_1 = -\frac{n}{\sum_{i=1}^{n}\log\left(1-e^{-x_i^\theta}\right)} \quad (11)$$

$$\hat{\beta}_2 = -\frac{m}{\sum_{j=1}^{m}\log\left(1-e^{-y_i^\theta}\right)} \quad (12)$$

The maximum likelihood for scale parameter (θ) is:

$$\frac{\partial\log L}{\partial\theta} = \frac{(n+m)}{\theta}+\left[\sum_{i=1}^{n}\log x_i+\sum_{j=1}^{m}\log y_j\right]-\sum_{i=1}^{n}x_i^\theta(1)\log x_i-\sum_{j=1}^{m}y_j^\theta(1)\log y_j+(\beta_2-1)\sum_{j=1}^{m}\frac{y_i^\theta\log y_i^\theta\log y_j e^{-y_i^\theta}}{\left(1-e^{-y_i^\theta}\right)}$$

Estimation of (S-S) Model by least square method: The estimator by this method obtained by minimizing sum square for the difference between [$F(x_i)$] and some non parametric estimators of [$F(x_i)$] which may be:

$$\hat{F}(x_i) = \frac{i}{n+1} = p_i$$

Let:

$$F_1(x, \alpha, \theta) = \left(1-e^{-x_i^\theta}\right)^{\beta_1}$$

$$\beta_1\ln\left(1-e^{-x_i^\theta}\right) = \ln p_i$$

$$T = \sum_{i=1}^{n}\left[\beta_1\ln\left(1-e^{-x_i^\theta}\right)-\ln p_i\right]^2$$

$$\frac{\partial T}{\partial\beta_1} = 2\sum_{i=1}^{n}\left[\beta_1\ln\left(1-e^{-x_i^\theta}\right)-\ln p_i\right]\ln\left(1-e^{-x_i^\theta}\right) = 0$$

$$\hat{\beta}_{1LS} = \frac{\sum_{i=1}^{n}\left[\ln p_i\ln\left(1-e^{-x_i^\theta}\right)\right]}{\sum_{i=1}^{n}\left[\ln\left(1-e^{-x_i^\theta}\right)\right]^2} \quad (14)$$

Using same method, we find the least square estimator of (β_2) for (y) were (y) is stress-random EWD (β_2, θ).

$$\hat{\beta}_{2LS} = \frac{\sum_{j=1}^{m}\left[\ln p_j \ln\left(1-e^{-y_j^{\theta}}\right)\right]}{\sum_{j=1}^{m}\left[\ln\left(1-e^{-y_j^{\theta}}\right)\right]^2}$$

Then from Eq. 14 and 15:

$$\hat{R}_{LS} = \frac{\hat{\beta}_{1LS}}{\hat{\beta}_{1LS}+\hat{\beta}_{2LS}} \quad (16)$$

RESULTS AND DISCUSSION

We apply simulation procedure to find numerical results for estimated reliability of stress-strength model using different sample size and different set of initial values of (β_1, β_2, θ) were:

Step 1:

$$F(x) = \left(1-e^{-x_i^{\theta}}\right)^{\beta_1}$$

$$u_i = \left(1-e^{-x_i^{\theta}}\right)^{\beta_1}$$

$$u_i^{\frac{1}{\beta_1}} = \left(1-e^{-x_i^{\theta}}\right)$$

$$x_i = \left[-\ln\left(1-u_i^{\frac{1}{\beta_1}}\right)\right]^{\frac{1}{\theta}}$$

Step 2: Let (v_j) be values of the random variable (y_j):

$$y_j = \left[-\ln\left(1-v_j^{\frac{1}{\beta_2}}\right)\right]^{\frac{1}{\theta}}$$

Use:

$$R = \frac{\beta_1}{\beta_1+\beta_2}$$

From Eq. 16 and:

$$\hat{R}_{MLE} = \frac{\hat{\beta}_{1MLE}}{\hat{\beta}_{1MLE}+\hat{\beta}_{2MLE}}$$

The comparison done using statistical measure mean square error. Using random sample for (x_i) and (y_j) with sizes (n, m) as:

$$(n, m) = \begin{bmatrix} (15, 10) & (15, 30) & (15, 50) & (15, 100) \\ (30, 10) & (30, 30) & (30, 50) & (30, 100) \\ (50, 10) & (50, 30) & (50, 50) & (50, 100) \\ (75, 10) & (75, 30) & (75, 50) & (75, 100) \end{bmatrix}$$

Table 1: MSE for ($\hat{R}_{MLE}$ and $\hat{R}_{LS}$) with ($\beta_1 = 4, \beta_2 = 3, \theta = 4$)

(n, m)	R	$\hat{R}_{MLE}$	MSE$\hat{R}_{MLE}$	$\hat{R}_{LS}$	MSE$\hat{R}_{LS}$	Best
(15, 10)	0.665	0.7456	0.06571	0.64322	0.01166	LS
(15, 30)	0.665	0.7362	0.06448	0.64465	0.02005	LS
(15, 50)	0.665	0.7338	0.07806	0.6438	0.04621	LS
(15, 100)	0.665	0.6455	0.07491	0.65531	0.04609	LS
(30, 10)	0.665	0.6482	0.7332	0.64312	0.0455	LS
(30, 30)	0.665	0.64403	0.03564	0.64101	0.0433	MLED
(30, 50)	0.665	0.64441	0.01071	0.6432	0.0197	MLE
(30, 100)	0.665	0.6651	0.0278	0.6429	0.01966	LS
(50, 10)	0.665	0.6631	0.02766	0.641812	0.01863	LS
(50, 30)	0.665	0.6662	0.06581	0.64337	0.01667	MLE
(50, 50)	0.665	0.6454	0.0431	0.64259	0.0116	MLE
(50, 100)	0.665	0.6423	0.0411	0.6473	0.011504	MLE
(75, 10)	0.665	0.6453	0.0411	0.0429	0.011609	LS
(75, 30)	0.665	0.6429	0.0322	0.64317	0.011505	LS
(75, 50)	0.665	0.6433	0.0311	0.6422	0.011604	LS
(75, 100)	0.665	0.6411	0.0301	0.6435	0.011052	LS

Table 2: MSE for ($\hat{R}_{MLE}$ and $\hat{R}_{LS}$) with ($\beta_1 = 2, \beta_2 = 4, \theta = 4$)

(n, m)	R	$\hat{R}_{MLE}$	MSE$\hat{R}_{MLE}$	$\hat{R}_{LS}$	MSE$\hat{R}_{LS}$	Best
(15, 10)	0.8455	0.83556	0.03847	0.808144	0.0052	LS
(15, 30)	0.8455	0.83467	0.04152	0.80089	0.0054	LS
(15, 50)	0.8455	0.83178	0.006	0.81562	0.0036	LS
(15, 100)	0.8455	0.8396	0.0008	0.82266	0.00349	MLE
(30, 10)	0.8455	0.83995	0.00688	0.83875	0.00211	LS
(30, 30)	0.8455	0.84015	0.00066	0.8452	0.00206	MLE
(30, 50)	0.8455	0.8424	0.000168	0.839967	0.00202	MLE
(30, 100)	0.8455	0.8416	0.000162	0.83972	0.00166	MLE
(50, 10)	0.8455	0.8396	0.00072	0.83961	0.00144	MLE
(50, 30)	0.8455	0.8355	0.000268	0.8396	0.00053	LS
(50, 50)	0.8455	0.8397	0.000513	0.8355	0.00028	LS
(50, 100)	0.8455	0.8369	0.00044	0.8375	0.00026	LS
(75, 10)	0.8455	0.8379	0.070011	0.8277	0.000205	LS
(75, 30)	0.8455	0.8229	0.06452	0.8256	0.000204	LS
(75, 50)	0.8455	0.8228	0.0641	0.8185	0.000201	LS
(75, 100)	0.8455	0.8222	0.0633	0.8113	0.000188	LS

$$(\beta_1, \beta_2) = [(2.5, 3) \quad (2, 4) \quad (1.5, 3.5) \quad (4, 4)]$$

While the shape parameters:

$$x_i = [15 \quad 30 \quad 50]$$
$$y_j = [30 \quad 50 \quad 100]$$

From estimated values of reliability function for stress-strength model for different sets of initial values ($\beta_1 = 2.5$, $\beta_2 = 3$, $\theta = 4$) and different sets of sample size (n, m) in Table 1, we find $\hat{R}_{LS}$ is best with percentage [(11/16)×100% = 69%] while $\hat{R}_{MLE}$ is best with [(5/16)×100% = 31%]. For the second set chosen of initial values (Table 2), also, we find that $\hat{R}_{LS}$ is best with percentage [(11/16)×100% = 69%] while $\hat{R}_{LS}$ is best with [(5/16)×100% = 31%]. For the third table, we find that $\hat{R}_{MLE}$ and $\hat{R}_{LS}$ with equal percentage [(8/16)×100% = 50%] while $\hat{R}_{LS}$ is best with [(8/16)×100% = 50%] (Table 3). The fourth table, we find that all values of $\hat{R}_{MLE}$ are the best (Table 4).

Table 3: MSE for $(\hat{R}_{MLE}$ and $\hat{R}_{LS})$ with $(\beta_1 = 1.5, \beta_2 = 3.5, \theta = 4)$

(n, m)	R	$\hat{R}_{MLE}$	$MSE\hat{R}_{MLE}$	$\hat{R}_{LS}$	$MSE\hat{R}_{LS}$	Best
(15, 10)	0.7624	0.84752	0.05310	0.7582	0.1925	MLE
(15, 30)	0.7624	0.8036	0.0644	0.8013	0.0336	LS
(15, 50)	0.7624	0.8044	0.0632	0.8082	0.0352	LS
(15, 100)	0.7624	0.8052	0.0604	0.7792	0.03433	LS
(30, 10)	0.7624	0.8172	0.0601	0.7761	0.0441	LS
(30, 30)	0.7624	0.8394	0.0052	0.7863	0.0473	MLE
(30, 50)	0.7624	0.8421	0.0053	0.7861	0.0855	MLE
(30, 100)	0.7624	0.8531	0.0072	0.0726	0.0672	MLE
(50, 10)	0.7624	0.8533	0.0061	0.7531	0.0663	MLE
(50, 30)	0.7624	0.8541	0.0041	0.7442	0.0656	MLE
(50, 50)	0.7624	0.8561	0.0043	0.7435	0.045	MLE
(50, 100)	0.7624	0.8572	0.0040	0.7434	0.043	MLE
(75, 10)	0.7624	0.8606	0.0040	0.7424	0.032	LS
(75, 30)	0.7624	0.8612	0.0036	0.7431	0.031	LS
(75, 50)	0.7624	0.8622	0.0035	0.7422	0.022	LS
(75, 100)	0.7624	0.8634	0.0033	0.7366	0.021	LS

Table 4: MSE for $(\hat{R}_{MLE}$ and $\hat{R}_{LS})$ with $(\beta_1 = 4, \beta_2 = 4, \theta = 4)$

(n, m)	R	$\hat{R}_{MLE}$	$MSE\hat{R}_{MLE}$	$\hat{R}_{LS}$	$MSE\hat{R}_{LS}$	Best
(15, 10)	0.634	0.6203	0.01066	0.5162	0.05691	MLE
(15, 30)	0.634	0.6208	0.0076	0.5008	0.0639	MLE
(15, 50)	0.634	0.6244	0.00646	0.5006	0.10538	MLE
(15, 100)	0.634	0.6195	0.00595	0.4992	0.10463	MLE
(30, 10)	0.634	0.6281	0.00499	0.4998	0.10572	MLE
(30, 30)	0.634	0.6365	0.00328	0.50022	0.04709	MLE
(30, 50)	0.634	0.6286	0.007015	0.4998	0.06159	MLE
(30, 100)	0.634	0.6093	0.00208	0.49975	0.10408	MLE
(50, 10)	0.634	0.6234	0.00664	0.4999	0.14073	MLE
(50, 30)	0.634	0.6271	0.00216	0.5001	0.14062	MLE
(50, 50)	0.634	0.6225	0.00195	0.5003	0.04435	MLE
(50, 100)	0.634	0.6268	0.00182	0.5136	0.0167	MLE
(75, 10)	0.634	0.6247	0.00153	0.5144	0.0032	MLE
(75, 30)	0.634	0.6351	0.00116	0.5056	0.0182	MLE
(75, 50)	0.634	0.6557	0.0005	0.5493	0.005013	MLE
(75, 100)	0.634	0.6225	0.0003	0.5448	0.005002	MLE

CONCLUSION

We can conclude that the MLE estimators for reliability function of the studied stress-strength model is best as compared with least square estimators.

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