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A Pseudo B-Ideal, Pseudo H-Ideal and a Pseudo Essence of a Pseudo BH-Algebra

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Abstract: In this study, we define the notion of a pseudo B-ideal, a pseudo H-ideal and a pseudo essence of a pseudo BH-algebra. Also, we study some properties and relationship between them.

Key words: BH-algebra, ideal of BH-algebra, pseudo BH-algebra, pseudo ideal of a pseudo BH-algebra, pseudo B-ideal, pseudo H-ideal, pseudo essence, pseudo 0-commutative, pseudo G-part, pseudo BCA-part, pseudo closed ideal

INTRODUCTION

Jun *et al*. (1998) introduced the notion of BH-algebra which is a generalization of BCH-algebra and the notion of ideal of a BH-algebra. Kim and Ahn (2011) introduced the notion of essenceof BH-algebra. Abbass and Dahham (2012) introduced the notion of completely closed ideal of a BH-algebra. By Abbass and Mahdi (2014) introduced the notion of a closed ideal, p-ideal, q-ideal to a BH-algebra and BCA-part. Jun and Kim (2015) introduced the notion of a pseudo BH-algebra.

MATERIALS AND METHODS

In this sudy, some basic concepts about a BH-algebra, ideal of BH-algebra, essence BH-algebra, 0-commutative BH-algebra, G-part of BH-algebra, BCA part of BH-algebra pseudo, BH-algebra, pseudo subalgebra of a pseudo BH-algebra and pseudo ideal of a pseudo BH-algebra are given.

Definition 1; Jun *et al.* (1998): A BH-algebra is a nonempty set $\mathfrak{X}$ with constant 0 and a binary operation "*" satisfying the following conditions:

- $x*x = 0, \forall x \in X$
- $x*0 = x, \forall x \in X$
- $x*y = 0$ and $y*x = 0 \Rightarrow x = y, \forall x, y \in \mathfrak{X}$

Definition 2; Abbass and Dahham (2012): A nonempty subset S of a BH-algebra $\mathfrak{X}$ is called a subalgebra of $\mathfrak{X}$, if for any x, y∈S, we have x*y∈S.

Definition 3; Abbass and Dahham (2012): A BH-algebra $\mathfrak{X}$ is said a 0-commutative if : $x*(0*y) = y*(0*x)$. For all x, y, z∈ $\mathfrak{X}$.

Definition 4; Abbass and Mhadi (2014): Let $\mathfrak{X}$ be a BH-algebra. Then the set $G(\mathfrak{X}) = \{x \in \mathfrak{X} : 0*x = x\}$ is called G-part.

Definition 5; Abbass and Mahdi (2014): Let $\mathfrak{X}$ be a BH-algebra. Then the set $\mathfrak{X}_+ = \{x \in \mathfrak{X} : 0*x = 0\}$ is called the BCA-part of $\mathfrak{X}$.

Definition 6; Kim and Ahn (2011): Let, $\mathfrak{X}$ be a BH-algebra. For any subsets G and H of $\mathfrak{X}$, we define $G*H = \{x*y: x \in G, yy \in H\}$.

Theorem 1; Kim and Ahn (2011): Let a subsets A, B and E of a BH-algebra, we have:

- $A \subseteq B \Rightarrow A*E \subseteq B*E$ and $E*A \subseteq E*B$
- $(A \cap B)*E \subseteq (A*E) \cap (B*E)$
- $E*(A \cap B) \subseteq (E*A) \cap (E*B)$
- $(A \cup B)*E = (A*E) \cup (B*E)$
- $E*(A \cup B) = (E*A) \cup (E*B)$

Definition 7; Kim and Ahn (2011): If A is a nonempty subset of a BH-algebra $\mathfrak{X}$ satisfies $A*\mathfrak{X} = A$, then A is called essence of $\mathfrak{X}$.

Theorem 2; Kim and Ahn (2011): Let $\mathfrak{X}$ be a BH-algebra. Then every a essence of $\mathfrak{X}$ is a subalgebra of $\mathfrak{X}$.

Theorem 3; Kim and Ahn (2011): Let $\mathfrak{X}$ be a BH-algebra. Then, every essence of $\mathfrak{X}$ contains the zero element 0.

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Definition 8; Jun *et al.* (1998): Let, $\mathfrak{X}$ be a BH-algebra and $I(\neq\emptyset)\subseteq\mathfrak{X}$. Then, I is called an ideal of $\mathfrak{X}$ if it satisfies:

- $0\in I$
- If $x*y\in I$ and $y\in I \Rightarrow x\in I$, for all $x\in\mathfrak{X}$

Definition 9: An ideal I of a BCH-algebra is called a closed ideal of $\mathfrak{X}$ if for every $x\in I$, we have $0*x\in I$. We generalize the concept of an ideal to a BH-algebra.

Definition 10: An ideal I of a BH-algebra $\mathfrak{X}$ is called a closed ideal of $\mathfrak{X}$ if: $0*x\in I$, for all $x\in I$.

Definition 11; Abbass and Dahham (2012): Let $\mathfrak{X}$ be a BH-algebra and I be a subset of $\mathfrak{X}$. Then I is called a BH-ideal of $\mathfrak{X}$ if it satisfies the following conditions:

- $0\in I$
- $x*y\in I$ and $y\in I$ imply $x\in I$
- $x\in I$ and $y\in\mathfrak{X}$ imply $x*y\in I$, $I*\mathfrak{X}\subseteq I$

Definition 12; Jun and Kim (2015): A pseudo BH-algebra is a nonempty set $\mathfrak{X}$ with a constant 0 and two binary operations "*" and "#" satisfying the folloeing condition:

- $x*x = x\#x = 0$
- $x*0 = x\#0 = x$
- $x*y = y\#x = 0 \Rightarrow x = y$, $\forall x, y\in\mathfrak{X}$

Definition 13; Jun and Kim (2015): Let $(\mathfrak{X}, *, \#, 0)$ be a pseudo BH-algebra, then a nonempty subset S of a pseudo BH-algebra $\mathfrak{X}$ is called a pseudo subalgebra of $\mathfrak{X}$, if for any $x, y\in S$, we have $x*y, x\#y\in S$.

Definition 14; Jun and Kim (2015): Let $(\mathfrak{X}, *, \#, 0)$ be a pseudo BH-algebra, then I is called pseudo ideal of $\mathfrak{X}$, if it satisfies:

- $0\in I$
- $x*y, x\#y\in I, y\in I \Rightarrow x\in I$, $\forall x, y\in\mathfrak{X}$

Definition 15; Jun and Kim (2015): A pseudo ideal I of a pseudo BH-algebra $\mathfrak{X}$ is called a pseudo closed ideal of $\mathfrak{X}$, if for every $x\in I$, we have $0*x, 0\#x\in I$.

RESULTS AND DISCUSSION

In this study, we define a new types of a pseudo ideals, a pseudo essence subset and a pseudo essence ideal of a pseudo BH-algebra. Also, we study some propostions to other some types of a pseudo ideals of a pseudo BH-algebra.

Definition 1: A pseudo BH-algebra $\mathfrak{X}$ is said a pseudo 0-commutative if:

- $x*(0\#y) = y*(0\#x)$
- $x\#(0*y) = y\#(0*x)$. For all $x, y, z\in\mathfrak{X}$

Example 1: Let $\mathfrak{X} = \{0, 1, 2\}$ be a set with the following Cayley Table 1. Then $\mathfrak{X}$ is a pseudo BH-algebra.

Table 1: Pseudo 0-commutative

*	0	1	2
0	0	1	2
1	1	0	1
2	2	1	0

#	0	1	2
0	0	1	2
1	1	0	2
2	2	2	0

Definition 2: Let $\mathfrak{X}$ be a pseudo BH-algebra. Then thae set $G(\mathfrak{X}) = \{x\in\mathfrak{X} : 0*x = 0\#x = x\}$ is called a pseudo G-part of $\mathfrak{X}$.

Example 2: Let $\mathfrak{X} = \{0, 1, 2, 3\}$ be a set with the following Cayley Table 2.

Table 2: Pseudo G-part of $\mathfrak{X}$

*	0	1	2	3
0	0	1	2	3
1	1	0	2	3
2	2	1	0	1
3	3	3	3	0

#	0	1	2	3
0	0	1	2	3
1	1	0	2	3
2	2	2	0	2
3	3	3	1	0

Definition 3: Let $\mathfrak{X}$ be a pseudo BH-algebra. Then the set $\mathfrak{X}_+ = \{x\in\mathfrak{X} : 0*x = 0\#x = 0\}$ is called a BCA-part of $\mathfrak{X}$.

Example 3: Let $\mathfrak{X} = \{0, 1, 2, 3\}$ be a set with the following Cayley Table 3.

Table 3: Pseudo BCA-part of $\mathfrak{X}$

*	0	1	2	3
0	0	0	0	0
1	1	0	2	3
2	2	1	0	1
3	3	3	3	0

#	0	1	2	3
0	0	0	0	0
1	1	0	2	3
2	2	2	0	2
3	3	3	1	0

Definition 4: A nonempty subset I of a pseudo BH-algebra $\mathfrak{X}$ is called a pseudo B-ideal of $\mathfrak{X}$ if it stisfies:

- $0\in I$
- $x*(z\#(0*y)), y\in I$ imply $x*z\in I$
- $x\#(z-(0\#y)), y\in I$ imply $x\#z\in I$

Example 4: Let $\mathfrak{X}$ = {0, 1, 2, 3} be a set with the following cayley (Table 4). Then, $\mathfrak{X}$ is a pseudo BH-algebra and let I = {0, 1} be a subset of $\mathfrak{X}$, then, it is a pseudo B-ideal of $\mathfrak{X}$.

Table 4: Pseudo B-ideal of $\mathfrak{X}$

*	0	1	2
0	0	0	0
1	1	0	2
2	2	1	0

#	0	1	2
0	0	0	0
1	1	0	2
2	2	2	0

Proposition 1: Let, $\mathfrak{X}$ be a pseudo BH-algebra such that $\mathfrak{X} = \mathfrak{X}_+$, then, every pseudo ideal of $\mathfrak{X}$ is a B-ideal of $\mathfrak{X}$.

Proof: Let, I be a pseudo ideal of $\mathfrak{X}$ and x* (z # (0*y)), y∈I. For all x, y, z∈ $\mathfrak{X}$. Since, $\mathfrak{X} = \mathfrak{X}_+$ ⇒x* (z#0)∈I. Since, $\mathfrak{X}$ is a pseudo BH-algebre⇒x*z∈I. Thus, x*z∈I. Similarly, x# (z*(0#y)), y∈I imply x#z∈I. Hence, I is a pseudo B-ideal of $\mathfrak{X}$.

Proposition 2: Let $\mathfrak{X}$ be a pseudo BH-algebra. If a pseudo B-ideal of $\mathfrak{X}$ is a pseudo G-part of $\mathfrak{X}$ then, it is a pseudo ideal of $\mathfrak{X}$.

Proof: Let, $\mathfrak{X}$ be a pseudo ideal of $\mathfrak{X}$ andlet x*y, x #y∈I, y∈I. For all x, y, z∈ $\mathfrak{X}$. Since, I is pseudo G-pert of $\mathfrak{X}$ ⇒x*0∈I.⇒x* (0 # (0*y))∈I and y∈I. Since, I is a pseudo B-ideal of $\mathfrak{X}$ ⇒x*0∈I. Since, $\mathfrak{X}$ is a pseudo BH-algebra⇒x∈I. Similarly, x #y∈I, y∈I⇒x∈I. Hence, I is a pseudo ideal of $\mathfrak{X}$.

Proposition 3: Let $\mathfrak{X}$ be a pseudo BH-algebra such that y = z # (0*y) and y = z*(0#y), then every pseudo B-ideal of $\mathfrak{X}$ is a pseudo ideal of $\mathfrak{X}$.

Proof: Let I be a pseudo B-ideal of $\mathfrak{X}$ and x*(z # (0*y)), y∈I. For all x, y, z∈ $\mathfrak{X}$. Since, y = z # (0*y) then x*y, y∈I imply x∈I. Similarly, x # (z*(0 # y)), y∈I imply x∈I. Hence, I is a pseudo ideal of $\mathfrak{X}$.

Definition 5: A non empty subset I of a pseudo BH-algebra $\mathfrak{X}$ is called a pseudo H-ideal of $\mathfrak{X}$ if it satisfies:

- 0∈I
- (x*y) # (x*z)∈I and y∈I⇒x∈I
- (x # y)* (x # z)∈I and y∈I⇒x∈I. For all x, y, z∈ $\mathfrak{X}$

Example 5: Let, $\mathfrak{X}$ = {0, 1, 2, 3} be a set with the following Cayley Table 5. Then, $\mathfrak{X}$ is a pseudo BH-algebra and let I = {0, 1} be a subset of $\mathfrak{X}$, then it is a pseudo H-ideal of $\mathfrak{X}$.

Table 5: Pseudo H-ideal of $\mathfrak{X}$

*	0	1	2	3
0	0	1	2	3
1	1	0	2	3
2	2	1	0	2
3	3	0	0	0

#	0	1	2	3
0	0	1	2	3
1	1	0	2	2
2	2	0	0	1
3	3	0	0	0

Proposition 4: Every pseudo H-ideal of a pseudo BH-algebra $\mathfrak{X}$ is a pseudo ideal of $\mathfrak{X}$.

Proof: Let, I be a pseudo H*-ideal of $\mathfrak{X}$ and let x*y, x # y∈I and y∈I. For all x, y, z∈ $\mathfrak{X}$. Since, $\mathfrak{X}$ is a pseudo BH-algebra⇒(x*y)∈I = ((x*y)#0)∈I = (x*y)#(x*x)∈I and y∈I. Since, I is a pseudo H-ideal⇒x∈I. Similarly, x#y∈I and y∈I⇒x∈I. Hence, I is a pseudo ideal of $\mathfrak{X}$.

Definition 6: Le, t $\mathfrak{X}$ be a pseudo BH-algebra. For a subsets A and B of $\mathfrak{X}$, then A*B and A#B are defined as follows:

- A*B = {x*y: x∈A, y∈B}
- A#B = {x#y: x∈A, y∈B}

Proposition 5: Let, $\mathfrak{X}$ be a pseudo BH-algebra.

- If 0∈B⊆ $\mathfrak{X}$. Then ∀B⊆ $\mathfrak{X}$, we have B⊆ A*B and B⊆A#B
- If 0∈A⊆ $\mathfrak{X}$. Then ∀B⊆ $\mathfrak{X}$, we have B⊆ A*B and B⊆A#B

Proof: Let, x∈A, Since, $\mathfrak{X}$ is a pseudo BH-algebra, then, x = x*0∈A*B and x =x#0∈A#B. Hence, (A⊆A*B) (A⊆A#B). Similarly of (1).

Definition 7: If A is a nonempty subset of a pseudo BH-algebra $\mathfrak{X}$ satisfies A* $\mathfrak{X}$ = A and A# $\mathfrak{X}$ = A, then A is called a pseudo essence subset of $\mathfrak{X}$. If A is a pseudo ideal of $\mathfrak{X}$, then it is called a pseudo essence ideal of $\mathfrak{X}$.

Example 6: Let, $\mathfrak{X}$ = {0, 1, 2, 3} be a set with the following Cayley Table 6.

Table 6: Pseudo essence of $\mathfrak{X}$

*	0	1	2	3
0	0	0	0	0
1	1	0	2	3
2	2	1	0	1
3	3	3	3	0

#	0	1	2	3
0	0	0	0	0
1	1	0	2	3
2	2	2	0	1
3	3	3	1	0

Then $\mathfrak{X}$ is a pseudo BH-algebra. Let, A = {0, 1}, B = {0, 2} and C = {0, 1, 2} then A, B and C are a pseudo essence subset of $\mathfrak{X}$. But D = {0, 3} is not a pseudo essence subset of $\mathfrak{X}$, since, 3*2 = 1∉D and 3#2 = 2∉D.

Proposition 6: Let, $\mathfrak{X}$ be a pseudo BH-algebra. Then every pseudo essence ideeal of $\mathfrak{X}$ is a pseudo is a pseudo essence subset of $\mathfrak{X}$.

Proof: Let, A be a pseudo ideal of $\mathfrak{X}$ and let x, y∊A. Since, x*y∊A⊆A*A⊆A* $\mathfrak{X}$ = A and x#y∊A⊆A#A⊆A# $\mathfrak{X}$ = A. Hence, A is a pseudo essence subset of $\mathfrak{X}$.

Remark 1: The converse of prposition (6) may be not true in general as follows in example (1), since, A is a pseudo essence 1*3 = 0∊A and 1∊A but 3∉A and 1#3 = 1∊A and 1∊A but 3∉A.

Proposition 7: Let, $\mathfrak{X}$ be a pseudo BH-algebra. Then, every pseudo essence ideal of $\mathfrak{X}$ is a pseudo closed ideal of $\mathfrak{X}$.

Proof: Let, A be a pseudo essence ideal of $\mathfrak{X}$, then, 0∊A. Let, x∊A, then, 0*x∊A*A⊆A* $\mathfrak{X}$ = A. Thus, 0*x∊A, similarly, 0#x∊A. Hence, A is a pseudo essence closed of $\mathfrak{X}$.

Definition 8: A nonempty subset I of a pseudo BH-algebra $\mathfrak{X}$. Then, I is called pseudo BH-ideal a of $\mathfrak{X}$ if it satisfies:

- 0∊I
- x*y, x # y∊I and y∊I imply x∊I
- x∊I and y∊I and imply x*y, x#y∊I, I* $\mathfrak{X}$, I# $\mathfrak{X}$ ⊆I. For all x, y∊ $\mathfrak{X}$

Proposition 8: Let, $\mathfrak{X}$ be a pseudo BH-algebra. Then every a pseudo essence ideal of $\mathfrak{X}$ is a pseudo BH-ideal of $\mathfrak{X}$.

Proof: Let, A be pseudo essence ideal of $\mathfrak{X}$. Since, A* $\mathfrak{X}$ = A, then A* $\mathfrak{X}$⊆A and A# $\mathfrak{X}$ = A, then A# $\mathfrak{X}$⊆A. Thus, A is a pseudo essence ideal of $\mathfrak{X}$ and A* $\mathfrak{X}$, A# $\mathfrak{X}$ ⊆A. Hence, A is a pseudo pseudo BH-ideal of $\mathfrak{X}$.

CONCLUSION

In this study, the notions of pseudo B-ideal, pseudo H-ideal and pseudo essence of a pseudo BH-algebra are introduced. Furthermore, the results are examined in terms of the relationship between pseudo B-idea, pseudo H-ideal and pseudo essence of a pseudo BH-algebra.

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