

Effective Detection and Classification of Drowsiness using Clustering and Support Vector Machines

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Abstract: The study presents drowsiness classification by analyzing the Electroencephalograph (EEG) signals. The EEG signals of different drowsiness situations such as active, drowsy and sleep are captured from many individuals using a multichannel electrode system. We consider the dominant electrode pair FPZ-CZ and PZ-OZ in this study. Denoising of the signal is done for clarity and band pass filtering is done with the required cutoff frequency. In order to get the five sub bands (delta, theta, alpha, beta and gamma) of EEG signal, third order Daubechies wavelet transform is used to decompose. Statistical analysis and energy features are calculated of these sub bands for the effective classification. Classification is done in two stages which starts from clustering of the feature data using K-means and later effective classification using the Support Vector Machine (SVM) which uses the Gaussian kernel. Work has been carried out in the famous Physiobank Sleep Database. Result is very promising and it overcomes many limitations by using the unsupervised learning and the effective classification.

Key words: Drowsiness, EEG signals, k-means clustering, SVM classification, decompose, Daubechies

INTRODUCTION

Drowsiness is a mental state between active and sleep situation. It is one of the major reason in the road accidents happening worldwide. Various studies all over the world have identified the impact of drowsiness in the driving scenario. Drowsiness makes the driver with less vision capability, longer response time and poor judgment scenarios. Thus, the effect of drowsy driving is a serious issue in the society because its aftermath are very dangerous and pathetic.

It actually affects the driver from slowing down the reaction time, affects the decision making ability and decreases the attentions to the surroundings. Researchers analyzed the drowsiness situation and found out that various reasons affects the drowsiness to a driver and it includes lack of sleep or less sleep, alcohol consumption and tiredness due to unhealthy body conditions.

Hence, researchers started their studies as a countermeasure to overcome this situation. Even back by Hunn (1993) found out that EEG signals can be used as an assessment method for finding fatigue pilots. After that various research have been carried in association with brain wave variations in the scene of fatigue or drowsy situations. The famous work by McKeown *et al.* (1998) explained about the various sleep stages and its classification. Lal and Craig (2001) studied the brain wave variations during the drowsiness and proposed EEG analysis for the drowsiness detection and analysis.

Electroencephalogram signals are the indication of brain activity. EEG analysis is done with the help of clinical experts who are familiar with the rhythmic changes of the brain waves. In order to get the EEG signals, electrodes are placed on the head skin to get good contact with the scalp, so that, it can register the neuronal activity changes as electrical potentials (Michel *et al.*, 2014). Figure 1 shows the mapping picture of the electrode placements on the scalp for the EEG recordings.

Spectral analysis of the EEG signal helps to split the signal into five different sub bands depending upon its frequency. Thus, the 5 primary frequency bands of an EEG signal are Δ , θ , α , β and γ . Table 1 shows the frequency bands and its relevance in our context of drowsiness analysis.

The main aim of this study is to effectively classify drowsiness data. We used the sleep database available from Physionet (Goldberger *et al.*, 2000; Kemp *et al.*, 1992). We use the EEG recordings from the electrode pairs of FPZ-CZ and OZ-PZ for the analysis. Signal is further decomposed into 5 sub bands using the 3rd order Daubechies wavelet up to five levels. Features such as mean, median, mode, variance, standard deviation and wavelet energy are calculated from these sub bands and are stored. k-means clustering method is applied on this as the data requires unsupervised learning to classy into three classes. Three classes are awake, drowsy and sleepy situations. These predicted class details are used as a

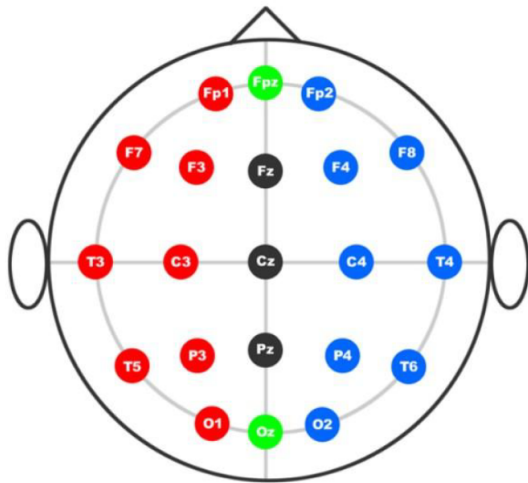


Fig. 1: Mapping of the electrode placements on the scalp for EEG recording

Table 1: Frequency sub bands of the EEG signal and its activity

Signals	Frequency (Hz)	Relevance/activity
Delta	0.5-4	Power increases during different conditions (Hamorny <i>et al.</i> , 1996)
Theta	4-8	Power increases during the drowsiness (Hamorny <i>et al.</i> , 1996)
Alpha	8-12	Most prominent rhythm. Power ecreases during the drowsiness (Setz <i>et al.</i> , 2010)
Beta	12-30	Power fluctuates during different onditions (Hamorny <i>et al.</i> , 1996), alert and attentive to external stimuli,dominant in deep sleep
Gamma	>30	Very rare and low amplitude mainly used in diagnosing brain diseases (Gurudath and Riley, 2014)

Table 2: Related work on drowsiness detection and analysis

Related work	Analysis
Amditis <i>et al.</i> (2010)	Uses camera input as well as vehicle data such as velocity, steering angle, etc.
Jo <i>et al.</i> (2011)	Drowsiness detected based on eye blinking and PCA-LDA related techniques are used
Lee and Chung (2012)	Uses eye features, bio signal variation in vehicle temperature and vehicle speed
Dai <i>et al.</i> (2010)	Mobile which uses accelerometer and orientation sensor
Mandal <i>et al.</i> (2017)	Vision based system which uses face images, eye movements, eye openness estimation and fatigue level classification
Mittal <i>et al.</i> (2016)	Analyzing the driving pattern by leaming the behavior of the driving style mainly based on head movement
Viola <i>et al.</i> (2005)	Real time system which uses infra-red cameras which analyzes head movement, normal eye movement and gaze movement and facial expression
Hariri <i>et al.</i> (2011) and Li <i>et al.</i> (2010)	Face detection and yawn analysis is done Power spectrum analysis on EEG and Fast ICA algorithm visual features with artificial neural networks

priori data for the classification method which uses support vector machines with Gaussian kernel. Details of the research is explained in detail in the study. In the study related works are described followed by the dataset

description from Physionet. In the later section the methodologies are explained and then the feature extraction and classification is described. Discussion on the results shows the efficiency and the success of the methods proposed.

Literature review: Various studies have been carried out in the detection, analysis and classification of drowsiness. Still the research is going on to effectively classify the drowsiness state for efficient decision making and alertness. Gurudath and Riley (2014) tried to detect the driver drowsiness using artificial neural networks. Some studies show that various physiological signals such as EEG, electrocardiography, electrooculography and electromyography can also be used to monitor the health of the driver to avoid fatigue situation due to drowsiness (Dong *et al.*, 2011). Some methods which uses regression methods and fuzzy neural networks shows improved efficiency (Arjunan *et al.*, 2009; Ebrahimi *et al.*, 2008) but they are computationally very complex and require big datasets for the analysis. Channels of EEG also gives importance in the analysis of the drowsiness state. Research and study shows (Amditis *et al.*, 2010)] that 5 channels of the EEG FPZ, FP1, CZ, OZ and PZ give well descriptors for the drowsiness. Table 2 shows some of the related studies in the field of drowsiness detection and alarming system.

Description of the dataset used: Dataset used in this study is from the sleep database available in the Physionet database. It is the sleep recordings and hypnograms in the European Data Format (EDF) done at the sleep center, MCH Westeinde Hospital, Netherlands by Bob Hemp and team (Goldberger *et al.*, 2000). There are 2 types of files with the extensions of rec. and hyp. which says the original recordings and the corresponding hypnograms respectively which is described in European data format (Kemp *et al.*, 1992). The dataset contains 8 different set of recordings. These recordings were collected from males and females in the age group of 21-35 years without any kind of medications or pressure. The recording contain horizontal Electro Oculo Graphy (EOG) and Electro Encephalo Graphy (EEG) readings for the electrode pairs FPZ-CZ and PZ-OZ EEG sampled at 100 Hz. Files with the sc* (sc = sleep cassette) recordings also contain the Electro Myo Graphy (EMG) envelope, oro-nasal airflow, rectal body temperature and an event marker, all sampled at 1 Hz. The files with the st* (st = sleep telemetry) recordings contain sub mental EMG sampled at 100 Hz and an event marker sampled at 1 Hz. In our research we are using the EEG data from the 2 electrode pairs FPZ-CZ and PZ-OZ for our analysis.

MATERIALS AND METHODS

In this research, the EEG signals from the physionet sleep database is used for the effective classification of drowsiness. For this purpose, filtering and removal of unwanted signals are done first. Later the sub band decomposition of the signal is carried using the wavelet method. Features are extracted from the decomposed data and are used for the clustering method and further for the classification of the classes using support vector machines.

Features: In order to classify a signal there should be some attributes which are specific and can represent that characteristic. Here, 5 statistical features such as mean, median, mode, variance and standard deviation together with the wavelet energy feature are extracted from each of the decomposed sub band waves.

Mean represents the central value of a discrete set of values. Mathematical representation of the mean is mentioned in Eq. 1:

$$\bar{x} = \frac{1}{n} \left(\sum_{i=1}^n x_i \right) = \frac{x_1 + x_2 + \dots + x_n}{n} \tag{1}$$

Where:

- n = The number of samples
- x = The sample values

Median is the value separating the higher half of the data from the lower half. Mode refers to the value that appears most often in a sample set. Standard deviation is a measure used to indicate how far the data points are close to the central value of the sample set. It is represented as SD or with the Greek letter σ . Equation 2 is the mathematical representation of standard deviation:

$$sd = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N - 1}} \tag{2}$$

where, $\{x_1, x_2, \dots, x_N\}$ are the observed values and \bar{x} is the mean value of the observations. A lower standard deviation shows that the sample points are towards the mean value and vice versa. Variance is the square of the standard deviation (σ^2) and it measures how far the data points spread out from the central value of a sample set.

Wavelet energy corresponds to the percentage of energy corresponding to the approximation and to the details (Bafroui and Ohadi, 2014). Wavelet energy at level j is as in Eq. 3:

$$E_j = \sum_k |D_j(k)|^2 \tag{3}$$

k being the data point. These features represent a strong feature set with alone and with combination also in the classification purpose of our data.

k-means clustering: In machine learning area, learning can be done in 2 different ways. If the classes are known then the learning can be done with supervised which uses feedback based algorithm. In the other hand, if the classes are unsure about the classes then the learning can be done with unsupervised method which automatically finds the correlation among the input data and divide them into related sets. Here, unsupervised learning is used due to the fact that the classes for the features are unknown. So, the well-known unsupervised learning method k-means clustering is used to cluster the features of the different sub bands into three classes namely sleepy, active and drowsy.

Main goal of this algorithm is to make the data into groups specified by the variable K (in our case it is 3). Grouping is done based on the similarity feature based on the centroid values iteratively. Initial run is done with the random selected centroids and iterates in 2 steps: data assignment and updating the centroid step.

Data assignment step: K centroids are assigned 1st and every data point is assigned to the nearest centroid. If the centroids are given by c_i and every data point is assigned to the group based on the Eq. 4:

$$\arg_{c_i \in C} \min \text{dist}(c_i, x)^2 \tag{4}$$

where, $\text{dist}(\cdot)$ is the standard Euclidean distance.

Updating the centroid step: In this step the centroids are recomputed based on the centroid of the grouped of the data points. It is done by the Eq. 5:

$$c_i = \frac{1}{|S_i|} \sum_{x_i \in S_i} x_i \tag{5}$$

Both of the above steps will iteratively be executed continuously till the criterion is satisfied. Criteria could be either there is not data points changing among groups or maximum number of iterations reached.

Support Vector Machines (SVM): Support Vector Machine (SVM) is a classification algorithm based on the statistical learning theory and the Vapnik-Chervonenkis

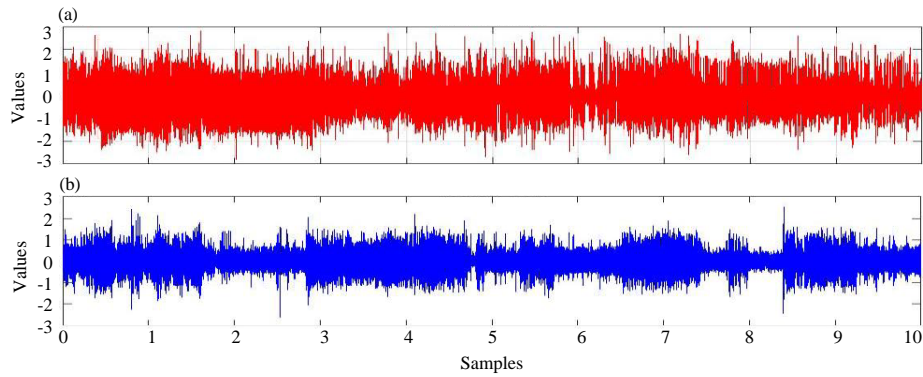


Fig. 2: Sample data of electrode pair FPZ-CZ and PZ-OZ: a) EEG data FPZ-CZ and b) EEG data PZ-OZ

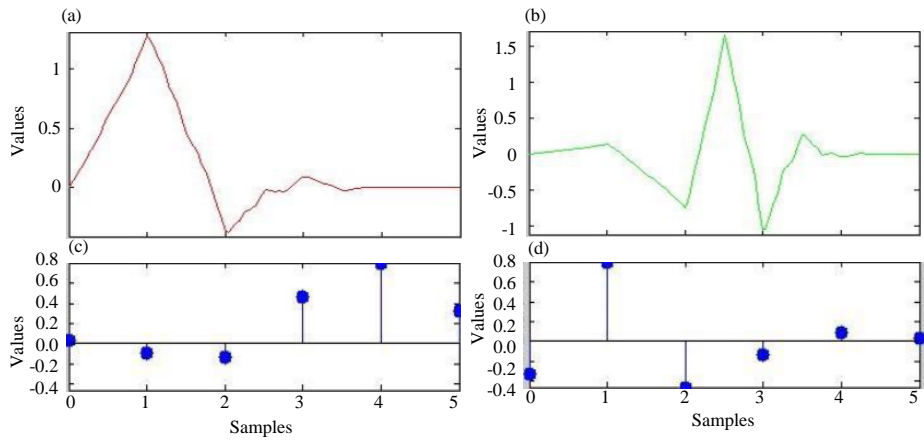


Fig. 3: Decomposition filters of 3 dB wavelet: a) Scaling function π ; b) Wavelet function π ; c) Decomposition low-pass filter and d) Decomposition high-pass filter

(Nowak, 2004) dimension which maps the input patterns into a higher dimension feature space through some nonlinear mapping methods which in effect constructs decision surface. In SVM a kernel function is important which is responsible for mapping the samples to a feature space. In this research, we use the Gaussian kernel which well known for its magical power of smoothness. Equation 6 for the Gaussian kernel. Kernel scale (σ) used here is 0.35:

$$k(x, y) = \exp\left(-\frac{\|x-y\|^2}{\sigma^2}\right) \quad (6)$$

Feature extraction and classification: The signal obtained from the dataset contains various other signals such as electrooculography apart from the EEG signal of all electrodes. We consider all signal except the EEG signal with electrode pair FPZ-CZ and PZ-OZ as noise and will remove them from the data. The signal is further filtered at 100 Hz. For the analysis part we split the signal into blocks of forty 2nd block. A sample data is shown in Fig. 2.

In this analysis third order Daubechies wavelet (Wickerhauser *et al.*, 2003) is used for the decomposition of signals are carried out up to five levels. EEG signal is passed through the low pass and high pass filters derived from the wavelet iteratively up to five levels. The output from each of these filters in each step is decimated by a factor of 2. It subsequently reduces the number of points but doubles the scale which is the strength of the wavelet transform. Figure 3 shows the 3 dB wavelet family and its corresponding low pass and high pass decomposition filters.

By evaluating the frequency spectrum of each of the coefficients it corresponds to the 5 sub bands of the EEG signal namely Δ , θ , α , β and γ . Figure 4 shows the corresponding sub bands of the selected signal.

Figure 5 shows the cluster output of 3 classes for the mean feature. In this research, clustering is done individually for all features for two electrode pairs. Similarly some combinations of features are also tested with clustering method for better results. combinations tried and gave considerable results are mean-mode, median-mode and variance-standard deviation.

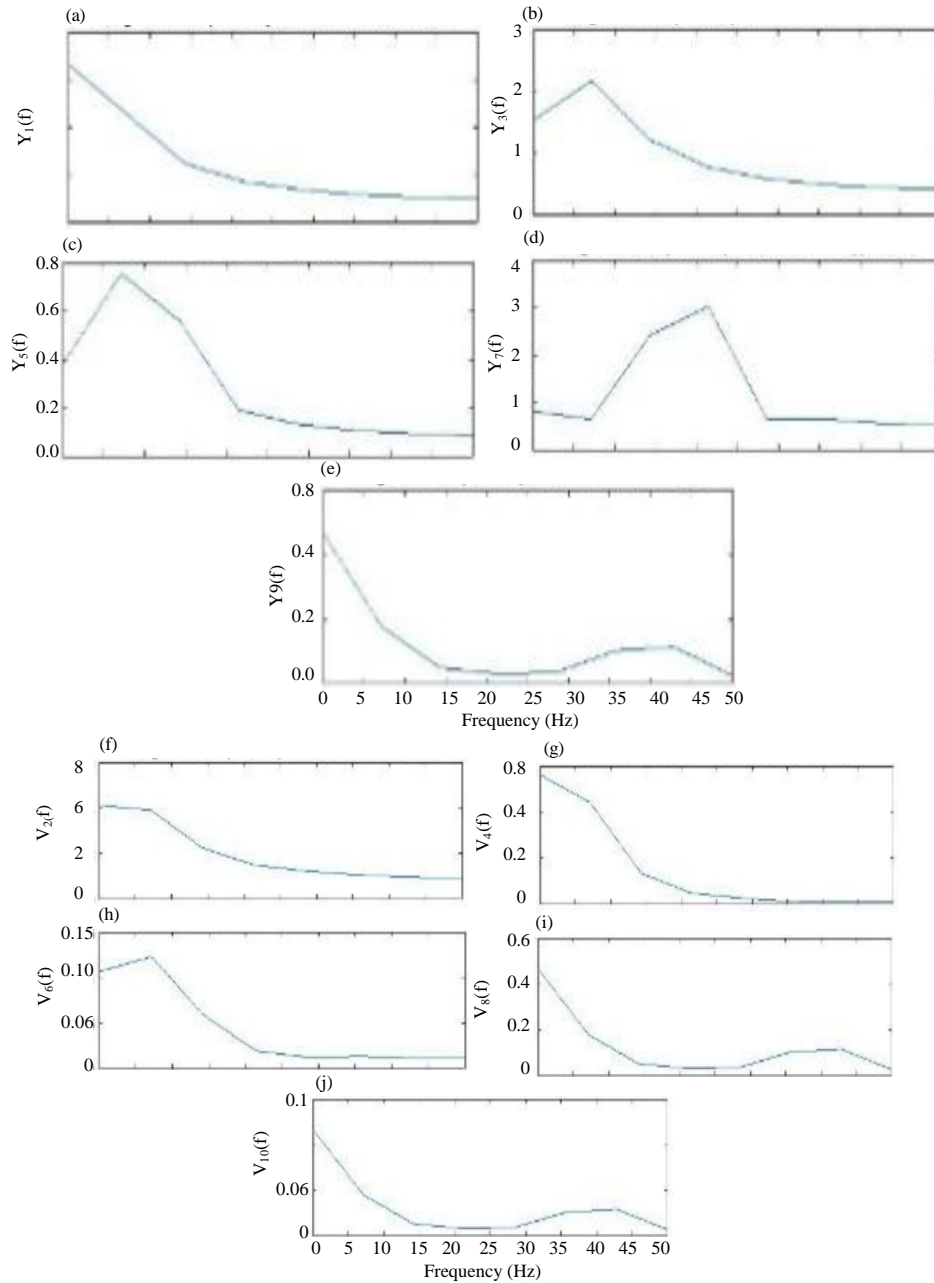


Fig. 4: Sub band waves of EEG signals for the electrode pair FPZ-CZ and PZ-OZ: a) Single sided amplitude spectrum of DELTA-PPZ-CZ; b) Single sided amplitude spectrum of THETA-PPZ-CZ; c) Single sided amplitude spectrum of ALPHA-PPZ-CZ; d) Single sided amplitude spectrum of BETA x7(t) PPZ-CZ; e) Single sided amplitude spectrum of GAMMA PPZ-CZ; f) Single sided amplitude spectrum of DELTA-PPZ-OZ; g) Single sided amplitude spectrum of THETA-PPZ-OZ; h) Single sided amplitude spectrum of ALPHA-PPZ-OZ; I) Single sided amplitude spectrum of BETA x8(t) PPZ-OZ and J) Single sided amplitude spectrum of GAMMA PPZ-OZ

Classification using support vector machine: Classification of the features is done using the support vector machine with the Gaussian kernel.

All individual clustered features are given to the SVM classifier and the results are noted. Similarly, the

combinations of features are also given to the SVM for the classifier and the recognition is noted. In our case, the statistical features (mean, mode, median, standard deviation and variance) and the wavelet energy feature are given for classification individually as well as

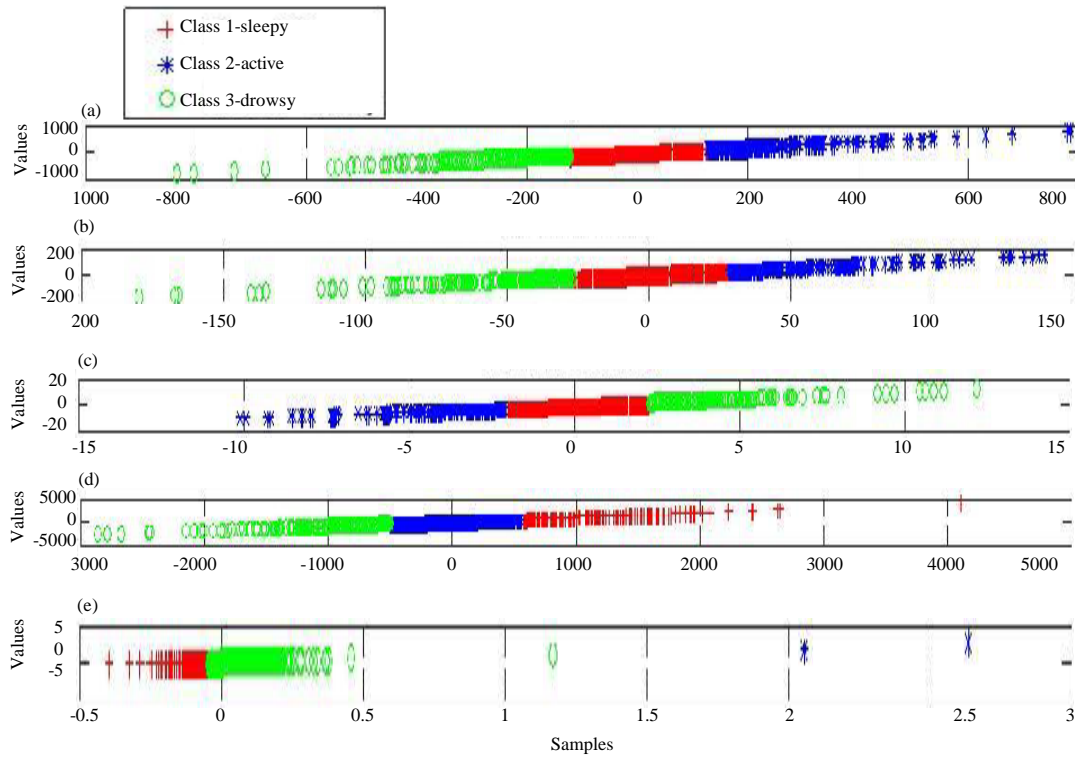


Fig. 5: a-d) k-means clustering data for the statistical feature mean

combining them together also. Detailed analysis of the accuracy rate and feature combinations are explained in the results study. Step wise algorithm for the entire procedure is as described.

Algorithm 1; SVM algorithm:

- Step 1: Start
- Step 2: Input- Read the signal from the electrode pair FPZ-CZ and PZ-OZ
- Step 3: Data is filtered
- Step 4: Divide the signals into blocks with 40 sec duration
- Step 5: Apply Wavelet Transform and divide the signal into five sub-bands
- Step 6: For each sub-band, extract the statistical and energy features
- Step 7: Store all the features extracted for entire signal
- Step 8: Apply K-means clustering to the features extracted and make clusters for active drowsy and sleepy data
- Step 9: Apply SVM classification to get the effective classification accuracy of the data
- Step 10: Stop

RESULTS AND DISCUSSION

Statistical features are extracted from the decomposed sub bands of the EEG signal. These features are effectively classified into three clusters using the k-means clustering. Clustered output is classified using the SVM classifier. Table 3 shows the classification accuracy for the features is shown when it was given individually to the SVM.

Table 3: Accuracy of SVM classification with individual features

Features	Accuracy obtained (%)
Mean	91.0
Median	69.9
Mode	70.9
Variance	84.7
Standard deviation	78.4
Wavelet energy	84.4

Table 4: Accuracy of SVM classification with combined features

Feature combinations	Accuracy obtained (%)
Mean-mean*mean	75.0
Mean-median	75.2
Mean-mode	92.2
Mean-mean*mode	87.5
Mean-mean*variance	90.6
Mean-mean*median	81.3
Median-mode	86.0
Variance-standard deviation	85.4
Mean-wavelet energy	87.5

Table 4 shows that the classification accuracies are better for variance, wavelet energy and mean. Out of these feature mean gives the best accuracy of 91%. Then, we analyzed the classification with combining the features together. Table 4 summarizes the classification accuracies with the combined features.

We tried almost all combinations and the selected combination results are shown in the Table 4 which gave considerable good results. In the individual feature classification study, mean outperformed all other features.

True class	1	3984	23	37
	2	2	1187	419
	3		144	2172
		1	2	3
		Predicted class		

Fig. 6: Confusion matrix for mean-mode combination

True class	1	99%	1%	1%		99%	1%	
	2	<1%	74%	26%		74%	26%	
	3		6%	94%		94%	6%	
		1	2	3		True positive class	True negative class	
		Predicted class						

Fig. 7: False predictive value and true positive rate for the classification of mean-mode feature

So, we used maximum features with mean. We tried the combinations of mean with mean multiplied by mean, mode, median and variance. Out of which the best accuracy acquired is 90.6% with the feature combination of mean with mean multiplied by variance. From the accuracy results mentioned in the table it is clear that the best accuracy is for the combination feature of mean and mode with 92.2%.

Performance of a classification model can be described using a confusion matrix. It is a table created on a set of test data for which the true values are known. Confusion matrix which shows the true class versus predicted class for the feature combination of mean and mode (Fig. 6).

Another way of representing the confusion matrix is with the True Positive Rate (TPR) and Positive

Predictive Value (PPV). TPR also called as sensitivity which measures the proportion of positives that are correctly identified. PPV is used to describe the performance of a statistical measure which is measured as the proportion of positive results which are true positive. Figure 7 shows the confusion matrix for the mean-mode combination with the characteristics of true positive rate and positive predictive value.

CONCLUSION

We used the statistical features and wavelet energy as the feature set for representing the data for classification. Unsupervised learning is done using k-means clustering method and the classification is done using the support vector machine with the Gaussian kernel. Best accuracy percentage is gained with the mean-mode combination feature with 92.2%. This successful method can be used to automatically classify the driver health condition and predict whether there is a chance of drowsiness in his behavior. Based on the information of occurring drowsiness can be alerted to the driver and effective measure can be taken to avoid any fatigue situations. This research can be extended with an extensive large and labeled dataset which have the details of drowsiness can be used to build an effective drowsiness prediction system which will help the drivers to take precautions well before the drowsy situations.

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