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Performance Outcomes in Learning Additional Mathematics by Grey Theory Approach

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Abstract: Grey system theory has evolved, since, it was first introduced roughly 40 years ago. It has been used in many applications and environments by different researchers worldwide. The aim of this study is to demonstrate the said theory in education environment by looking at the performance outcome in learning a mathematical subject at secondary school level through analysis and forecasting. The results show that the GM (1, 1) Model produced 3.8% mean relative percentage error and future research will focus on how to optimize the forecasting.

Key words: Grey relational analysis, grey modelling, examination forecasting, GM (1, 1) Model, environment, applications

INTRODUCTION

Grey system theory was first introduced in the early 1980's by Deng Julong (Hui *et al*., 2009) as a method for making quantitative predictions and its ability to deal with the systems that have partially unknown parameters. It has been widely and successfully applied to various fields (Bhattacharyya, 2015). Some of the main contribution to grey system theory are grey forecasting, grey control, Grey Relational Analysis (GRA) and grey Modelling (GM). GRA can be used for system analysis as analternative to statistical methods. GM is developed based on requirement for system modelling with limited data which constitutes a problem for most of the traditional statistical modelling methods (Slavek and Jovic, 2012).

In this study, GRA is used to establish a ranking scheme that rank the order of the grey relationship amongst the factors involved according to the order of their magnitudes and GM is forecasting purposes.

MATERIALS AND METHODS

Assume the non-negative sequence of raw data is $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)\}$; then its first order Accumulation Generated Operation (AGO) sequence $X^{(1)}$ is:

$$X^{(1)} = \left\{x^{(1)(1)}, x^{(1)(2)}, ..., x^{(1)(n)}\right\}$$

where, $x^{(1)}(k) = \Sigma x^{(0)}(k)$, k = 2, 3, ..., n and $x^{(1)}(1) = x^{(0)}(1)$. Using those relationships, we are able to write down the basic mathematical equations on the grey relational analysis and grey modelling.

Grey relational analysis: GRA is a new analysis method which is based on geometrical mathematics. It is suitable for solving complicated interrelationships between factors and variables and has been successfully applied on many fields (Sallehuddin *et al*., 2008). GRA analyzes the relational grade between two discrete sequences based on their similarity and variability (Slavek and Jovic, 2012). Accordingly, there are three main steps in GRA, namely data pre-processing which involves data representative, $x = (x_o, x_i)$ and data normalization, X (k), grey relational coefficient (γ_i) determination and grey relational grade (Γ_i) calculation. There are three kinds of influence factors: benefit-type, defect-type and medium-type. The standard transformation formula for defect-type or smaller-the-better influence factor (Tsai *et al*., 2003) are given by:

$$X(k) = \left[\max x(k)\text{-}x(k)|\right]/\left[\max x(k)\text{-}\min x(k)\right]$$

$\gamma_i = (\Delta\min+p\Delta\max)/(\Delta x(k)+p\Delta\max)$ where p = 0.5 will offers moderate distinguishing effect and stability and $\Delta\min = 0$ and $\Delta\max = 1$ (Sallehuddin *et al*., 2008). $\Gamma_i = (1/n)\Sigma\gamma_i(k)$ represents grey relational grade that shows the correlation between the reference sequence and comparability sequence. Generally, Γ_i>0.9 indicates a marked influence, Γ_i>0.8 a relatively marked influence,

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Γ_i>0.7 a noticeable influence and Γ_i<0.6 a negligible influence (Fu *et al*., 2001). In this study, the original data series (x) is made up of the reference series (x_o) represent the student numbers for each grading and comparative series (x_i) represent the examination grades category (Grades C, D, E and G).

Grey modelling: The modelling is performed in order to establish a set of grey variation equations and Grey differential equations (Slave and Jovic, 2012). GM (1, 1) is the most widely used time series grey forecasting model and it is not necessary to employ all the data from the original series in order to construct the model (Cui *et al*., 2013) which indicates a first-order differential equation of one variable defined as:

$$dx^{(1)}/dt+ax^{(1)} = u$$

The above equation is called the grey difference equation or the image equation of GM (1, 1) and is used for infinite information (Slave and Jovic, 2012). Parameters a, u are called the developing coefficient and grey input respectively and can be estimated by least squares method (Mondal and Pramanik, 2015) as:

$$[a, u]^T = \left(B^TB\right)^{-1}\left(B^TY\right)$$

Solve the grey difference equation of GM (1, 1), the predicted GM (1, 1) Model for the predicted AGO or time response sequence and the inverse AGO sequence or restored data respectively as:

$$\dot{x}^{(1)}(k) = \left[x^{(0)}(1)-(u/a)\right]\exp^{-a(k-1)}+u/a \quad (1)$$

$$\dot{x}^{(0)}(k) = \left[x^{(0)}(1)-(u/a)\right](1-\exp^{a})\exp^{-ak} \quad (2)$$

Residual error testing of GM (1, 1) could be carried out by calculating the absolute error (Hui *et.al*., 2009) and the Mean Relative Percentage Error (MRPE), respectively (Mondal and Pramanik, 2015) as:

$$\Delta(k) = \left|\dot{x}^{(0)}(k)-x^{(0)}(k)\right| \text{ where } k = 1, 2, ..., n$$

$$\text{MRPE} = (1/n)\Sigma\left(\left|\dot{x}^{(0)}(k)-x^{(0)}(k)\right|/x^{(0)}(k)\right.$$

where, k = 1, 2, ..., n

Another way for forecasting is to use a different modelling types such as GM (2, 1) Model and Verhulst Model (Mondal and Pramanik, 2015). GM (2, 1) Model is a single sequence second-order linear dyanamic Model and is fitted by differential equations as shown below:

Table 1: Original O-Level examination results in Additional Mathematics

Years	Total No. of students	Grade C	Grade D	Grade E	Grade G (Fail)
2012	6.532	502	714	938	1.515
2013	6.602	508	808	976	1.386
2014	6.463	579	771	896	1.581
2015	5.936	583	823	826	1.637
2016	5.220	476	595	723	1.536

$$d^2X^{(1)}/dt^2+a\,dX^{(1)}/dt = u$$

$$[a, u]^T = \left(B^TB\right)^{-1}\left(B^TY\right)$$

$$\dot{x}^{(1)}(k+1) = \left(u/a^2-x^{(0)}(1)/a\right)e^{-ak}+ \left(u/a\right)(k+1)+\left[x^{(0)}(1)-u/a)\left((1+a)/a\right)\right] \quad (3)$$

$$\dot{x}^{(0)}(k+1) = \dot{x}^{(1)}(k+1)-\dot{x}^{(1)}(k) \quad k = 1, 2, ..., n \quad (4)$$

Basically having a strong basic knowledge and understandings in Mathematics during the secondary school played an important aspects in the later parts of one’s life. Often we can hear the same statement from students that Additional Mathematics is not an easy subject to score during an examination. To demonstrate the statement, an O-Level examination results in Additional Mathematics (Table 1) from a state in Malaysia are used as experimental data which contain five observations (years) with the respective affecting factors (the grading). The dataset and results are shown in Table 1 that showed the number of failures in Grade G are quite high every year, Grade C is a pass and Grade D and E are categorially weak passes.

RESULTS AND DISCUSSION

Grey relational analysis: Assume the theoretical value of the number of failures X_0 to be zero, meaning that we wanted to minimize the number of students obtaining Grade C, D, E and G in the said subject every year. Table 2 and 3 show the Grey relational generation and coefficient, respectively.

Table 4 shows that Grade D has the strongest correlation to the number of students obtaining Grade C, D, E and G. In practice, we might curving the border line cases of these group of students (Grade D) to a higher grading (Grade C). Thus, producing a better overall examination result for the state. Grade G is showing the weakest correlation where in practice these students need to re-sit for the subject in the following years if they wished to improve their results, so that, they will have a better chances in getting a technical or vocational type of jobs.

Table 2: Grey relational generation

	Grades			
Years	C	D	E	G
X_0	0.00	0.00	0.00	0.00
2012	0.76	0.48	0.15	0.49
2013	0.70	0.07	0.00	1.00
2014	0.04	0.23	0.32	0.22
2015	0.00	0.00	0.59	0.00
2016	1.00	1.00	1.00	0.40

Table 3: Grey relational coefficient

	Grades			
Years	C	D	E	G
2012	0.39	0.51	0.77	0.50
2013	0.42	0.88	1.00	1.00
2014	0.93	0.68	0.61	0.39
2015	1.00	1.00	0.49	0.33
2016	0.33	0.33	0.33	0.45

Table 4: Grey relational grade and rank

	Grades			
Perameters	C	D	E	G
Γ	0.614	0.68	0.64	0.53
Rank	3	1	2	4

Table 5: GM (1,1) results

	Years				
Perameters	2012	2013	2014	2015	2016
Actual raw data $x^{(0)}(k)$	1515	1386	1581	1637	1536
Predicted AGO sequence data $\dot{x}^{(1)}$ (k)	1515	2901	4482	6119	7655
Inverse or restored AGO sequence raw data $\dot{x}^{(0)}$ (k)	155	1429	1470	1512	1556
Absolute relative or residual error Δ(k)	0	43	111	125	20

Grey modelling: Using Eq. 1 and 2, Table 5 shows the calculations of the sequence raw data and the residual errors for Grade G are large for each year:

$$(B^TB)^{-1} = \begin{bmatrix} 8.17\times10^{-8} & 3.7\times10^{-4} \\ 3.7\times10^{-4} & 1.919 \end{bmatrix}$$

$$B^TY = \begin{bmatrix} -28,151,900 \\ 6140 \end{bmatrix} u/a =$$

$$-(1366.46/0.02821) = -48,438.85$$

$$x^{(1)}(k+1) = (1,515+48,438.85)\exp^{(0.02821k)}-$$

$$48,438.85 = 49,953.85 \exp^{(0.02821k)}-48,438.85$$

$$x^{(0)}(k+1) = (1,515+48,438.85)(1-\exp^{-0.02821})$$

$$\exp^{0.02821k} = 49,953.85(0.02782)\exp^{0.02821k}$$

$$MRPE = (1/5)[0.1906) = 0.03812$$

Another forecasting model that way selected by this study was the GM (2, 1) as shown in Table 6.

Table 6: GM (2,1) results

	Years				
Perameters	2012	2013	2014	2015	2016
Actual raw data $x^{(0)}(k)$	1515	1386	1581	1637	1536
Generated AGO sequence data $\dot{x}^{(1)}$ (k)	1515	2901	4482	6119	7655
Predicted IAGO sequence data $\dot{x}^{(1)}$ (k)	1515	3020	4483	5838	6903
Predicted IAGO sequence raw data $\dot{x}^{(0)}$ (k)	1515	1505	1463	1355	1065
Residual error Δ(k)	0	119	118	282	471

$$(B^TB)^{-1} = \begin{bmatrix} 2.88x10-5 & 0.04421 \\ 0.04421 & 68.1097 \end{bmatrix}$$

$$B^TY = \begin{bmatrix} -66,037 \\ 21 \end{bmatrix} u/a = -1489.19/-0.97345 =$$

$$1,529.81\ u/a^2 = -1,571.53(1+a)/a =$$

$$-0.0273\ x^{(0)}(1)/a = -1,556.32$$

Using Eq. 4 and rewrite Eq. 3 as follows:

$$\dot{x}^{(1)}(k+1) = [-1571.53+1556.32]e^{0.97345k} +$$

$$1529.81(k+1)^{+} + [(1515-1529.81)](-0.0273)$$

$$= -15.21^{e0.97345k} + 1529.81(k+1) + 0.4043$$

$$MRPE = (1/5)(0.6394) = 0.1279$$

CONCLUSION

GM (1, 1) Model is performing better than GM (2, 1) for the given datasets. Future research will focus on the optimization approaches in order to provide a more accurate forecast.

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