Journal of Engineering and Applied Sciences 14 (7): 2141-2145, 2019
ISSN: 1816-949X

ON K-Metro Domination Number of C_n^2

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Abstract: A dominating set D of a graph G = G(V, E) is called metro dominating set of G if for every pair of vertices u, v there exists a vertex w in D such that d(u, w)≠d(v, w). The k-metro domination number of square of a cycle, $\lambda\beta_k$ (C_n^2) is the order of a smallest k-dominating set of (C_n^2) which resolves as a metric set. In this study, we caculate the k-metro domination number of C_n^2.

Key words: Domination, metric dimension, metro domination, smallest, k-metro, resolves

INTRODUCTION

All the graph considered here are simple, finite and connected. A set of nodes S resolves a graph G if every node of G is determined uniquely by its vector of distance to the nodes in S. The work in this study undertakes a general study of resolving sets in square cycles of graphs. Harary and Melter (1976) introduce the notion of metric dimension in the year 1976. A vertex x∊V(G) resolves a pair of vertices u, w∊V(G) if d(v, x)≠d(w, x). A set of vertices S⊆V(G) resolves G and S is resolving set of G, if every pair of distinct vertices of G areresolved by some vertex in S. A resolving set S of G with minimum cardinality is a metric basis of G and its cardinality is the metric dimension of G, denoted by β(G). Harary and Melter (1976) introduce the notion of the metric dimension. A vertex will be represented by a work place in the graph and the connection between the two places will be represented by edges of the graph. Identifying the minimum number of machines to be placed at certain vertices to trace each and every vertices exactly once is a classical problem. Using the network concept the above problem can be solved. The vertices of a network where machines are placed is called land marks. It is necessary to determine a collection of vertices at which to place detection device, so that, if there is an object at any vertex in the graph, it can be detected and its position uniquely identified. In order to detect an object which might be at any vertex in V(G). It is necessary to have a dominating set. The additional problem of uniquely indentifying the location of the object requires a metric dimension feature. These two concept motivated for the investigating of the new graph invariant called locating domination number by Slater *et al*. (2005).

MATERIALS AND METHODS

Definition 2.1; Locating number: A sub set D of V(G) called a dominating set, if every vertex V-D is adjacent to at least one vertex in D. The minimum cardinality of a dominating set is called the domination number of the graph G. The metric dimension of a graph G is the cardinality of minimal subset S of V such that for each pair of vertices u,v of V there is a vertex w in S such that the length of the shortest path from w to u is different from the length of a shortest path from w to v. The metric dimension of G is also called locating number of G. A dominating set D is called a locating dom-inating set or simply LD-set if for each pair of vertices u, v∊V-D, N D(u) ≠N D(v) where, N D(u) = N(u)∩D, an LD set of the graph G is called the locating domination number of G denoted by γL(G).

Definition 2.2; Metro domination number: A dominating set D of V(G) having the property that for each pair of vertices u, v there exists a vertex w in D such that d(u, w) ≠d(v, w) is called the metro dominating set of G or simply MD set. The minimum cardinality of a metro dominating set of G is called metro domination number of G and is denoted by $\gamma_\beta(G)$.

RESULTS AND DISCUSSION

We recall the following results which we use in the next sections.

Theorem 3.1; Harary and Melter (1976): The metric dimension of non trivial complete graph of order n is n-1.

Theorem 3.2; Alishahi and Shalmaee (2015): For any non trivial graph G on n≥2 vertices, βk(G) = n-1 if and only if diam(G)≤k where, k≥1 is any integer.

Theorem 3.3; Raghunath *et al*. (2014): Graph with metric dimension two cannot have a sub graph isomorphic to K_5 or K_{33}.

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Theorem 3.4; Raghunath *et al.* (2005): If C_n is cycle of order n, then, $\beta(C_n) = 2$.

Theorem 3.5; Caro *et al.* (2000): For any two positive integers n, k with k<n, $\beta_b(C_n^k) = \beta(C_n^k)$.

Theorem 3.6; Shanmukha *et al.* (2002): Let, G be a graph on n vertices, Then, $\gamma_\beta(G) = n-1$ if and only if G is K_n or $k_{1,n-1}$ for $n \geq 1$.

Theorem 3.7; Shanmukha *et al.* (2002): If $\gamma_\beta(G) = 2$ then, G cannot have K_4 as a sub graph of G.

Remark 3.8: If C_n is a cycle with n vertices then:

$$\beta(C_n^k) = \begin{cases} k+1 \text{ if } n \neq 0 \pmod{2k} \\ 2k+1 \text{ if } n \equiv 0 \pmod{2k} \\ 2k \text{ if } n \equiv 1 \pmod{2k} \end{cases}$$

Main results

Lemma 4.1: For any integer n $\gamma_2(C_n^2) = \lceil n/9 \rceil, n \geq 6$.

Proof: Let $v_1, v_2, v_3, ..., v_n$ be the vertices of the square path C_n^2. Let, D be the minimum 2-dominating set of C_n^2. Let, W = V-D. Any vertex $v_i \in D$ will be adjacent to four vertices of W. Also, all the vertices of W are either adjacent to at least one of the vertex of D or it will be at the distance less than or equal to four from one of the vertex of D. Hence, each $v_i \in D$, dominates at most eight vertices of W. Thus:

$$\gamma_2(C_n^2) \geq \left\lceil \frac{n}{9} \right\rceil \quad (1)$$

To prove the reverse inequality, we define the following dominating sets:

$$D = \left\{ V_{9k+1}; 0 \leq k \leq \left\lfloor \frac{n-1}{9} \right\rfloor \right\}, n \geq 6$$

We note that the above dominating set serve as a 2-dominating set. Thus:

$$\gamma_2(C_n^2) \leq \left\lceil \frac{n}{9} \right\rceil \quad (2)$$

From Eq. 1 and 2:

$$\gamma_2(C_n^2) = \left\lceil \frac{n}{9} \right\rceil$$

Lemma 4.2: For any integer:

$$n, \gamma_3(C_n^2) = \left\lceil \frac{n}{13} \right\rceil, n \geq 10$$

Proof: Let, $v_1, v_2, v_3, ..., v_n$ be the vertices of the square path C_n^2. Let, D be the minimum 3-dominating set of C_n^2. Let, W = V-D. Any vertex $v_i \in D$ will be adjacent to four vertices of W. Also, all the vertices of W are either adjacent to at least one of the vertex of D or it will be at the distance less than or equal to six from one of the vertex of D. Hence, each $v_i \in D$, dominates at most twelve vertices of W. Thus:

$$\gamma_3(C_n^2) \geq \left\lceil \frac{n}{13} \right\rceil \quad (3)$$

To prove the reverse inequality, we define the following dominating sets:

$$D = \left\{ V_{12k+1}; 0 \leq k \leq L \left\lfloor \frac{n-1}{13} \right\rfloor \right\}, n = 10$$

We note that the above dominating set serve as a 3-dominating set. Thus:

$$\gamma_3(C_n^2) \leq \left\lceil \frac{n}{13} \right\rceil$$

From Eq. 1 and 2:

$$\gamma_3(C_n^2) = \left\lceil \frac{n}{13} \right\rceil$$

Lemma 4.3: For any integer:

$$n, \gamma_4(C_n^2) = \left\lceil \frac{n}{17} \right\rceil, n \geq 14$$

Proof: Let, $v_1, v_2, v_3, ..., v_n$ be the vertices of the square path C_n^2. Let, D be the minimum 4-dominating set of C_n^2. Let, W = V-D. Any vertex $v_i \in D$ will be adjacent to four vertices of W. Also, all the vertices of W are either adjacent to at least one of the vertex of D or it will be at the distance less than or equal to eight from one of the vertex of D. Hence, each $v_i \in D$ dominates at most 17 vertices of W. Thus:

$$\gamma_4(C_n^2) \geq \left\lceil \frac{n}{17} \right\rceil \quad (5)$$

To prove the reverse inequality, we define the following dominating sets:

$$D = \left\{ V_{16k+1}; 0 \leq k \leq L \left\lfloor \frac{n-1}{17} \right\rfloor \right\}, n \geq 14$$

We note that the above dominating set serve as a 3-dominating set. Thus:

$$\gamma_4(C_n^2) \leq \left\lceil \frac{n}{17} \right\rceil \quad (6)$$

From Eq. 1 and 2:

$$\gamma_4(C_n^2) = \left\lceil \frac{n}{17} \right\rceil$$

Theorem 4.4: For any integer n:

$$\gamma_{\beta 2}(C_n^2) = \begin{cases} 3 \text{ if } 6 \le n \le 18 \\ 4 \text{if } n \equiv 1 \text{ (mod 4)}, n \le 36, n = 26 \\ \left\lceil \frac{n}{9} \right\rceil \text{if } n \ge 19 \end{cases}$$

Proof: By Lemma 4.1, it is clear that:

$$\gamma_2(C_n^2) = \left\lceil \frac{n}{9} \right\rceil, n \ge 6$$

this 2-dominating set serves as a metric set by remark 3.9. Thus:

$$\gamma_4(C_n^2) \ge \left\lceil \frac{n}{9} \right\rceil$$

To prove the reverse inequality, we define the following dominating sets:

$$D = \{v_1, v_3, v_n\} \; n = 6, 7$$
$$D = \{v_1, v_4, v_n\} \; n = 8, 10, 11, 12$$
$$D = \{v_1, v_4, v_6, v_9\} \; n = 9$$
$$D = \{v_1, v_4, v_8, v_{11}\} \; n = 13$$
$$D = \{v_1, v_{10}, v_n\} \; n = 14, 15$$
$$D = \{v_1, v_7, v_{15}\} \; n = 16$$
$$D = \{v_1, v_7, v_{10}, v_{15}\} \; n = 17$$
$$D = \{v_1, v_9, v_{17}\} \; n = 18, 20, 22, 24$$
$$D = \{v_1, v_{10}, v_{19}\} \; n = 19, 23, 27$$
$$D = \{v_1, v_{10}, v_{12}, v_{20}\} \; n = 21$$
$$D = \{v_1, v_{10}, v_{14}, v_{23}\} \; n = 25$$
$$D = \{v_1, v_{10}, v_{19}, v_{26}\} \; n = 26$$
$$D = \left\{ V_{9k+1}; 0 \le k \le \left\lceil \frac{n-9}{9} \right\rceil \right\}, n \equiv 0 \text{(mod 2) and } n \equiv 3 \text{(mod 4)}, n \ge 28$$
$$D = \left\{ V_{9k+1}; 0 \le k \le \left\lceil \frac{n}{9} \right\rceil - 2 \right\} \cup v_{\left\lceil \frac{n+3}{2} \right\rceil} \cup v_{\left\lceil \frac{n+3}{2} \right\rceil} + 9, n \equiv 1 \text{(mod 4)}, n \ge 28$$

We note that, the above dominating sets serves as minimum 2-dominating set and also by remark 3.9, it serves as a metric set. Thus:

$$\gamma_2(C_n^2) \le \left\lceil \frac{n}{9} \right\rceil \quad (8)$$

From Eq. 1 and 2:

$$\gamma_2(C_n^2) = \left\lceil \frac{n}{9} \right\rceil$$

Theorem 4.5: For any integer n:

$$\gamma\beta_3(C_n^2) = \begin{cases} 3 \text{ if } 10 \le n \le 26 \\ 4 \text{ if } n \equiv 1 \text{ (mod 4)}, n \le 52 \\ \left\lceil \frac{n}{13} \right\rceil \text{ if } n \ge 27 \end{cases}$$

Proof: By lemma 4.2, it is clear that:

$$\gamma_3(C_n^2) = \left\lceil \frac{n}{13} \right\rceil, n \ge 10$$

this 3-dominating set serves as a metric set by remark 3.9. Thus:

$$\gamma\beta_3(C_n^2) \ge \left\lceil \frac{n}{13} \right\rceil$$

To prove the reverse inequality, we define the following dominating sets:

$$D = \{v_1, v_7, v_n\} \; n = 10, 11, 12, 15$$
$$D = \{v_1, v_6, v_8, v_{13}\} \; n = 13$$
$$D = \{v_1, v_7, v_{13}\} \; n = 14$$
$$D = \{v_1, v_7, v_{15}\} \; n = 16, 18$$
$$D = \{v_1, v_7, v_{10}, v_{15}\} \; n = 17$$
$$D = \{v_1, v_7, v_{19}\} \; n = 19$$
$$D = \{v_1, v_7, v_{17}\} \; n = 20$$
$$D = \{v_1, v_7, v_{12}, v_{17}\} \; n = 21$$
$$D = \{v_1, v_{14}, v_n\} \; n = 22, 23, 24$$
$$D = \{v_1, v_7, v_{14}, v_{21}\} \; n = 25$$
$$D = \{v_1, v_{13}, v_{21}\} \; n = 26$$
$$D = \left\{ V_{13k+1}; 0 \le k \le \left\lceil \frac{n-13}{13} \right\rceil \right\}, n = 27, 31, 35, 39$$
$$D = \left\{ V_{12k+1}; 0 \le k \le \left\lceil \frac{n-13}{13} \right\rceil \right\}, n = 28, 30, 32, 34, 36$$
$$D = \left\{ V_{13k+1}; 0 \le k \le \left\lceil \frac{n-13}{13} \right\rceil \right\} \cup v_{\left\lceil \frac{n+3}{28} \right\rceil} \cup v_{\left\lceil \frac{n+3}{2} \right\rceil} + 13, n = 29, 33, 37$$
$$D = \left\{ V_{12k+1}; 0 \le k \le \left\lceil \frac{n-11}{13} \right\rceil \right\}, n = 38$$
$$D = \left\{ V_{13k+1}; 0 \le k \le \left\lceil \frac{n-13}{13} \right\rceil \right\}, n \equiv 0 \text{(mod 2) and } n \equiv 3 \text{(mod 4)}, n \ge 40$$
$$D = \left\{ V_{13k+1}; 0 \le k \le \left\lceil \frac{n}{13} \right\rceil - 2 \right\} \cup v_{\left\lceil \frac{n+3}{2} \right\rceil} \cup v_{\left\lceil \frac{n+3}{2} \right\rceil} + 13, n \equiv 1 \text{ (mod 4)}, n \ge 40$$

We note that the above dominating sets serves as minimum 3-dominating sets and also by remark 3.9, it serves as a metric set. Thus:

$$\gamma\beta_3(C_n^2) \leq \left\lceil \frac{n}{13} \right\rceil \quad (10)$$

From Eq. 1 and 2:

$$\gamma\beta_3(C_n^2) = \left\lceil \frac{n}{13} \right\rceil$$

Theorem 4.6: For any integer n:

$$\gamma\beta_4(C_2^2) = \begin{cases} 3 \text{ if } 14 \leq n \leq 34 \\ 4 \text{ if } n \equiv 1 (\text{mod } 4), n \leq 68 \\ \left\lceil \frac{n}{17} \right\rceil \text{if } n \geq 35 \end{cases}$$

Proof: By Lemma 4.3, it is clear that

$$\gamma_4(C_n^2) = \left\lceil \frac{n}{17} \right\rceil, n \geq 14 \quad (11)$$

this 4-dominating set serves as a metric set by remark 3.9. Thus:

$$\gamma_{\beta 4}(C_n^2) \geq \left\lceil \frac{n}{17} \right\rceil$$

To prove the reverse inequality, we define the following dominating sets:

$D = \{v_1, v_9, v_n\}$ n = 14, 15
$D = \{v_1, v_9, v_{16}\}$ n = 16
$D = \{v_1, v_9, v_{16}, v_{17}\}$ n = 17
$D = \{v_1, v_9, v_{17}\}$ n = 18
$D = \{v_1, v_9, v_{19}\}$ n = 19, 20, 22, 24, 26
$D = \{v_1, v_9, v_{14}, v_{21}\}$ n = 25
$D = \{v_1, v_{13}, v_{27}\}$ n = 27, 28, 30
$D = \{v_1, v_{13}, v_{16}, v_{23}\}$ n = 29
$D = \{v_1, v_{15}, v_{31}\}$ n = 31.32, 34
$D = \{v_1, v_{13}, v_{18}, v_{25}\}$ n = 33

$$D = \left\{V_{17k+1}; 0 \leq k \leq \left\lceil \frac{n-17}{17} \right\rceil\right\}, n = 35, 39, 41, 47$$

$$D = \left\{V_{18k+1}; 0 \leq k \leq \left\lceil \frac{n-17}{17} \right\rceil\right\}, n = 36, 40, 44, 48$$

$D = \{v_1, v_{18}, v_{34}, v_{41}\}$ n = 45

$$D = \left\{V_{17k+1}; 0 \leq k \leq \left\lceil \frac{n-17}{17} \right\rceil\right\}, n = 0(\text{mod } 2) \text{ and}$$

$n \equiv 3(\text{mod}4), n \geq 52$

$$D = \{V_{13k+1}; 0 \leq k \leq \left\lceil \frac{n}{17} \right\rceil - 2\} \cup v_{\left\lceil \frac{n+3}{2} \right\rceil} \cup v_{\left\lceil \frac{n+3}{2} \right\rceil} + 17$$

$n \equiv 1(\text{mod } 4), n \geq 52$

We note that, the above dominating sets serves as minimum 4-dominating sets and also by remark 3.9, it serves as a metric set. Thus:

$$\gamma_{\beta_4}(C_n^2) \leq \left\lceil \frac{n}{17} \right\rceil \quad (12)$$

From Eq. 1 and 2:

$$\gamma_{\beta_4}(C_n^2) = \left\lceil \frac{n}{17} \right\rceil$$

As the generalization of the Theorem 4.4 , Thorem 4.5 and Theorem 4.6, the following theorem follows (Fig. 1 and 2)(Buckley and Harary, 1990; Dirac, 1952; Dreyer, 2000; Khuller *et al*., 1996).

Theorem 4.7: For any integer n:

$$\gamma\beta_k(C_n^2) = \begin{cases} 3 \text{ if } 4k\text{-}2 \leq n \leq 8k+2 \\ 4 \text{ if } n \equiv 1 (\text{mod } 4) \\ \left\lceil \frac{n}{4k+1} \right\rceil \text{if } n \geq 8k+3, k \geq 2 \end{cases}$$

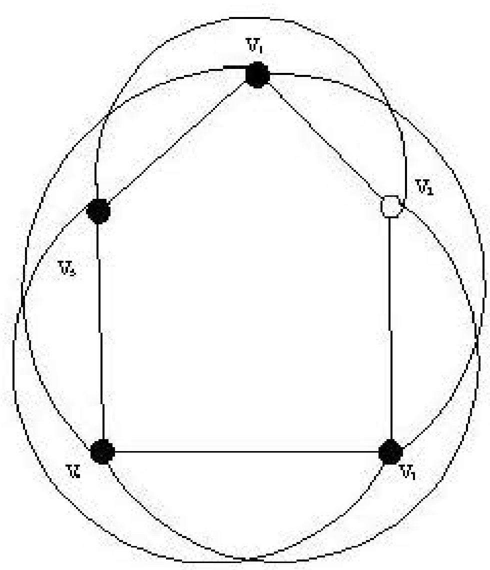

Fig. 1: $\gamma\beta_2(C_n^2) = 4$

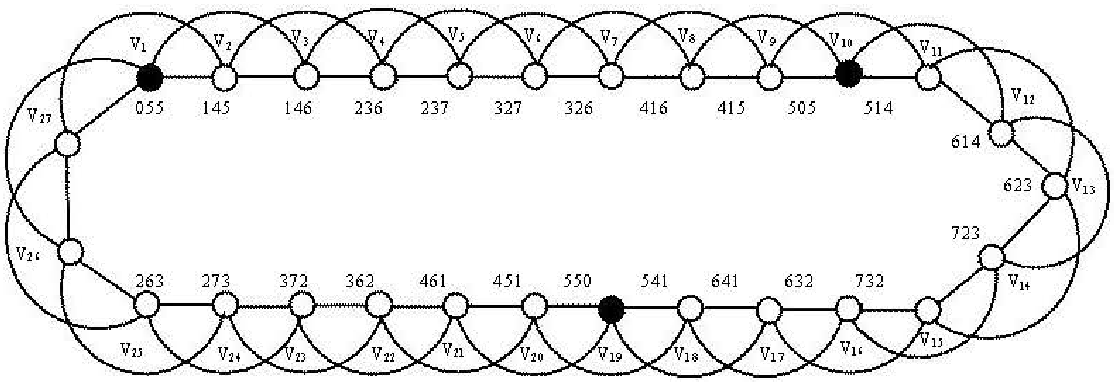

Fig. 2: $\gamma\beta_2(C^2_{27}) = 3$

CONCLUSION

In this study, we obtain k-metro domination number of square of cycle.

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