

## Soft Simply\* Generalized Continuous Mappings in Soft Topological Spaces

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**Abstract:** In this study, we introduce a new class of sets, called soft simply\* generalized closed (briefly,  $SS^{M^*}$  g-closed) sets based on soft simply\* open set, soft delta ( $S\delta$ -) open set in soft topological spaces. This class of sets (soft simply\* generalized closed set) properly contains the classes of soft closed sets, soft  $\alpha$ -closed sets, soft generalized  $\alpha$ -closed (briefly, sg  $\alpha$ -closed) sets, soft generalized closed sets, soft  $\alpha$  generalized closed (briefly,  $s\alpha$ g-closed) sets and the class of soft simply\* closed sets as applications of soft simply\* generalized closed sets and study some of their properties, also, we introduce new types of continuous functions namely, simply\* continuous soft simply\* irresolute functions and we study some of their properties.

**Key words:** Soft topology, soft simply open set, soft simply\* open set, soft  $\delta$  set, soft simply\*continuous function, soft simply\* g-continuous function, soft simply\*g-irresolute, soft simply\*generalized closed set

### INTRODUCTION

Molodtsov (1999) defined the concept of soft set theory as a mathematical tool for dealing with uncertainties. He introduced the fundamental results of this new soft set theory and also presented some applications of the soft set theory in several fields, for example, smoothness of functions, game theory, operations research by Shabir and Naz (2011) initiated the study of soft topological spaces. They defined the soft topology on the collection  $\tau_{\sim}$  of soft sets over a universe set  $X$  and gave basic notions of soft topological spaces such as soft open sets. By Kannan (2012) defined soft, generalized closed and open sets in soft topological spaces and studied some of their properties. Subhashini and Saker defined soft pre-open sets and soft pre-closed (Chen, 2013). He introduced the concept of soft, semi-open sets and studied some of its properties. Arockiarani and Lancy (2013) defined soft  $\beta^-$ -open sets. Finally, Akdag and Ozkan (2014) defined soft  $\alpha$ --open (soft  $\alpha$ -closed) sets. Akdag and Ozkan (2014) introduced the concept of soft b-open sets and soft b-continuous functions. Also, Yuksel *et al.* (2013) studied behavior relative to soft subspaces of soft, generalized closed sets and continued investigating the properties of soft, generalized closed and open sets. In this study, we introduced new class of sets namely soft simply\* open set, soft simply\* generalized closed based on simply\* open set in soft topological spaces. These classes of sets properly contain the class of soft closed sets, soft  $\alpha$ -closed sets, soft generalized-closed sets, soft  $\alpha$ -generalized closed sets, soft, generalized  $\alpha$ -closed sets

and soft simply\* closed sets. As applications of soft, generalized sets, we also introduce and study some the new soft continuous functions, called soft simply\* generalized continuous and soft simply\* generalized irresolute functions. And, we study some of their properties. Let  $(F, A)$  be a soft set over  $X$ , then, we denote to soft closure and soft interior (resp. sot b-interior, soft b-closure, soft  $\alpha$ -interior, soft  $\alpha$ -closure, soft simply\* interior, soft simply\* closure soft semi interior and soft semi closure) of soft set  $(F, A)$  will be denoted by  $cl_s(F, A)$  and  $int_s(F, A)$  (resp.  $bint_s(F, A)$ ,  $bcl_s(F, A)$ ,  $\alpha int_s$ ,  $\alpha cl_s(F, A)$ ,  $(F, A)^{ss^*0}$ ,  $(F, A)^{ss^*}$  and  $scls(F, A)$ ), respectively.

**Definition 1.1; Molodtsov (1999):** Let,  $X$  be an initial universe set and  $P(X)$  the power set of  $X$  that is the set of all subsets of  $X$  and let,  $E$  be a set of parameters. A pair  $(F, E)$  where,  $F$  is a map from  $E$  to  $P(X)$  is called a soft set over  $X$ . And, we shall denote to the family of all soft sets of  $(F, E)$  over  $X$  by  $SS(F, E)$ .

#### Definition 1.2:

- Null soft set is denoted by  $\emptyset_{\sim}$ , if for all  $e \in E$ ,  $F(e) = \emptyset$
- Absolute soft set is denoted by  $X$ , if for all  $e \in E$ ,  $F(e) = X$

Let  $(F, E), (G, E) \in SS(X, E)$ . Then, we say that the pair  $(F, E)$  is a soft subset of  $(G, E)$  if  $F(p) \subseteq G(p)$  for every  $p \in E$ . Symbolically, we write  $(F, E) \subseteq (G, E)$ . Also, we say that the soft sets  $(F, E)$  and  $(G, E)$  are soft equal if  $(F, E) \subseteq (G, E)$  and  $(G, E) \subseteq (F, E)$ . Symbolically, we write  $(F, E) = (G, E)$ .

**Definition 1.3; Molodtsov (1999):** For two soft sets (F, E) and (G, B) over a common universe set X. We define:

Union of two soft sets of (F, A) and (G, B) is the soft set (H, C) where,  $C = A \sqcup B$  and  $\forall e \in C$ , then,  $H(e) = \{f(e), \text{ if } e \in A-B \text{ or } G(e)\}$ , if  $e \in B-A$  or  $F(e) \sqcup G(e)$ , if  $e \in A \cap B$ . We write  $(F, A) \sqcup (G, B) = (H, C)$ .

Intersection of F, A and G, B is the soft set (H, C) where,  $C = A \cap B$  and  $\forall e \in C$ ,  $H(e) = F(e) \cap G(e)$ . We write  $(F, A) \cap (G, B) = (H, C)$ .

**Definition 1.4; Zorlutuna et al. (2012):** Let (F, A)  $\in$  SS(X, E). Then, the complement of soft set (F, A) is the soft set where the map  $H: A \rightarrow P(X)$  defined as follows: for ever. Symbolically, we write  $(H, A) = (F, A)^c$

**Definition 1.5; Zorlutuna et al. (2012):** Let, X be an initial universe set E a set of parameters and  $\tau \sim \in$  SS(X, E) and then, we say that the family  $\tilde{\tau}$  defines a soft topology on  $\tilde{X}$  if the following axioms are true:

- $\emptyset \sim, X \sim \in \tau \sim$
- If  $(G, E) = (H, E) \in \tau \sim$  then  $(G, E) \cap (H, E) \in \tau \sim$
- If  $(G_i, E) \in \tau \sim$  for every  $i \in I$  then,  $\bigcup (G_i, E) \in \tau \sim$

Then triple (X,  $\tau \sim$ , E) is called a “soft topological space” or “soft space”. The members of  $\tau \sim$  are called soft open sets in X. Also, a soft set (F, E) is called soft closed if the complement  $(F, E)^c$  belongs to  $\tau \sim$ . The family of soft closed sets is denoted by  $\tau \sim^c$ .

**Definition 1.6; Shabir and Naz (2011):** Let (X,  $\tilde{\tau}$ , E) be a soft topological space over X and (F, E) be a soft set over X:

The soft closure of F, E is the soft set  $cl_s(F, E) = \bigcap \{(G, E); (G, E) \text{ is soft closed and } (F, E) \subseteq (G, E)\}$ . This soft interior of (F, E) is the soft set  $int_s(F, E) = \bigcup \{(H, E); (H, E) \text{ is soft open set and } (H, E) \subseteq (F, E)\}$ . Clearly,  $cl_s(F, E)$  is the smallest soft closed set over X which contains (F, E) and  $int_s(F, E)$  is the largest soft open set over X which is contained in (F, E).

**Definition 1.7:** A soft subset (F, E) of a soft topological space (X,  $\tau \sim$ , E) is said to be:

- Soft semi-open (Chen, 2013) if  $(F, E) \subseteq cl_s(int_s(F, E))$
- Soft pre-open (Ilango and Ravindran, 2013) if  $(F, E) \subseteq int_s(cl_s(F, E))$
- Soft  $\alpha$ -open (Ilango and Ravindran, 2013) if  $(F, E) \subseteq int_s(cl_s(int_s(F, E)))$
- Soft  $\beta$ -open (Akdag and Ozkan, 2014) if  $(F, E) \subseteq cl_s(int_s(cl_s(F, E)))$

- Soft regular open (Yuksel et al., 2014) (resp. Soft regular closed) if  $(F, E) = int_s(cl_s(F, E))$  (resp.  $(F, E) = cl_s(int_s(F, E))$ )
- Soft b-open set (Yuksel et al., 2014) if  $(F, E) \subseteq int_s(cl_s(F, E)) \cap cl_s(int_s(F, E))$  and soft-closed if  $(F, E) \supseteq int_s(cl_s(F, E)) \cap cl_s(int_s(F, E))$
- Soft simply open set (Sayed and El-Bably, 2017) if  $(F, E) \subseteq (G, E) \cap (V, E)$  where (G, E) is soft open set and (V, E) is soft nowhere dense set

**Definition 1.8:** Let (X,  $\tilde{\tau}$ , A) and (X,  $\tilde{\sigma}$ , B) be two soft topological spaces. A soft function  $f: (X, \tilde{\tau}, A) \rightarrow (X, \tilde{\sigma}, B)$  is said to be:

- Soft continuous (Chen, 2013) if the inverse image of each soft open set in (X,  $\tilde{\sigma}$ , B) is soft open set in (X,  $\tilde{\tau}$ , A)
- soft  $\alpha$ -continuous (Chen, 2013) if the inverse image of each soft  $\alpha$ -open set in (X,  $\tilde{\sigma}$ , B) is soft  $\alpha$ -open set in (X,  $\tilde{\tau}$ , A)
- Soft semi continuous (Chen, 2013) if for each soft open set (G, B) of (X,  $\tilde{\sigma}$ , B), the inverse image  $f^{-1}$  is soft semi open set of (X,  $\tilde{\tau}$ , A)
- Soft b-continuous (Akdag and Ozkan, 2014) (briefly, sb-continuous) if the inverse image of each soft open set of (X,  $\tilde{\sigma}$ , B) is a sb-open set in (X,  $\tilde{\tau}$ , A)
- Soft irresolute (Chen, 2013) if for each soft semi open set (G, B) of (X,  $\tilde{\sigma}$ , B), the inverse image  $f^{-1}(G, B)$  is soft semi open set of (X,  $\tilde{\tau}$ , A)

**Definition 1.9; Mahmood (2014):** Let (X,  $\tilde{\tau}$ , E) be a soft topological space a soft subset (F, E) of X is said to be soft regular generalized  $\alpha$ -closed (briefly, soft rg $\alpha$ -closed) if  $\alpha cl_s(F, E) \subseteq (G, E)$ . Whenever  $(F, E) \subseteq (G, E)$  and  $(G, E) \in SR(X, E)$ .

**Definition 1.10; Mahmood (2014):** Let (X,  $\tilde{\tau}$ , E) be a soft topological space a soft subset (F, E) of X is said to be soft generalized b-closed (briefly, soft gb-closed) if  $bcl_s(F, E) \subseteq (G, E)$ . Whenever  $(F, E) \subseteq (G, E)$  and  $(G, B) \in \tau$ .

**Definition 1.11; Mahmood (2014):** Let (X,  $\tilde{\tau}$ , E) be a soft topological space a soft subset (F, E) of X is said to be soft generalized  $\alpha$ -closed (briefly, soft g  $\alpha$ -closed) if  $\alpha cl_s(F, E) \subseteq (G, B)$ . Whenever  $(F, E) \subseteq (G, B)$  and  $(G, B) \in S\alpha O(X, E)$ .

**Definition 1.12; Mahmood (2014):** Let (X,  $\tilde{\tau}$ , E) be a soft topological space a soft subset (F, E) of X is said to be

soft regular generalized b-closed (briefly, soft r g b-closed) if  $bcl_s(F, E) \subseteq (G, B)$ . Whenever  $(F, E) \subseteq (G, E)$  and  $(G, B) \in SRO(X, E)$ .

**Definition 1.13; Kannan (2012):** Let  $(X, \tau, E)$  be a soft topological space a soft subset  $(F, E)$  of  $X$  is said to be soft, generalized closed (brief, soft g-closed) if  $cl_s(F, E) \subseteq (G, B)$ . Whenever  $(F, E) \subseteq (G, E)$  and  $(G, B) \in \tilde{\tau}$ .

**MATERIALS AND METHODS**

**Soft delta open set:** In this study, we introduce a new class of soft open set called soft delta open set.

**Definition 2.1:** A soft subset  $(A, E)$  of a soft topological space  $(X, \tau, E)$  is said to be soft delta open (briefly,  $S\delta$ -open) set if for every  $X \in (A, E)$  their soft open set  $(G, E)$  such that  $x \in (G, E) \subseteq \text{int}_s(cl_s(G, E))$ ,  $(G, E) \subseteq (A, E)$ .

**Definition 2.1:** Let  $(X, \tau, E)$  be a soft topological space over  $X$  and  $(F, E)$  be a soft set over  $X$ .

The soft delta closure of  $F, E$  (briefly,  $\delta cl_s$ ) is the soft set  $\delta cl_s(F, E) = \tilde{\cap} \{(G, E)\}$  such that  $(G, E)$  is soft  $\delta$ -closed set such that  $(G, E)$  is a soft  $\delta$ -closed and  $(F, E) \subseteq (G, E)$ .

The soft delta interior of  $F, E$  (briefly,  $\delta \text{int}_s$ ) is the soft set  $\delta \text{int}_s(F, E) = \tilde{\cup} \{(H, E)\}$  such that  $(H, E)$  is soft  $\delta$ -open set and  $(H, E) \subseteq (F, E)$ . Clearly,  $\delta cl_s(F, E)$  is the smallest soft  $\delta$ -closed set over  $X$  which contains  $(F, E)$  and  $\delta \text{int}_s(F, E)$  is the largest soft  $\delta$ -open set over  $X$  which is contained in  $(F, E)$ .

**Soft simply\* open sets:** In this study, we introduce a new class of soft open set in soft topological space namely soft simply\* (briefly,  $SS^{M^*}$ ) open set.

**Definition 3.1:** A soft subset  $(A, E)$  of soft topological space  $(X, \tau, E)$  is said to be soft simply\* open (briefly,  $SS^{M^*}$ -open) set if  $\delta \text{int}_s(cl_s(A, E)) \subseteq cl_s(\delta \text{int}_s(A, E))$ .

**Definition 3.2:** A soft subset  $(A, E)$  of soft topological space  $(X, \tau, E)$  is said to be soft simply\* open (briefly,  $SS^{M^*}$ -closed) set if  $\text{int}_s(\delta cl_s(A, E)) \subseteq \delta cl_s(\text{int}_s(A, E))$ . We shall denote the class of soft simply\* open, soft regular open, soft  $\odot$ -open, soft, semi-open set, soft pre-open set, soft delta open set and soft b-open (resp. soft simply\* closed, soft regular, soft  $\alpha$ -closed, soft, semi-closed, soft pre-closed set, soft, closed set and soft b-closed) sets of a universe set  $X$  by  $SS^{M^*}O(X, E)$ ,  $SRO(X, E)$ ,  $S\alpha O(X, E)$ ,  $SSO(X, E)$ ,  $SPO(X, E)$ ,  $S\delta O(X, E)$  and  $Sbo(X, E)$  (resp.  $SS^{M^*}C(X, E)$ ,  $SRC(X, E)$ ,  $SS^{M^*}\alpha C(X, E)$ ,  $SSC(X, E)$ ,  $SPC(X, E)$ ,  $S\delta C(X, E)$  and  $SbC(X, E)$ ), respectively. The following diagram gives the relationship between soft simply\* open set and some other types of soft near open sets (Fig. 1).

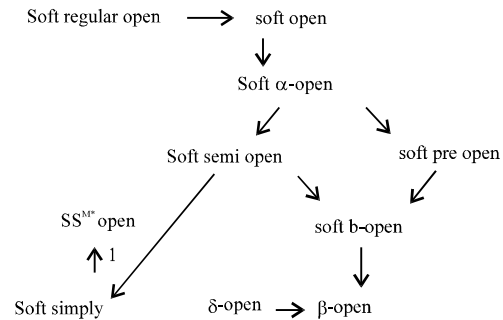


Fig. 1: Relationship between soft simply\* open set and some other types of soft near open sets

**Example 3.1; Implications 1, 2:** Let  $X = \{a, b, c\}$ ,  $E = \{e_1, e_2\}$  and  $\tau = \{\tilde{X}, \emptyset, (F, E), (G, E), (H, E)\}$  be a soft topological space where  $(F, E) = \{(e_1, \{a\})\}$ ,  $(e_2, \{a\})$ ,  $(G, E) = \{(e_1, \{b\})\}$  and  $(H, E) = \{(e_1, \{a, b\})\}$ , if  $(L, E) = \{(e_1, \{b, c\}), (e_2, \{a, c\})\}$  be a soft subsets of  $(X, \tau, E)$ . Then  $(L, E)$  is a soft simply\* open set but not a soft simply open set, also if we let  $(W, E) = \{(e_1, \{a, b\})\}$  ( $e_2, X$ ), be a soft subset of  $(X, \tau, E)$  then  $(W, E)$  is a soft simply\* open set but not soft  $\delta$ -open set.

**Definition 3.3:** A soft subset  $(F, E)$  of soft topological space  $(X, \tau, E)$  is said to be:

The soft simply\* closure of  $F, E$  (briefly,  $(F, E)^{SS^*}$  is the soft set  $(F, E)^{SS^*} = \tilde{\cap} (G, E)$ :  $(G, E)$  and  $(G, E)$  is a soft simply\*closed set and  $(F, E) \subseteq (G, E)$ .

The soft simply\* interior of  $F, E$  (briefly  $(F, E)^{SS^*0}$  is the soft set  $(F, E)^{SS^*0} = \tilde{\cup} (H, E)$ :  $(H, E)$  is a soft simply\* open set and  $(H, E) \subseteq (F, E)$ . Clearly,  $(F, E)^{SS^*0}$  is the smallest soft simply\* closed set over  $X$  which contains  $(F, E)$  and  $(F, E)^{SS^*0}$  is the largest soft simply\* open set over  $X$  which is contained in  $(F, E)$ .

**Remark 3.1:** We consider that for any soft set  $(F, A)$  in a soft topological space  $(X, E)$ . Then  $(F, A)$  satisfies as the following:

1.  $(F, E)^{SS^*} \subseteq Scl_s(F, E) \subseteq \alpha cl_s(F, A) \subseteq cl_s(F, A) \subseteq Rcl_s(F, A)$
2.  $bcl_s(F, E) \subseteq Scl_s(F, E) \subseteq \alpha cl_s(F, A) \subseteq cl_s(F, A) \subseteq Rcl_s(F, A)$

**Soft simply\* generalized closed sets:** In this study, we introduce a new class of sets, namely soft simply\*generalized closed (briefly,  $SS^{M^*}$ g-closed) sets and study their fundamental properties.

**Definition 4.1:** A soft subset  $(A, E)$  of soft topological space  $(X, \tau, E)$  is said to be soft simply\* generalized closed set (briefly,  $SS^{M^*}$ g-closed) if  $(A, E)^{SS^*} \subseteq (U, E)$  whenever  $(A, E) \subseteq (U, E)$  and  $(U, E) \in \tilde{\tau}$ .

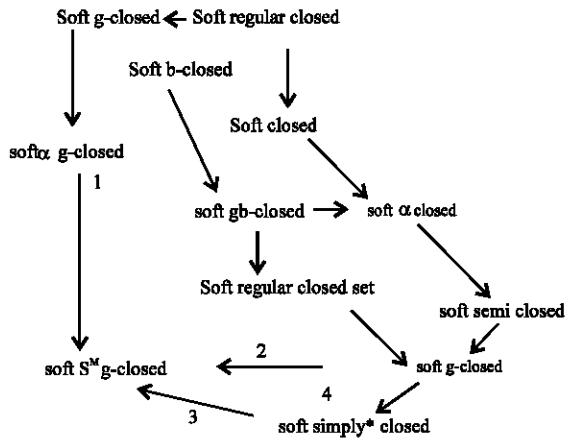


Fig. 2: Classes of closed sets

**Definition 4.2:** A soft subset  $(A, E)$  of soft topological space  $(X, \tilde{\tau}, E)$  is said to be soft simply\* generalized open (briefly,  $SS^{M^*}g$ -open) if  $(F, E) \in (A^{SS^*}, E)$  whenever  $(F, E) \subseteq (A, E)$  and  $(F, E) \in SC(X, E)$ . The complement of  $SS^{M^*}g$ -closed sets is called,  $SS^{M^*}g$ -open. We shall denote the class of all  $SS^{M^*}g$ -closed sets and  $SS^{M^*}g$ -open sets by  $SS^{M^*}GC(X, E)$  and  $SS^{M^*}GO(X, E)$ , respectively.

**Remark 4.1:** The following diagram is shown  $SS^{M^*}g$ -closed sets properly contains the classes of closed sets soft  $\alpha$ -closed sets  $g$ -closed sets, soft  $g$   $\alpha$ -closed sets, soft- $\alpha$   $g$ -closed sets and  $SS^{M^*}$ -closed sets (Fig. 2).

**Example 4.1; Implication 1, 2, 3:** Let  $(X, \tilde{\tau}, E)$  be soft topological space. Where let  $X = \{a, b\}$ ,  $E = \{e_1, e_2\}$  and  $\tilde{\tau} = \tilde{X} \cup \{e_1, \{a\}, (e_2, \{b\}), (e_1, \{a, b\}), (e_2, \{a, b\})\}$  the soft subset  $(G, E) = (e_1, \{a, b\}), (e_2, \{a\})$  is a soft simply\*generalized closed but not soft  $\alpha$ -generalized closed, since,  $(B, E) = (e_1, \{a, b\}), (e_2, \{a, b\})$  such that  $(G, E) \subseteq (B, E)$ . Is a soft open set  $(G, E)^{SS^*} = (G, E) = (e_1, \{a, b\}), (e_2, \{a\})$  but  $\alpha cl_s(G, E) = X \subseteq (B, E)$ . Thus, the soft set  $(G, E)$  is a soft simply\* generalized closed but it is not soft  $\alpha$ -generalized closed set.

**Example 4.2; From example 3.1:** If  $(A, E) = (e_1, \{a\}), (e_2, \{a\})$  be a soft subset of  $(X, \tilde{\tau}, E)$ . Then  $(A, E)$  is soft simply\*  $g$ -closed set but not soft simply\* open set also if, we let  $(B, E) = \{(e_1, \{a\}), (e_2, \{a\})\}$  be a soft subset of  $(X, \tilde{\tau}, E)$  such that  $(B, E) \subseteq (A, E)$  and  $(G, E)$  is open set,  $\alpha cl_s(B, E) = X \subseteq (G, E)$ .

**Theorem 4.1:** For any soft topological space  $(X, \tilde{\tau}, E)$  the following statements hold:

- The intersection and union of a  $SS^{M^*}g$ -closed set need not a  $SS^{M^*}g$ -closed set
- The intersection and union of a  $SS^{M^*}g$ -open set need not a  $SS^{M^*}g$ -open set

**Theorem 4.2:** For any soft topological space  $(X, \tilde{\tau}, E)$  the following statements hold:

- The union of a soft simply\* open set is a soft simply\* soft set
- The intersection of a soft simply\* open set is not soft simply\* open set
- The union of soft simply\* open set is a soft simply\* open set
- The intersection of soft simply\* open set is a soft simply\* open set

**Poof:** Let  $(A_1, E), (A_2, E)$  be two soft simply\* open sets. Then,  $\delta int_s(cl_s(A_1, E)) \subseteq cl_s(\delta int_s(A_1, E))$  and  $\delta int_s(cl_s(A_2, E)) \subseteq cl_s(\delta int_s(A_2, E)) \subseteq cl_s$  such that  $\delta int_s(cl_s(A_1, E)) \cap (A_2, E)$  but  $\delta int_s(cl_s(A_1, E)) \cap \delta int_s(cl_s(A_2, E)) \subseteq \delta int_s(cl_s(A_1, E)) \cap cl_s(A_2, E) \subseteq cl_s(\delta int_s(A_1, E)) \cap (A_2, E)$ . Thus the union of two simply\* open sets is a soft simply\* open set.

Let  $(A_1, E), (A_2, E)$  be two a soft simply\* open sets. Then,  $\delta int_s(cl_s(A_1, E)) \subseteq cl_s(\delta int_s(A_1, E))$  and  $\delta int_s(cl_s(A_2, E)) \subseteq cl_s(\delta int_s(A_2, E))$  such that  $\delta int_s(cl_s(A_1, E)) \cap (A_2, E) \subseteq cl_s(\delta int_s(A_2, E)) \subseteq cl_s(\delta int_s(A_1, E)) \cap \delta int_s(cl_s(A_2, E)) \subseteq cl_s(\delta int_s(A_1, E)) \cap cl_s(\delta int_s(A_2, E)) \subseteq cl_s(\delta int_s(A_1, E)) \cap (A_2, E)$ . This means that the intersection of two soft simply\* open sets is not a soft simply\* open set.

Let  $(A_1, E), (A_2, E)$  be two soft simply\* closed sets. Then,  $int_s(\delta cl_s(A_1, E)) \subseteq \delta cl_s(int_s(A_1, E))$  and  $int_s(\delta cl_s(A_2, E)) \subseteq \delta cl_s(int_s(A_2, E))$  such that  $int_s(\delta cl_s(A_1, E)) \cap int_s(\delta cl_s(A_2, E)) \subseteq int_s(\delta cl_s(A_1, E)) \cap int_s(A_2, E) \subseteq \delta cl_s(int_s(A_1, E)) \cap (A_2, E)$ . This means that  $int_s(\delta cl_s(A_1, E)) \cap int_s(A_2, E) \subseteq \delta cl_s(int_s(A_1, E)) \cap (A_2, E)$ . And hence the union of soft simply\* closed set is soft simply\* closed set. Similarly by 2.

**Remark 4.2:** For any soft topological space  $(X, \tilde{\tau}, E)$  the following statements are hold:

- The union and the intersection of a soft simply\* generalized open set need not soft simply\* generalized open set
- The union and the intersection of a soft simply\* generalized closed set need not soft simply\* generalized closed set

The following example show the above remark.

**Example 4.3; From example 3.1:** We have, Let  $(A, E) = \{(e_1\{a, c\})\}$ ,  $(G, E) = \{(e_2\{a, c\})\} \in SS^{M^*}GO(X, E)$  but  $(A, E) \bar{\cap}(G, E) = \{(e_1\{a, c\})\}$ ,  $\{(e_2\{a, c\})\} \notin SS^{M^*}GO(X, E)$  also  $(B, E) = \{(e_1\{a, c\})\}$ ,  $(e_2, X)$ ,  $(V, E) = \{(e_1, X), (e_2, \{a, c\})\} \in SS^{M^*}GO(X, E)$ , since,  $(B, E)$ .

Let  $(A, E) = \{(e_1\{b\})\}$ ,  $(e_1\{X\})$ ,  $(e_1\{X\})$ ,  $(e_2\{b\}) \in SS^{M^*}GC(X, E)$ . But  $(A, E) \bar{\cap}(G, E) = \{(e_1\{b\})\}$ ,  $(e_2\{b\}) \notin SS^{M^*}GC(X, E)$  also if  $(D, E) = \{(e_1\{b\})\}$ ,  $(V, E) = (e_2\{b\}) \in SS^{M^*}GC(X, E)$  but  $(A, E) \bar{\cap}(G, E)$ . From the above example, we have the following result.

**Theorem 4.3:** For a soft subset  $(A, E)$  of a soft topological space  $(X, \tilde{\tau}, E)$ . Then,  $(A, E)$  is a soft simply\* closed set if and only if  $int_s(\delta cl_s(A, E)) \bar{\subseteq} \delta cl_s(cl_s(A, E))$ .

**Proof:** Let  $(A, E)$  be a soft closed set, then  $(A, E)^c$  is a soft simply\* open set in soft in soft  $(X, \tilde{\tau}, E)$ . Thus,  $\delta int_s(cl_s(X-(A, E))) \bar{\subseteq} \delta cl_s(int_s(X-(A, E)))$ , this tends to  $\delta int_s(X-int_s(A, E)) \bar{\subseteq} cl_s(X-\delta cl_s(A, E))$ . This implies that  $X-int_s(\delta cl_s(A, E)) \bar{\subseteq} X-\delta cl_s(int_s(A, E))$ , thus,  $\delta cl_s(int_s(A, E)) \bar{\subseteq} int_s(\delta cl_s A, E)$ . Then  $(A, E)$  is soft simply\* closed set. Conversely, since,  $int_s(\delta cl_s(A, E)) \bar{\subseteq} \delta cl_s(A, E)$ . Then,  $cl_s(\delta int_s(A, E)) \bar{\cap} int_s(\delta cl_s(A, E)) \bar{\subseteq} \delta cl_s(A, E)$  this tends to  $\delta cl_s(int_s(A, E)) \bar{\subseteq} int_s(\delta cl_s(A, E))$ , therefore,  $(A, E)$  is soft simply\* closed set.

**Theorem 4.4:** For a soft subset  $(A, E)$  of a soft topological space  $(X, \tilde{\tau}, E)$  the following statements are equivalent:

- $(A, E) \in SS^{M^*}GO(X, E)$
- $(F, E) \in (A, E)^{SS^0}$ , whenever  $(F, E) \bar{\subseteq} (A, E)$  and  $(F, E) SC(X, E)$
- For each soft closed set  $(F, E)$ ,  $(F, E) \bar{\subseteq} (A, E)$  there exist soft simply\* open (briefly,  $SS^{M^*} \not\subseteq$  open) set  $(G, E)$ ,  $(G, E) \bar{\subseteq} X$  such that  $(F, E) \bar{\subseteq} (G, E) \bar{\subseteq} (A, E)$ .

**Proof:** (1)-(2) let  $(A, E) \in SS^{M^*}GO(X, E)$  and  $(F, E) \bar{\subseteq} (A, E)$  such that  $(F, E)$  is a soft closed set. Since,  $X-(A, E) \bar{\subseteq} X-(F, E)$  where  $X-(F, E)$  is a soft open set and  $(A, E) \in SS^{M^*}GC(X, E)$ . Then,  $X-(A, E) SS^+ \bar{\subseteq} X-(F, E)$ , we have  $(X-(A, E))^{SS^+} = X-(A, E)^{SS^0}$ , then  $X-(A, E)^{SS^0} \bar{\subseteq} X-(F, E)$ . Consequently,  $(F, E) \bar{\subseteq} (A, E)^{SS^0}$ .

(2)-(3) the proof is obvious if, we put  $(G, E) = (A, E)^{SS^0}$ . (3)-(1) Since,  $X-(F, E) \bar{\subseteq} X-(A, E)$ , since,  $X-(F, E)$  open set,  $X-(G, E) \in SS^{M^*}C(X, E)$  and  $X-(F, E) \bar{\subseteq} X-(G, E) \bar{\subseteq} X-(A, E)$  then,  $X-(F, E) \bar{\subseteq} (X-(A, E))^{SS^+}$ . This implies that  $X-(A, E) \in SS^{M^*}C(X, E)$ . Thus,  $(A, E) \in SS^{M^*}GO(X, E)$ .

**Theorem 4.5:** Let  $(X, \tilde{\tau}, E)$  be a soft topological space over  $X$  and  $(Y, E) \bar{\subseteq} X$  be a non-empty soft subset of  $X$  and  $(F, E)$  be a soft set over  $Y$ . If  $(Y, E)$  is soft open in  $X$  and  $(F, E)$   $SS^{M^*}$ g-closed set in  $(X, \tilde{\tau}, E)$ , then  $(F, E)$  is  $SS^{M^*}$ g-closed set relative to  $(Y, \tilde{\tau}_Y)$ .

**Proof:** Let  $(F, E) \bar{\subseteq} Y \bar{\cap}(G, E)$  and suppose that  $(G, E)$  is a soft simply\* open set in  $X$ . Then  $(F, E) \bar{\subseteq} (G, E)$  and hence,  $(F, E)^{SS^+} \bar{\subseteq} (G, E)$ . This implies that  $Y \bar{\cap}(F, E)^{SS^+} = (F, E)^{SS^+} \bar{\subseteq} Y \bar{\cap}(G, E)$ . Hence, we obtain  $(F, E)$  is a soft  $SS^{M^*}$ g-closed set relative to  $(Y, \tilde{\tau}_Y)$ .

**Theorem 4.6:** Let  $(X, \tilde{\tau}, E)$  be a soft topological space over  $X$  and  $(F, E)$  be a soft set over  $X$ . If a soft set  $(F, E)$  is a soft  $SS^{M^*}$ g-closed set in  $(X, \tilde{\tau}, E)$ , then  $(F, E)^{SS^+}-(F, E)$  contains only null a soft simply\* open set.

**Proof:** Suppose that  $(F, E)$  is soft  $SS^{M^*}$ g-closed. Let  $(H, E)$  be a soft simply\* of closed subset of  $(F, E)^{SS^+}-(F, E)$ . Then  $(H, E) \bar{\subseteq} (F, E)^{SS^+}-(F, E)^{SS^+}-(F, E)^c$  and so,  $(F, E) \bar{\subseteq} (H, E)^c$ . But  $(F, E)$  is a soft  $SS^{M^*}$ g-closed set. Therefore,  $(F, E)-(H, E)^c$ . Consequently  $(H, E) \bar{\subseteq} ((F, E)^{SS^+})^c$ . We have already  $(H, E) \bar{\subseteq} (F, E)^{SS^+}$ . From (1) and (2)  $(H, E) \bar{\subseteq} (F, E)^{SS^+} \bar{\cap} ((F, E)^{SS^+})^c = \emptyset$ . Thus,  $(H, E) = \emptyset$ . Therefore,  $(F, E)^{SS^+}-(F, E)$  contains only null soft regular closed set.

**Theorem 4.7:** Let  $(X, \tilde{\tau}, E)$  be a soft topological space over  $X$  and  $(F, E)$  be a soft set over  $X$ . If a soft set  $(F, E)$  is soft rg-closed set. Then  $(F, E)$  is soft  $SS^{M^*}$ g-closed set.

**Proof:** Suppose that  $(F, E) \bar{\subseteq} (G, E)$  where  $(G, E)$  is soft regular open. If  $(G, E)$  is a soft regular open, then  $(G, E)$  is a soft simply\* open. Thus,  $(F, E) \bar{\subseteq} (G, E)$  and  $(G, E)$  is a soft open. Since,  $(F, E)$  is a soft rg-closed then  $cl_s(F, E) \bar{\subseteq} (G, E)$ . And hence,  $(F, E)^{SS^+} \bar{\subseteq} (G, E)$ . Therefore,  $(G, E)$  is a soft  $SS^{M^*}$ g-closed set.

**Theorem 4.8:** Every soft  $\alpha$ -closed set is soft simply\* closed set.

**Proof:** Let  $(A, E)$  be soft  $\alpha$ -closed set in soft topological space  $(X, \tilde{\tau}, E)$ , then,  $\alpha cl_s(A, E) \bar{\subseteq} (A, E)$ , since,  $(A, E)^{SS^+} \bar{\subseteq} \alpha cl_s(A, E) \bar{\subseteq} (A, E)$ , hence,  $(A, E)^{SS^+} \bar{\subseteq} (A, E)$  but  $(A, E) \bar{\subseteq} (A, E)^{SS^+}$ , then, we have  $(A, E)^{SS^+} = (A, E)$ . Thus, a set  $(A, E)$  is a soft simply\* closed set.

**Theorem 4.9:** Every soft  $\alpha$  g-closed set is a soft  $SS^{M^*}$ g-closed set.

**Proof:** Let  $(F, E)$  be any soft  $\alpha$  g- closed set in  $X$  and  $(G, E)$  be any soft open set containing  $(F, A)$ . Since, is a soft  $\alpha$  g-closed, then  $\alpha cl_s(A, E) \subseteq (G, E)$ . But  $(A, E)^{SS^*} \subseteq \alpha cl_s(A, E)$ , for any soft set  $(A, E)$  in  $X$ , thus,  $(F, A)^{SS^*} \subseteq (G, E)$ . Hence  $(F, A)$  is soft  $SS^{M^*}$ g-closed set.

**Theorem 4.10:** Every soft rg  $\alpha$ -closed set is a soft  $SS^{M^*}$ g-closed set.

**Proof:** Let  $(F, A)$  be any soft rg  $\alpha$ -closed set in  $X$  and  $(G, B)$  be any soft regular open set containing  $(F, A)$ . Since each soft regular open set is soft open and  $(F, A)$  is a soft r g  $\alpha$ -closed then  $\alpha cl_s(F, A) \subseteq (G, B)$ . But  $(A, E)^{SS^*} \subseteq \alpha cl_s(A, E)$ , for any soft set  $(A, E)$  in  $X$ , thus,  $(A, E)^{SS^*} \subseteq (G, E)$ . Hence  $(F, A)$  is a soft  $SS^{M^*}$ g-closed set.

**Theorem 4.11:** Every soft g b-closed set is a soft  $SS^{M^*}$ g-closed set.

**Proof:** Let  $(F, A)$  be any soft g b-closed set in  $X$  and  $(G, B)$  be any soft open set containing  $(F, A)$ . Since,  $(F, A)$  is soft g b-closed, then  $bcl_s(F, A) \subseteq (G, E)$ . But  $(A, E)^{SS^*} \subseteq bcl_s(F, A)$  for any soft set  $(A, E)$  in  $X$ , thus,  $(A, E)^{SS^*} \subseteq bcl_s(F, A) \subseteq (G, E)$ . Hence,  $(F, A)$  is a soft  $SS^{M^*}$ g-closed set.

**Theorem 4.12:** Let  $(X, \tilde{\tau}, E)$  be a soft topological space over  $X$  and  $(F, E), (G, E)$  are a soft sets over  $X$ . If a soft set  $(F, E)$  is soft  $SS^{M^*}$ g-closed set  $(F, E) \subseteq (G, E) \subseteq (F, E)^{SS^*}$ , then  $(F, E)^{SS^*} - (F, E)$  contains only null soft simply\* closed set.

**Proof:** If  $(F, E) \subseteq (G, E)$ , then  $(G, E)^c \subseteq (F, E)^c$  (1). If  $(G, E) \subseteq (F, E)$ , then  $(G, E)^{SS^*} \subseteq ((F, E)^{SS^*})^{SS^*} = (F, E)^{SS^*}$  (2), that is  $(G, E)^{SS^*} \subseteq (F, E)^{SS^*}$ . From (1) and (2), we get  $(G, E)^{SS^*} \cap (G, E)^c \subseteq (F, E)^{SS^*} \cap (F, E)^c$  which implies  $(G, E)^{SS^*} - (G, E)^c \subseteq (F, E)^{SS^*} \cap (F, E)^c$ . Now  $(F, E)$  is a soft  $SS^{M^*}$ g-closed. Hence,  $(F, E)^{SS^*} - (F, E)^c$  contains only null a soft simply\* closed, neither does  $(G, E)^{SS^*} - (G, E)^c \subseteq (F, E)^{SS^*} - (F, E)^c$ .

**Theorem 4.13:** Every soft  $\alpha$ -closed set is a soft simply\* generalized closed set.

**Proof:** Let  $(A, E)$  be soft  $\alpha$ -closed set in soft topological space  $(X, \tilde{\tau}, E)$ , then  $\alpha cl_s(A, E) = (A, E)$  such that  $(A, E) \subseteq (U, E)$  where  $(U, E)$  is soft open set in soft topological space  $(X, \tilde{\tau}, E)$ , since,  $(A, E)^{SS^*} \subseteq \alpha cl_s(A, E) = (A, E)$ ,

hence,  $(A, E)^{SS^*} \subseteq (A, E)$  but  $(A, E) \subseteq (U, E)$  then, we have  $(A, E)^{SS^*} \subseteq (A, E) \subseteq (U, E)$ . Thus, a set  $(A, E)$  is a soft simply\* generalized closed set.

**Theorem 4.14:** Every soft generalized closed set is a soft simply\* generalized closed set.

**Proof:** Let  $(A, E)$  be soft generalized closed set in soft topological space  $(X, \tilde{\tau}, E)$  such that  $(A, E) \subseteq (U, E), (U, E)$  is soft open set in  $(X, \tilde{\tau}, E)$  then  $\alpha cl_s(A, E) \subseteq (U, E)$  and  $(A, E)^{SS^*} \subseteq \alpha cl_s(A, E) \subseteq (U, E)$ . Hence  $(A, E)^{SS^*} \subseteq (A, E)$ . Then the soft set  $(A, E)$  is soft simply\* generalized closed set.

**Theorem 4.15:** Every soft generalized  $\alpha$ -closed set is a soft simply\* generalized closed set.

**Proof:** Let  $(A, E)$  be soft generalized  $\alpha$ -closed set in soft topological space  $(X, \tilde{\tau}, E)$  such that  $(A, E) \subseteq (U, E), (U, E)$  is a soft open set in  $(X, \tilde{\tau}, E)$  then  $\alpha cl_s(A, E) \subseteq (U, E)$  and  $(A, E)^{SS^*} \subseteq \alpha cl_s(A, E) \subseteq (U, E)$ . Hence,  $(A, E)$  is soft simply\* generalized closed set.

**Theorem 4.16:** Every soft regular generalized b-closed set is a soft simply\* generalized closed set.

**Proof:** Let  $(A, E)$  be soft generalized b-closed set in soft topological space  $(X, \tilde{\tau}, E)$  such that  $(A, E) \subseteq (U, E)$  and  $(U, E)$  is a soft regular open set in  $(X, \tilde{\tau}, E)$  then  $bcl_s(A, E) \subseteq (U, E)$  and but  $(A, E)^{SS^*} \subseteq bcl_s(A, E)$  for any soft set in  $X$  and hence,  $(A, E)^{SS^*} \subseteq (U, E)$ . Hence,  $(A, E)$  is soft simply\* generalized closed set.

**Theorem 4.17:** In a soft topological space  $(X, \tilde{\tau}, E)$ ,  $\tilde{\tau} = \tilde{\tau}_1$  if and only if every soft set over  $X$  is a soft  $SS^{M^*}$ g-closed set.

**Proof:** Suppose that  $\tilde{\tau} = \tilde{\tau}_1$  and that  $(F, E) \subseteq (G, E), (G, E) \in \tilde{\tau}$ . Then  $(F, E)^{SS^*} \subseteq (G, E)^{SS^*} = (G, E)$  and  $(F, E)$  is a soft  $SS^{M^*}$ g-closed. Conversely, suppose that every soft set over  $X$  is soft  $SS^{M^*}$ g-closed. Let  $(G, E) \in \tilde{\tau}$ , then, since,  $(G, E) \subseteq (G, E)$  and  $(G, E)$  is a soft  $SS^{M^*}$ g-closed we have  $(G, E)^{SS^*} \subseteq (G, E)$ ,  $(F, E)^{SS^*} \subseteq (G, E)$  and  $(G, E) \in \tilde{\tau}$ . Thus,  $\tilde{\tau}_1 \subseteq \tilde{\tau}$ . If  $(H, E) \in \tilde{\tau}_1$ , then  $(H, E)^c \in \tilde{\tau} \subseteq \tilde{\tau}_1$  and hence,  $(H, E)^c \in \tilde{\tau}$ . Finally,  $\tilde{\tau} = \tilde{\tau}_1$ .

**Theorem 4.18:** A soft set  $(F, E)$  is a soft  $SS^{M^*}$ g-closed if and only if  $(F, E)^{SS^*} - (F, E)$  is soft  $SS^{M^*}$ g-open.

**Proof:**  $\Rightarrow$  Suppose that  $(F, E)$  is a soft  $SS^{M^*}$ -g-closed and  $(H, E) \prec (F, E)^{SS^*} \prec (F, E)$ ,  $(H, E)$ , is a soft closed. By theorem 4.6  $(H, E) = \varphi$  and hence,  $(H, E) \subseteq (F, E)^{SS^*} \prec (F, E)^{SS^*0}$ . By theorem 4.12,  $(F, E)^{SS^*} \prec (F, E)$  is a soft  $SS^{M^*}$ -g-open. Conversely, suppose that  $(F, E) \subseteq (G, E)$  where  $(G, E)$  is a soft open set. Now  $(F, E)^{SS^*} \cap (G, E)^c \subseteq (F, E)^{SS^*} \cap (F, E)^c = (F, E)^{SS^*} \prec (F, E)$  and since,  $(F, E)^{SS^*} \cap (G, E)^c$  is a soft closed and  $(F, E)^{SS^*} \prec (F, E)^c$  is a soft  $SS^{M^*}$ -g-open it follows that  $(F, E)^{SS^*} \cap (G, E)^c \subseteq (F, E)^{SS^*} \prec (F, E)^c = \varphi$ . Therefore,  $(F, E)^{SS^*} \cap (G, E)^c = \varphi$  or  $(F, E)^{SS^*} = (G, E)$ . Thus, we get  $(F, E)$  is a soft  $SS^{M^*}$ -g-closed.

**Theorem 4.19:** Every soft semi closed and soft simply\* closed set is a soft simply\* generalized closed set.

**Proof:** Let  $(A, E)$  be soft semi closed in soft topological space  $(X, \tilde{\tau}, E)$ . Then,  $(A, E)^{SS^*} \subseteq (A, E)$ ,  $(A, E) \subseteq (U, E)$  where  $(U, E)$  is soft open set such that, since,  $(A, E)^{SS^*} \subseteq \text{int}(\text{cl}(A, E)) \subseteq (A, E)^{SS^*} \subseteq (A, E)$ . Thus,  $(A, E)^{SS^*} \subseteq (U, E)$  therefore,  $(A, E)$  is soft simply\*generalized closed set.

### RESULTS AND DISCUSSION

**Soft simply\* generalized continuous and soft simply\* generalized irresolute functions:** In this study, we introduce new types of soft continuous functions, called soft simply\* generalized continuous (briefly,  $SS^{M^*}$ -generalized) and soft simply\* generalized irresolute (briefly,  $SS^{M^*}$ -generalized irresolute) functions and study some their properties.

**Definition 5.1:** A soft function  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, E)$  is called. Soft simply\* generalized continuous (briefly,  $SS^{M^*}$ -g-continuous) if for every  $(G, E) \in (Y, \tilde{\sigma}, E)$ , then  $f^{-1}(G, E) \in SS^{M^*}GO(X, E)$ , then. Soft simply\* generalized irresolute (briefly,  $SS^{M^*}$ -g-irresolute) if for every  $(G, E) \in SS^{M^*}GO(Y, E)$ , then  $f^{-1}(G, E) \in SS^{M^*}GO(X, E)$ . Soft simply\* continuous (briefly,  $SS^{M^*}$ -continuous) if for every  $(G, E) \in (Y, \tilde{\sigma}, E)$ , then  $f^{-1}(G, E) \in SS^{M^*}O(X, E)$ .

**Remark 5.1:** The following diagram shows the relation between the new class of functions and the other types of continuous mappings which have been in definition (Fig. 3). In Fig. 2, the implications are not reversible as the following example.

**Example 5.1; Implications 1-3:** Let  $X = \{a, b, c\}$ ,  $Y = \{X, y, z\}$ ,  $E = \{e_1, e_2\}$ ,  $K = \{k_1, k_2\}$  and let  $(X, \tilde{\tau}, E)$  and  $(Y, \tilde{\sigma}, E)$  be two soft topological spaces where,  $\tilde{\tau} = \{\tilde{x}, \tilde{\varphi}(e_1,$

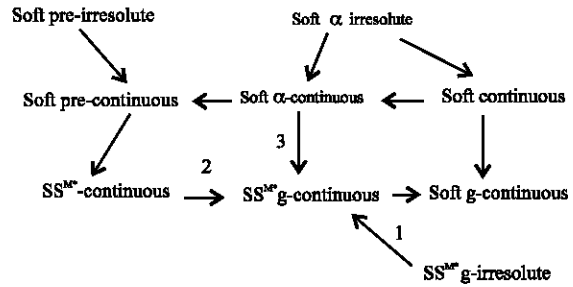


Fig. 3: Relation between the new class of functions

$\{a\}$ ),  $(e_1, \{a\}), (e_1, \{a, b\}), (e_2, \{a, b\})$  represents the soft topology and  $(Y, \tilde{\sigma}, E)$  is defined by  $\tilde{\sigma} = \{\tilde{Y}, \tilde{\varphi}(\{k_1, \{Y\}\}, \{k_2, \{Y\}\})\}$ . Define the soft functions  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, E)$  and  $p: E \rightarrow K$  as follows:  $f(a) = a, f(b) = b, f(c) = c$  and  $p(e_1) = k_1, p(e_2) = k_2$  assume  $(G, K) = \{\{k_1, \{Y\}\}, \{k_2, \{Y\}\}\}$ . Then,  $(G, K)$  is a soft open set in  $(Y, \tilde{\sigma}, E)$  and it is clear that  $f^{-1}(G, K) = \{(e_1, \{c\}), (e_2, \{c\})\}$  is a soft simply\* generalized open set but it is not neither soft  $\alpha$ -open set and nor soft simply\* open set. Therefore, the function  $f$  is soft simply\* generalized continuous function but it is not neither soft  $\alpha$ -continuous function nor  $SS^{M^*}$ -continuous function and also but it is not  $SS^{M^*}$ -g-irresolute function, since, there exist a soft simply\* generalized open set  $(V, K) = \{\{k_1, \{z\}\}, \{k_2, \{z\}\}\}$  but  $f^{-1}(V, K) = \{(e_1, \{c\}), (e_2, \{a\})\} \notin SS^{M^*}O(X, E)$ .

**Remark 5.2:** The composition of two  $SS^{M^*}$ -g-continuous functions need not be  $SS^{M^*}$ -g-continuous as shown by the following example:

**Example 5.2:** Let,  $X = \{a_1, b_1, c_1\}$ ,  $Y = \{a, b, c\}$ ,  $Z = \{i, j, k\}$ ,  $E = \{e_1, e_2\}$ ,  $\tilde{\tau} = \{\tilde{x}, \tilde{\varphi}(\{e_1, \{a_1, b_1\}\}, \{e_2, \{a_1, b_1\}\}), \{(e_1, \{b_1\}), (e_2, \{a_1, c_1\})\}, \{(e_1, \{b_1\}), (e_2, \{a_1\})\}, \{(e_1, \{a_1, b_1\}), (e_2, \{X\})\}, \{(e_1, \{X\}), (e_2, \{a_1, b_1\})\}, \{(e_1, \{b_1, c_1\}), (e_2, \{a_1, c_1\})\}, \tilde{\sigma} = \{\tilde{Y}, \tilde{\varphi}(\{e_1, \{a\}\}, \{e_2, \{a\}\}), \{(e_1, \{b\}), (e_2, \{b\})\}, \{(e_1, \{a, b\}), (e_2, \{b\})\}, \tilde{\gamma} = \{\tilde{Z}, \tilde{\varphi}(\{e_1, \{I\}\}, \{e_2, \{k\}\})\}$ , Let  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, E)$  and  $g: (Y, \tilde{\sigma}, E) \rightarrow (Z, \tilde{\gamma}, E)$  are defined as  $f(a_1) = a, f(b_1) = b, f(c_1) = c$  and  $g(a) = i, g(b) = j, g(c) = k$ , since,  $f$  and  $g$  are  $SS^{M^*}$ -g-continuous but  $g \circ f$  is not  $SS^{M^*}$ -g-continuous, since, there exist a soft open set  $(A, E) = \{(e_1, \{I\}), (e_2, \{k\})\}$  is a soft open set but  $f^{-1}(A, E) = \{(e_1, \{a\}), (e_2, \{a\})\} \notin SS^{M^*}GO(X, E)$ .

**Theorem 5.1:** A soft function  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\sigma}, E)$  is a  $SS^{M^*}$ -g-continuous if and only if  $f^{-1}(G, E) \in SS^{M^*}GC(X, E)$  for every  $(G, E) \in SC(Y, \tilde{\sigma}, E)$  where  $SC(Y, \tilde{\sigma}, E)$  is the set of all soft closed sets of  $Y$ .

**Proof:** Obvious.

**Theorem 5.2:** A soft function  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\alpha}, E)$  is a  $SS^{M^*}$ -g-continuous function if and only if  $f^{-1}(G, E) \in SS^{M^*}GO(X, E)$  for every  $(G, E) \in SO(Y, \tilde{\alpha}, E)$ .

**Proof:** Obvious from the definition.

**Theorem 5.3:** A soft function  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\alpha}, E)$  is a  $SS^{M^*}$ -g-irresolute if and only if  $f^{-1}(G, E) \in SS^{M^*}GC(X, E)$  for every  $f^{-1}(G, E) \in SS^{M^*}GC(Y, E)$ .

**Proof:** Since,  $f$  is  $SS^{M^*}$ -g-irresolute every  $(G, E) \in SS^{M^*}GC(Y, E)$ , then  $Y-(G, E) \in SS^{M^*}GO(Y, E)$ , thus,  $f^{-1}(Y-(G, E)) = X-f^{-1}(G, E)$  is a  $SS^{M^*}GO(X, E)$ . Conversely, let  $(F, E) \in SS^{M^*}GO(Y, E)$  then  $Y-(F, E) \in SS^{M^*}GC(Y, E)$  then  $f^{-1}(Y-(F, E)) = X-f^{-1}(F, E) \in SS^{M^*}GC(X, E)$ , then  $f^{-1}(F, E) \in SS^{M^*}GO(X, E)$ . Then, a soft function  $f$  is a  $SS^{M^*}$ -g-irresolute function.

**Theorem 5.4:** Let  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\alpha}, E)$  and  $g: (X, \tilde{\tau}, E) \rightarrow (Z, \tilde{\gamma}, E)$  are  $SS^{M^*}$ -g-irresolute functions. Then,  $Gof: (X, \tilde{\tau}, E) \rightarrow (Z, \tilde{\gamma}, E)$  is  $SS^{M^*}$ -g-irresolute function.

**Proof:** Let  $(A, E) \in SS^{M^*}GO(Z, E)$ , since,  $(X, \tilde{\tau}, E) \rightarrow (Z, \tilde{\gamma}, E)$ ,  $g: (X, \tilde{\tau}, E) \rightarrow (Z, \tilde{\gamma}, E)$ ,  $SS^{M^*}$ -g-irresolute. Then,  $g^{-1}(A, E) \in SS^{M^*}GO(Y, E)$ , since,  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\alpha}, E)$  is a  $SS^{M^*}$ -g-irresolute then  $f^{-1}(g^{-1}(A, E)) = (gof)^{-1}(A, E) \in SS^{M^*}GO(X, E)$  and hence,  $gof$  is a  $SS^{M^*}$ -g-irresolute function.

**Theorem 5.5:** Let  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\alpha}, E)$  is a  $SS^{M^*}$ -g-irresolute and  $g: (X, \tilde{\tau}, E) \rightarrow (Z, \tilde{\gamma}, E)$  is a  $SS^{M^*}$ -g-continuous then, the composition then  $gof: (X, \tilde{\tau}, E) \rightarrow (Z, \tilde{\gamma}, E)$  is a  $SS^{M^*}$ -g-continuous function.

**Proof:** Let  $(A, E)$  be soft open set in  $(Z, \tilde{\gamma}, E)$ , since,  $g$  is a  $SS^{M^*}$ -g-continuous function, then  $g^{-1}(A, E) \in SS^{M^*}GO(Y, E)$ . Since,  $f$  is a  $SS^{M^*}$ -g-irresolute function this implies that  $f^{-1}(g^{-1}(A, E)) = (gof)^{-1}(A, E) \in SS^{M^*}GO(X, E)$  consequently  $gof$  is a  $SS^{M^*}$ -g-continuous function.

**Theorem 5.6:** Let  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\alpha}, E)$  is a  $SS^{M^*}$ -g-continuous and  $g: (X, \tilde{\tau}, E) \rightarrow (Z, \tilde{\gamma}, E)$  is a  $SS^{M^*}$ -continuous then the composition then  $gof: (X, \tilde{\tau}, E) \rightarrow (Z, \tilde{\gamma}, E)$  is need not a  $SS^{M^*}$ -g-continuous function.

**Proof:** Obvious.

**Theorem 5.7:** Every soft simply\* continuous is soft simply\*generalized continuous.

**Proof:** Let  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\alpha}, E)$  be soft simply\* continuous and  $(V, E)$  is soft closed set in  $(Y, E)$ . Since,  $f$  is a soft continuous, then  $f^{-1}(V, E)$  is a soft closed set in  $(X, E)$ . And since, every soft closed set is soft simply\* generalized closed set, thus  $f^{-1}(V, E)$  is a soft simply\* generalized closed set in  $(X, E)$ . Hence,  $f$  is soft simply\* generalized continuous.

**Theorem 5.8:** Soft continuous function  $f: (X, \tilde{\tau}, E) \rightarrow (Y, \tilde{\alpha}, E)$  is soft simply\* generalized continuous function if  $SS^{M^*}GO(X, E) = SS^{M^*}GC(X, E)$ .

**Proof:** Let  $(A, E)$  be a soft open set in  $(Y, E)$ , since,  $f$  is soft continuous function. Then  $f^{-1}(A, E)$  is soft open set in  $(X, \tilde{\tau}, E)$ , since, every open set is soft simply\* generalized closed set in  $(X, E)$ . Therefore,  $f^{-1}(A, E)$  is soft simply\* generalized closed set in  $(X, E)$  and hence,  $f$  is a simply\* generalized continuous function.

### CONCLUSION

This study shows the soft simply\* generalized continuous mapping in soft topological space and some of their properties.

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