

The use of a Combined Parameterized Model for Analysis of Elastic Scattering Processes of ${}^4\text{He}+{}^{56}\text{Fe}$ and ${}^4\text{He}+{}^{58}\text{Fe}$ at Energy 25 MeV

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Abstract: The analysis of angular distributions of two elastic scattering processes ${}^4\text{He}+{}^{56}\text{Fe}$ and ${}^4\text{He}+{}^{58}\text{Fe}$ at energy 25 MeV, using Regge pole model based on strong absorption phenomenon the parameterized scattering matrix of McIntyre Model and Regge pole model is used to reproduce the cross-sections of the experimental data for the two elastic scattering processes at forward angles and backward angles. For backward angles the parameterized scattering matrix is modified by adding Regge pole factor in another combined model in order to account for reproducing the cross sections.

Key words: Heavy-ion scattering, strong absorption model, McIntyre, Regge pole, distributions, backward

INTRODUCTION

The study of elastic scattering provides a practical tool that allows a good understanding of the ‘global’ properties of a heavy-ion interaction. In particular, the knowledge of interaction radii, critical angular momenta, surface diffuseness’s were found successful to analyze the experimental data of angular distribution for elastic scattering of heavy nuclei. Based on the partial wave expansion model in the presence of strong absorption where for strongly absorbed nuclei, the S-matrix which represents the accumulative effect of the scattering potential on the incident wave, may be determined experimentally by a partial wave fitting to the measured angular distribution of elastic scattering. It was found that, the parameterized S-matrix is a source of important information which is needed in the theoretical calculations of transfer reactions based on Strong Absorption Model (SAM) (Badran *et al.*, 2015; Badran and Al-Masri, 2013; Badran and Istiti, 2017 and Badran *et al.*, 2010).

This research presents the numerical analysis of angular distributions of two elastic scattering processes ${}^4\text{He}+{}^{58}\text{Fe}$ and ${}^4\text{He}+{}^{56}\text{Fe}$ (at laboratory energies 25 MeV). The Regge pole factor will be introduced into the partial wave expansion formula of the scattering amplitude using the parameterized S-matrix as a background (Istiti, 2018).

MATERIALS AND METHODS

Theory: The amplitude $f(\theta)$ as a function of partial wave expansion for elastic scattering expressed by:

$$f(\theta) = \frac{1}{i2k} \sum_{l=0}^{\infty} (2l+1)[S_l - 1]P_l(\cos\theta) \quad (1)$$

Where:

- S_l = The complex amplitude of the l th scattered partial wave
- $P_l(\cos\theta)$ = The Legendre polynomial of order l
- k = The wave number

The scattering matrix amplitude is express by:

$$S_l = \eta_l e^{2i\delta_l} \quad (2)$$

The Coulomb phase shifts are given by the exact solution of the Rutherford scattering problem:

$$\sigma_l = \arg \Gamma(1 + i\eta_l) \quad (3)$$

The semi-classical strong absorption model has the sharp cutoff conditions (Badran *et al.*, 2015; Badran and Istiti, 2017):

$$\begin{aligned} \eta_l &= 0 & S_l &= 0 & \text{if } l \leq l_g \\ \eta_l &= 1 & S_l &= e^{(2i\sigma_l)} & \text{if } l > l_g \end{aligned} \quad (4)$$

Here, η_l is called the reflection coefficient of the outgoing l th partial wave determined by the boundary conditions at the nuclear surface. This means that waves up to the grazing angular momentum l_g are completely absorbed. The sum of Radii R of the projectile R_p and target nuclei R_T . This strong interaction radius has been defined:

$$R = r_0 (A_p^{1/3} + A_T^{1/3}) \quad (5)$$

Regge-pole factor expressed as:

$$S(l) = \left[1 + e^{-i\alpha} e^{(l\epsilon^{-1})/\Delta} \right]^{-1} \left[1 + \frac{l-l_0 - iz(l)}{l-l_0 - ip(l)/2} \right]$$

or

$$S(l) = \left[1 + e^{-i\alpha} e^{(l\epsilon^{-1})/\Delta} \right]^{-1} \left[1 + \frac{iD(l)}{l-l_0 - i\Gamma(l)/2} \right]$$

The $z(l)$ and $p(l)$ functions, in equation, represent the Regge zero and Regge pole at a complex l (l). Where amplitude of the pole is:

$$D(l) = D_0 [1 - \text{Re}S_l(\text{BG})] \tag{7}$$

and the width of the pole is:

$$\Gamma(l) = \Gamma_0 [1 - \text{Re}S_l(\text{BG})] \tag{8}$$

The phase of the pole is represented by ϕ_0 (Badran *et al.*, 2015; Badran and Al-Masri, 2013; Badran and Istiti, 2017; Badran *et al.*, 2010). We use Fortran code to obtain the best fit of experiment data of angular distribution and the final choice of extracted parameters depend on the value of χ^2 for each model is. The χ^2 is defined by the expression:

$$\chi^2 = \frac{1}{N} \left(\sum_i^N \left[\frac{(\sigma_{\text{theory}}^i - \sigma_{\text{exp}}^i)}{(\Delta\sigma_{\text{exp}}^i)} \right]^2 \right)$$

Here, σ_{theory}^i , σ_{exp}^i and $\Delta\sigma_{\text{exp}}^i$ are theoretical, experimental cross section and the corresponding error in cross section, respectively, N is the No. of experimental data points. However, the average value of 10% of the experimental measurement is taken for each experimental error of the energy of the elastic scattering under study (Badran *et al.*, 2015; Badran *et al.*, 2010).

RESULTS AND DISCUSSION

The available experimental data of scattering ^4He by different target nuclei ^{58}Fe and ^{56}Fe are analyzed using, Regge pole model by fitting the calculated results to the experimental data. We use Fortran code to obtain the best fit of experiment data of angular distribution and the final choice of extracted parameters depend on the value of χ^2 for each model is. The χ^2 is defined by the expression (Table 1 and Fig. 1):

$$\chi^2 = \frac{1}{N} \left(\sum_i^N \left[\frac{(\sigma_{\text{theory}}^i - \sigma_{\text{exp}}^i)}{(\Delta\sigma_{\text{exp}}^i)} \right]^2 \right)$$

Here, σ_{theory}^i , σ_{exp}^i and $\Delta\sigma_{\text{exp}}^i$ are theoretical, experimental cross section and the corresponding error in cross section, respectively, N is the No. of experimental data

Table 1: List of parameters for elastic scattering of α particles by the target nucleus $^{58,56}\text{Fe}$ at laboratory energies 25 MeV which are extracted from the analyses using McIntyre plus Regge pole model. The total reaction cross-sections σ_r (FM) and σ_r (GFM) are obtained from FM and GFM, respectively

Elastic scattering	$^4\text{He}+^{56}\text{Fe}$	$^4\text{He}+^{58}\text{Fe}$
E_{Lab} (MeV)	25	25
r_0 (fm)	1.43	1.46
μ (Rad)	0.295	0.29
d (fm)	0.70	0.55
l_0	10	13
ϕ_{l_0} ($^\circ$)	1	10
D_0	5	1
Γ_0	5	10
l_g	12	12
Δ	0.631	0.592
R (fm)	7.74	7.96
D/R	3.74×10^{-2}	3.50×10^{-2}
p	6.62	6.27
n	3.27	3.276
h	11.90	11.56
θ_R (Rad)	2.9×10^1	2.80×10^1
θ_{Nuc1} (Rad)	-5.70×10^1	-4.6×10^1
σ_r (mb) [GFM]	119.54	118.61
σ_r (mb) [FM]	127.28	126.76
χ^2	0.014	0.012

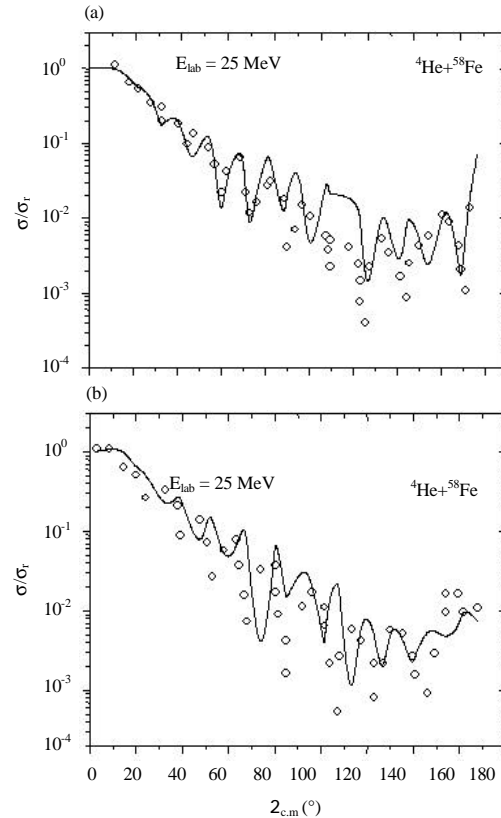


Fig. 1: The experimental data (symbols) of angular distribution of elastic scattering of α particles by the target nucleus $^{58,56}\text{Fe}$ at laboratory energies 25 MeV are shown and compared to the theoretical results obtained using the combined model of McIntyre plus Regge (solid line)

data points. However, the average value of 10% of the experimental measurement is taken for each experimental error of the energy of the elastic scattering under study (Badran *et al.*, 2015; Badran *et al.*, 2010).

Elastic scattering of ${}^4\text{He}+{}^{56,58}\text{Fe}$ target nuclei: McIntyre plus Regge pole are used to analyze the experimental data of angular distribution for elastic scattering of α particle by different target nuclei ${}^{56}\text{Fe}$, ${}^{58}\text{Fe}$, at energy 25 MeV (Badran *et al.*, 2015; Badran and Al-Masri, 2013; Badran and Istaiti, 2017; Badran *et al.*, 2010 and Istaiti, 2018).

CONCLUSION

We have analyzed the elastic scattering α particle by target nuclei ${}^4\text{He}+{}^{56}\text{Fe}$ and ${}^4\text{He}+{}^{58}\text{Fe}$ (at laboratory energies 25 MeV) using combined model of McIntyre and Regge Model, There are quantities determined type of diffraction pattern in the angular distribution; the diffraction parameter kR (l_0 the angular momentum) and (the strength of the Coulomb interactions which represents by Sommerfeld parameter n , The radius interaction region R is increasing when the atomic mass of target nucleus is increased with fix target the better formula of ($R = r_0 A_T^{1/3} + r_w$), The diffusivity parameter d , the parameters (l_0 , ϕ_0 , D_0 and Γ_0) used in backward angle has been obtained, l_0 the orbital angular momentum which represent the location for the pole, the D_0 amplitude and the Γ_0 width exhibit the similar behavior the width of the pole, ϕ_0 which estimate the phase angel determined the size of oscillation (Istaiti, 2018).

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