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Analysis of Fuzzy Priority Queue with Two Classes of Customers and Unequal Service Rates using Mixed Integer Non Linear Programming Approach

[1]B. Kalpana and [2]N. Anusheela
[1]Research and Development Centre, Bharathiar University, 641 046 Coimbatore, Tamil Nadu, India
[2]Department of Mathematics, Government Arts College Ooty, 643 002 Tamil Nadu, India

Abstract: In this study membership function of the steady state system performance measures of priority queues with two classes of customers is constructed using mixed integer non linear programming approach. Here, type 1 are considered to have priority over the type 2 customers. In this system with non preemptive priority, a low priority customer in service is not interrupted upon arrival of a higher priority customer at a queuing system and they are served with different service rates. Here, the arrival rate and service rate are considered to be fuzzy. The basic idea of this study is to use Zadeh's extension principle, α-cut approach and mixed integer non linear programming problem to construct the lower and the upper bound of the system performance measures. Using α-cut approach the fuzzy queues are transformed into crisp queues. For different values of α the membership function is constructed. The defuzzification of the performance measures is provided by Yager's ranking index. A numerical example is solved to demonstrate the validity of given model.

Key words: Mixed integer non linear programming, membership function, non preemptive priority queue, performance measures, priority queue, steady state

INTRODUCTION

Queueing models have a wide application in many organizations and one of such application area is real life situations having a policy of two class service channels. Priority queue are generally applied in many practical situations such as communication systems and computer networks. In crisp environments, many articles on this topic have been published (Miller, 1981; Gail *et al*., 1988; Reddy *et al*., 1993; Kao and Narayanan, 1990; Lee, 2001; Stanford, 1991; Takine, 1996). Mainly in the traditional queueing theory the arrival times and service times follow probability distributions. Priority discipline queuing models are of two types preemptive and non preemptive. In the system with preemptive priorities a customer being served (low priority) is sent back to the queuing area whenever a higher priority customer arrives at a system. The customer reenters the service where it is left off when there are no high priority customers. In the system with non preemptive priorities a customer being served is not interrupted, if a higher priority customer arrives at a queuing system and is completely served once the service is started.

On the other hand, the conversion of fuzzy queues to crisp queues has also been extensively discussed in literature and number of methods and approaches has been put in use. One of such methods is the zero one mixed integer programming approach. The researchers adopted this method with single channel priority queuing models. Few articles are published on fuzzy queues. Based on Zadeh's extension principle (Yager, 1986; Zadeh, 1978; Zimmermann, 2001) studies are analysed by the researchers. Fuzzy markov chain is discussed by Stanford (1982). A general approach for analyzing two fuzzy queues namely M/F/1 and FM/FM/1 have been proposed by Li and Lee (1989). Negi and Lee (1992) used α-cut and two variable simulation in analyzing fuzzy queues. Kao and Wilson (1999) and Kao *et al*. (1999) proposed a parametric programming approach in constructing the membership function. Chen (2005, 2006) used the same approach in analyzing FM/FM/1/N and FM/FM/1/∞ fuzzy systems. Previous studies have not considered two classes. This study adopts zero one mixed integer programming approach in solving fuzzy priority queue with two classes of customers and unequal service rates.

MATERIALS AND METHODS

Model description

Non preemptive priority queue with different service rates: Consider a non preemptive priority system with single server in which two types of customers arrive to receive their service. The inter arrival time of the customers are exponentially distributed with rate λ_1 and λ_2, respectively. Inter arrival times are assumed to be independent. The service times are independent and identically distributed according to an exponential

Corresponding Author: B. Kalpana, Department of Mathematics, CSI College of Engineering, 643 215 Ooty, Tamil Nadu, India

distribution with different service rates μ_1, μ_2, respectively. Customers are served according to FCFS basis within a priority class. In this model a low priority customer's service is not interrupted by the high priority customers. From the traditional queuing theory, we have:

$$Lq_1 = \frac{\lambda_1\left(\frac{\lambda_1}{\mu_1^2}+\frac{\lambda_1}{\mu_2^2}\right)}{(1-\rho_1)}$$

$$Lq_2 = \frac{\lambda_2\left(\frac{\rho_1}{\mu_1}+\frac{\rho_1}{\mu_2}\right)}{(1-\rho_1)(1-\rho)}$$

Where:

$\rho = \rho_1+\rho_2$

$\rho_1 = \frac{\lambda_1}{\mu_1}$

$\rho_2 = \frac{\lambda_2}{\mu_2}$

In the steady state it is necessary that, we have $\rho = \rho_1+\rho_2<1$.

Fuzzy non preemptive priority queue with different service rates: Consider a queuing system in which customers arrive according to a non preemptive priority with arrival rate $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$, respectively where $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$ are fuzzy numbers and service rates are exponentially distributed with fuzzy service rate $\tilde{\mu}_1$ and $\tilde{\mu}_2$. In this model the arrival rate and service rate are approximately known and are represented by convex fuzzy sets. A fuzzy set $\tilde{A}$ in its universal set X is convex if $\mu_{\tilde{A}}(\phi x_1+(1-\phi)x_2)\geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ where, $\mu_{\tilde{A}}$ is its membership function, $\phi\in[0, 1]$ and x_1, $x_2\in X$. Let, $\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}_2}(x_2), \eta_{\tilde{\mu}_1}(y_1), \eta_{\tilde{\mu}_2}(y_2)$ denote the membership functions of the arrival rate and service rate, respectively. Then, we have:

$$\tilde{\lambda}_1 = \left\{\left(x_1, \eta_{\tilde{\lambda}_1}(x_1)\right)|x_1\in S(\tilde{\lambda}_1)\right\}$$

$$\tilde{\lambda}_2 = \left\{\left(x_2, \eta_{\tilde{\lambda}_2}(x_2)\right)|x_2\in S(\tilde{\lambda}_2)\right\}$$

$$\tilde{\mu}_1 = \left\{\left(y_1, \eta_{\tilde{\mu}_1}(y_1)\right)|y_1\in S(\tilde{\mu}_1)\right\}$$

$$\tilde{\mu}_2 = \left\{\left(y_2, \eta_{\tilde{\mu}_2}(y_2)\right)|y_2\in S(\tilde{\mu}_2)\right\}$$

where, $s(\tilde{\lambda}_1), s(\tilde{\lambda}_2), s(\tilde{\mu}_1), s(\tilde{\mu}_2)$ are the supports of $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_1, \tilde{\mu}_2$ denote the universal sets of the arrival rates and service rates, respectively. Let, $p(x_1, x_2, y_1, y_2)$ denote the system performance measure of interest. When $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_1, \tilde{\mu}_2$ are fuzzy numbers, $p(\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_1, \tilde{\mu}_2)$ will be fuzzy as well. On the basis of Zadeh's extension principle (Zadeh, 1978; Yager, 1986) the membership function of the performance measure $p(\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_1, \tilde{\mu}_2)$ is defined as:

$$\eta_{p(\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_1, \tilde{\mu}_2)}(z) = \mathrm{Sup}_{\Omega}\min\left\{\begin{matrix}\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}_2}(x_2), \eta_{\tilde{\mu}_1}(y_1), \\ \eta_{\tilde{\mu}_1}(y_2)|z = p(x_1, x_2, y_1, y_2)\end{matrix}\right\}$$

where, the supremum is taken over the set:

$$\Omega = \left\{x_1\in X_1, x_2\in X_2, y_1\in Y_1, y_2\in Y_2|0<\frac{x_1}{y_1}+\frac{x_2}{y_2}<1\right\}$$

Assume that the performance measure of interest is the expected number of customers in the low priority and high priority queue. The membership function for low priority customer is:

$$\eta_{\tilde{E}(N_1)}(z) = \mathrm{Sup}\Omega\min\left\{\begin{matrix}\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}_2}(x_2)\eta_{\tilde{\mu}_1}(y), \\ \eta_{\tilde{\mu}_2}(y)|z = \frac{x_1\left(\frac{x_1}{y_1^2}+\frac{x_2}{y_2^2}\right)}{\left(1-\frac{x_1}{y_1}\right)}\end{matrix}\right\}$$

The membership function for high priority customer is:

$$\eta_{\tilde{E}(N_1)}(z) = \mathrm{Sup}_{\Omega}\min\left\{\begin{matrix}\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}_2}(x_2)\eta_{\tilde{\mu}_1}(y), \\ \eta_{\tilde{\mu}_2}(y)|z = \frac{x_2\left(\frac{x_1}{y_1^2}+\frac{x_2}{y_2^2}\right)}{\left(1-\frac{x_1}{y_1}\right)\left(1-\frac{x_1}{y_1}+\frac{x_2}{y_2}\right)}\end{matrix}\right\}$$

The membership functions are not in their usual forms making it difficult to imagine its shape.

The solution procedure: The α-cut approach is used to construct the membership function. Let $\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}2}(x2), \eta_{\tilde{\mu}_1}(y_1), \eta_{\tilde{\mu}_1}(y_2)$ denote the membership function of $\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\mu}_1, \tilde{\mu}_2$, respectively. We have:

$$\tilde{\lambda}_1 = \left\{\left(x_1, \eta_{\tilde{\lambda}_1}(x_1)\right)|x_1\in X_1\right\}$$

$$\tilde{\lambda}_2 = \left\{\left(x_2, \eta_{\tilde{\lambda}_2}(x_2)\right)|x_2\in X_2\right\}$$

$$\tilde{\mu}_1 = \left\{\left(y_1, \eta_{\tilde{\lambda}_1}(y_1)\right)|y_1\in Y_1\right\}$$

$$\tilde{\mu}_2 = \{(y_2, \eta_{\tilde{\lambda}_2}(y_2)) | y_2 \in Y_2\}$$

where, X_1, X_2, Y_1, Y_2 are crisp universal sets of the arrival rate of high priority, low priority, service rate of high priority and low priority, respectively. Denote the α-cuts of $\tilde{\lambda}_1$ and $\tilde{\mu}_1$ as crisp intervals are:

$$\lambda_1(\alpha) = [x_{1\alpha}^L, x_{1\alpha}^U] = \begin{bmatrix} \min_{x_1 \in X_1} \{x_1 / \eta_{\tilde{\lambda}_1}(x_1) \geq \alpha\}, \\ \min_{x_1 \in X_1} \{x_1 / \eta_{\tilde{\lambda}_1}(x_1) \geq \alpha\} \end{bmatrix}$$

$$\mu_1(\alpha) = [y_{1\alpha}^L, y_{1\alpha}^U] = \begin{bmatrix} \min_{y_1 \in Y_1} \{y_1 / \eta_{\tilde{\mu}_1}(y_1) \geq \alpha\}, \\ \min_{y_1 \in Y_1} \{y_1 / \eta_{\tilde{\mu}_1}(y_1) \geq \alpha\} \end{bmatrix}$$

These intervals indicate that the group arrival rate and service rate lie at possibility level α. The arrival rate and service rate can be represented by different levels of confidence intervals (Negi and Lee, 1992; Zimmermann, 2001). Consequently the FM_2/FM_1, $FM_2/1$ queue can be reduced to a family of crisp M_1, M_2/M_1,$M_2/1$ with different α-level sets. The fuzzy arrival rate and fuzzy service rate are assumed to be fuzzy numbers in this model as crisp can be represented by degenerated membership function. Using the convexity of a fuzzy number (Zimmermann, 2001) the bounds of these intervals are functions of and can be obtained as $x_{1\alpha}^L = \min \eta_{\tilde{\lambda}_1}^{-1}(\alpha)$, $x_{1\alpha}^U = \max \eta_{\tilde{\lambda}_1}^{-1}(\alpha)$, $y_{1\alpha}^L = \min \eta_{\tilde{\mu}_1}^{-1}(\alpha)$, $y_{1\alpha}^U = \max \eta_{\tilde{\mu}_1}^{-1}(\alpha)$, respectively. Now, we use the cuts of L_{q1} and L_{q2} to construct its membership function, since, the membership function is parametrized by α. Using Zadeh's extension principle, $\eta_{\tilde{L}_{q2}}(Z)$ is the minimum of $\eta_{\tilde{\lambda}_1}(x_1), \eta_{\tilde{\lambda}_2}(x_2), \eta_{\tilde{\mu}_1}(y_1), \eta_{\tilde{\mu}_1}(y_2)$. To derive the membership function we need at least one of the following cases to hold such that:

$$z = \frac{x_2\left(\frac{x_1}{y_1^2} + \frac{x_2}{y_2^2}\right)}{\left(1 - \frac{x_1}{y_1}\right)\left(1 - \left(\frac{x_1}{y_1} + \frac{x_2}{y_2}\right)\right)}$$

satisfies $\eta_{\tilde{L}_{q1}}(z) = \alpha$:

Case 1: $(\eta_{\tilde{\lambda}_1}(x_1) = \alpha, \eta_{\tilde{\lambda}_2}(x_2) \geq \alpha, \eta_{\tilde{\mu}_1} \geq \alpha, \eta_{\tilde{\mu}_2} \geq \alpha)$

Case 2: $(\eta_{\tilde{\lambda}_1}(x_1) \geq \alpha, \eta_{\tilde{\lambda}_2}(x_2) = \alpha, \eta_{\tilde{\mu}_1} \geq \alpha, \eta_{\tilde{\mu}_2} \geq \alpha)$

Case 3: $(\eta_{\tilde{\lambda}_1}(x_1) \geq \alpha, \eta_{\tilde{\lambda}_2}(x_2) \geq \alpha, \eta_{\tilde{\mu}_1} = \alpha, \eta_{\tilde{\mu}_2} \geq \alpha)$

Case 4: $(\eta_{\tilde{\lambda}_1}(x_1) \geq \alpha, \eta_{\tilde{\lambda}_2}(x_2) \geq \alpha, \eta_{\tilde{\mu}_1} \geq \alpha, \eta_{\tilde{\mu}_2} = \alpha)$

To find the membership function $\eta_{\tilde{L}_{q2}}(z)$ it is sufficient to find the left shape function and right shape function of $\eta_{\tilde{L}_{q2}}(z)$ which is equivalent to find the lower bound $(\tilde{L}_{q2})_\alpha^L$ and the upper bound $(\tilde{L}_{q2})_\alpha^U$ of the α-cuts of $\eta_{\tilde{L}_{q2}}(z)$. As the requirement of $\eta_{\tilde{\lambda}_1}(\tilde{x}_1)$ α-can be represented by $x = x_\alpha^L$ or $x = x_\alpha^U$ this can be formulated as the constraint of $x_1 = \beta_1 x_{1\alpha}{}^L + (1-\beta_1) x_{1\alpha}{}^U$ where, $\beta_1 = 0$ or 1. Similarly, $\eta_{\tilde{\lambda}_2}(x_1) = \alpha$ can be formulated as the constraint of $x_2 = \beta_2 x_{2\alpha}{}^L + (1-\beta_2) x_{2\alpha}{}^U$ where, $\beta_2 = 0$ or 1, $\eta_{\tilde{\mu}_1}(x_1) = \alpha$ can be formulated as the constraint of $y_1 = \beta_3 y_{3\alpha}{}^L$ where β_3 0 or 1 and $\eta_{\tilde{\mu}_1}(x_1) = \alpha$ can be formulated as the constraint of $y_2 = \beta_4 y_{4\alpha}{}^L + (1-\beta_4) x_{a\alpha}{}^U$ where $\beta_4 = 0$ or 1. From the above four cases the membership function $\eta_{\tilde{L}_{q2}}(z)$ can be constructed by finding the lower bound $(\tilde{L}_{q2})_\alpha^L$ and upper bound $(\tilde{L}_{q2})_\alpha^U$ of the cuts of $\tilde{L}_{q2}$ in that we set:

$$(L_{q2})_\alpha^{L_1} = \min_\Omega \left(\frac{x_2\left(\frac{x_1}{y_1^2} + \frac{x_2}{y_2^2}\right)}{\left(1 - \frac{x_1}{y_1}\right)\left(1 - \left(\frac{x_1}{y_1} + \frac{x_2}{y_2}\right)\right)} \right)$$

$$\text{s.t } x_1 = t_1 (x_1)_\alpha^L + (1-t_1)(x_1)_\alpha^U$$
$$x_{2\alpha}^L \leq x \leq x_{2\alpha}^U, y_{1\alpha}^L \leq y \leq y_{1\alpha}^U, y_{2\alpha}^L \leq y \leq y_{2\alpha}^U,$$
$$t_1 = 0 \text{ or } 1$$

$$(L_{q2})_\alpha^{L_2} = \min_\Omega \left(\frac{x_2\left(\frac{x_1}{y_1^2} + \frac{x_2}{y_2^2}\right)}{\left(1 - \frac{x_1}{y_1}\right)\left(1 - \left(\frac{x_1}{y_1} + \frac{x_2}{y_2}\right)\right)} \right)$$

$$\text{s.t } x_2 = t_2 (x_2)_\alpha^L + (1-t_2)(x_2)_\alpha^{(U)},$$
$$x_{1\alpha}^L \leq x \leq x_{1\alpha}^U, y_{1\alpha}^L \leq y \leq y_{1\alpha}^U, y_{2\alpha}^L \leq y \leq y_{2\alpha}^U,$$
$$t_2 = 0 \text{ or } 1$$

$$(L_{q2})_\alpha^{L_3} = \min_\Omega \left(\frac{x_2\left(\frac{x_1}{y_1^2} + \frac{x_2}{y_2^2}\right)}{\left(1 - \frac{x_1}{y_1}\right)\left(1 - \left(\frac{x_1}{y_1} + \frac{x_2}{y_2}\right)\right)} \right)$$

st

$$y_1 = t_3 (1-t_3)(y_1)_\alpha^U (y_1)_\alpha^L +$$
$$x_{1\alpha}^L \leq x \leq x_{1\alpha}^U, x_{2\alpha}^L \leq x \leq x_{2\alpha}^U, y_{2\alpha}^L \leq y \leq y_{2\alpha}^U,$$
$$t_3 = 0 \text{ or } 1$$

$$(L_{q2})_\alpha^{L_4} = \min_\Omega \left(\frac{x_2\left(\frac{x_1}{y_1^2} + \frac{x_2}{y_2^2}\right)}{\left(1 - \frac{x_1}{y_1}\right)\left(1 - \left(\frac{x_1}{y_1} + \frac{x_2}{y_2}\right)\right)} \right)$$

$$\text{s.t } y_2 = t_4 (y_2)_\alpha^L + (1-t_4)(y_2)_\alpha^U,$$
$$x_{1\alpha}^L \leq x \leq x_{1\alpha}^U, x_{2\alpha}^L \leq x \leq x_{2\alpha}^U, y_{1\alpha}^L \leq y \leq y_{1\alpha}^U,$$
$$t_4 = 0 \text{ or } 1$$

$$\left(L_{q_2}\right)_\alpha^{U_1} = \max_\Omega\left(\frac{x_2\left(\frac{x_1}{y_1^2}+\frac{x_2}{y_2^2}\right)}{\left(1-\frac{x_1}{y_1}\right)\left(1-\left(\frac{x_1}{y_1}+\frac{x_2}{y_2}\right)\right)}\right)$$

$$\text{s.t } x_1 = t_5\left(x_1\right)_\alpha^L + \left(1-t_5\right)\left(x_1\right)_\alpha^U,$$
$$x_{2\alpha}^L \le x \le x_{2\alpha}^U,\ y_{1\alpha}^L \le y \le y_{1\alpha}^U,\ y_{2\alpha}^L \le y \le y_{2\alpha}^U,$$
$$t_5 = 0 \text{ or } 1$$

$$\left(L_{q_2}\right)_\alpha^{U_2} = \max_\Omega\left(\frac{x_2\left(\frac{x_1}{y_1^2}+\frac{x_2}{y_2^2}\right)}{\left(1-\frac{x_1}{y_1}\right)\left(1-\left(\frac{x_1}{y_1}+\frac{x_2}{y_2}\right)\right)}\right)$$

$$\text{s.t } x_2 = t_6\left(x_2\right)_\alpha^L + \left(1-t_6\right)\left(x_2\right)_\alpha^U,$$
$$x_{1\alpha}^L \le x \le x_{1\alpha}^U,\ y_{1\alpha}^L \le y \le y_{1\alpha}^U,\ y_{2\alpha}^L \le y \le y_{2\alpha}^U,$$
$$t_6 = 0 \text{ or } 1$$

$$\left(L_{q_2}\right)_\alpha^{U_3} = \max_\Omega\left(\frac{x_2\left(\frac{x_1}{y_1^2}+\frac{x_2}{y_2^2}\right)}{\left(1-\frac{x_1}{y_1}\right)\left(1-\left(\frac{x_1}{y_1}+\frac{x_2}{y_2}\right)\right)}\right)$$

$$\text{s.t } y_1 = t_7\left(y_1\right)_\alpha^L + \left(1-t_7\right)\left(y_1\right)_\alpha^U,$$
$$x_{1\alpha}^L \le x \le x_{1\alpha}^U,\ x_{2\alpha}^L \le x \le x_{2\alpha}^U,\ y_{2\alpha}^L \le y \le y_{2\alpha}^U,$$
$$t_7 = 0 \text{ or } 1$$

$$\left(L_{q_2}\right)_\alpha^{U_4} = \max_\Omega\left(\frac{x_2\left(\frac{x_1}{y_1^2}+\frac{x_2}{y_2^2}\right)}{\left(1-\frac{x_1}{y_1}\right)\left(1-\left(\frac{x_1}{y_1}+\frac{x_2}{y_2}\right)\right)}\right)$$

$$\text{s.t } y_2 = t_8\left(y_2\right)_\alpha^L + \left(1-t_8\right)\left(y_2\right)_\alpha^U,$$
$$x_{1\alpha}^L \le x \le x_{1\alpha}^U,\ x_{2\alpha}^L \le x \le x_{2\alpha}^U,\ y_{1\alpha}^L \le y \le y_{1\alpha}^U,$$
$$t_8 = 0 \text{ or } 1$$

From the definitions of $\tilde{\lambda}_1$ and $\tilde{\mu}_1$, $x_1 \in \lambda_1(\alpha)$ and $y_1 \in \mu_1(\alpha)$ can be replaced by $x_1 \in [x_{1\alpha}^L, x_{1\alpha}^U]$, $x_2 \in [x_{2\alpha}^L, x_{2\alpha}^U]$ and $y_1 \in [y_{1\alpha}^L, y_{2\alpha}^U]$, $y_2 \in [y_{2\alpha}^L, y_{2\alpha}^U]$. At least one of x_1, x_2, y_1, y_2 must hit the boundaries of their α-cuts to satisfy $\eta_{\tilde{L}_{q_2}}(z) = \alpha$. The minimum and maximum of the objective functions of models shows that lower bound and upper bound can be found by solving the above eight models. These eight models are mixed integer non linear programming problem with 0-1 variables. There are several methods to solve these problems. The study of how the optimal solutions change as $x_{1\alpha}^L, x_{1\alpha}^U, x_{2\alpha}^L, x_{2\alpha}^U, y_{1\alpha}^L, y_{1\alpha}^U, y_{2\alpha}^L, y_{2\alpha}^U$ vary over the interval α∈[0, 1] is also involved and they fall into category of parametric programming.

The crisp interval obtained by solving the above models represents the α-cut of L_{q2} These α-cuts form a nested structure with respect to α. According to Zadeh's extension principle $\tilde{L}q_2$ defined is a fuzzy number that possesses convexity. Hence for two values α_1 and α_2 such that $0<\alpha_2<\alpha_1\le 1$, we have $(Lq)_{\alpha_1}^L \ge (Lq)_{\alpha_2}^L$ and $(Lq)_{\alpha_1}^U \ge (Lq)_{\alpha_2}^U$. Accordingly the membership function $\eta_{\tilde{L}_q}(z)$ can be obtained from the solutions of the above models. If both $(Lq)_\alpha^L$ and $(Lq)_\alpha^U$ are invertible with respect to α then a left shape function $[(Lq)_\alpha^U]^{-1}$ and a right shape function $[(Lq)_\alpha^L]^{-1}$ can be obtained. From which the membership function $\eta_{\tilde{L}_q}$ is constructed as:

$$\eta_{\tilde{L}_{q_1}}(z) = \begin{cases} L(z), & \left(L_{q_2}\right)_{\alpha=0}^L \le z \le \left(L_{q_2}\right)_{\alpha=1}^L \\ R(z), & \left(L_{q_2}\right)_{\alpha=1}^U \le z \le \left(L_{q_2}\right)_{\alpha=0}^U \end{cases}$$

Or else the values of $(L_q)_\alpha^L$ and $(L_q)_\alpha^U$ cannot be obtained analytically, the numerical solutions for $(L_q)_\alpha^L$ and $(L_q)_\alpha^U$ at different possibility level α can be collected to approximate the shapes of L(z)and R(z). The set of intervals $\{[(Lq)_\alpha^L, (Lq)_\alpha^U] \mid \alpha \in [0, 1]\}$ reveals the shape of $\eta_{\tilde{L}_q}$ even though the exact function is not known explicitly. Membership functions of other performance measures can be derived in the similar manner. Since, the above performance measures are described by membership functions they conserve all of fuzziness of arrival rate and service rate. If we prefer one crisp value for each performance measures rather than a fuzzy set for practical use we defuzzify the fuzzy value by ranking method namely Yager ranking index. For a given convex fuzzy number the Yager ranking index is defined by:

$$O(\tilde{\wedge}) = \int_0^1 \frac{\wedge_\alpha^L + \wedge_\alpha^U}{2} d\alpha$$

where, $(\wedge_\alpha^L, \wedge_\alpha^U)$ is the α-cut of $\tilde{\wedge}$. It is a robust ranking technique which possesses the properties of compensation, linearity and additive.

Numerical example: Let the arrival, service rates are triangular fuzzy numbers given by $\tilde{\lambda}_1 = [1, 2, 3], \tilde{\lambda}_2 = [4, 5, 6], \tilde{\mu}_1 = [14, 15, 16], \tilde{\mu}_2 = [18, 19, 20]$ per min, respectively. It is easy to find that $[x_{1\alpha}^L, x_{1\alpha}^U] = [1+\alpha, 3-\alpha]$, $[x_{2\alpha}^L, x_{2\alpha}^U] = [4+\alpha, 6-\alpha]$, $[y_{1\alpha}^L, y_{1\alpha}^U] = [14+\alpha, 16-\alpha]$, $[y_{2\alpha}^L, y_{2\alpha}^U] = [18+\alpha, 20-\alpha]$. The α-cuts of $\tilde{L}_{q_1}$:

$$\left(L_{q_1}\right)_\alpha^L = \frac{1108+1776\alpha+737\alpha^2+71\alpha^3+2\alpha^4}{96000-28400\alpha+2920\alpha^2-127\alpha^3+2\alpha^4}$$

$$\left(L_{q_1}\right)_\alpha^U = \frac{8208-5640\alpha+1211\alpha^2-87\alpha^3+2\alpha^4}{49896+18180\alpha+2206\alpha^2+111\alpha^3+2\alpha^4}$$

With the help of MATLAB 8.6, the inverse functions of $(L_{q_1})^L_\alpha$ and $(L_{q_1})^L_\alpha$ exist, yield the membership function:

$$\text{Membership function}_{L_{q1}}(z)=\begin{cases} L(z); \dfrac{277}{24000}\le z\le \dfrac{3694}{70395} \\ R(z); \dfrac{3694}{70395}\le z\le \dfrac{38}{231} \end{cases}$$

where, L(z)=(127*x+71)/(8*(x-1))+(6^(1/2)*((56307*x*((q)/(x-1)^3)^(1/3)-53442112*x-8*x*((q)/(x-1)^3)^(2/3)+(3331*((q)/(x-1)^3)^(1/3))/2+4*((q)/(x-1)^3)^(2/3)+40000*x^2+(1667*x^2*((q)/(x-1)^3)^(1/3))/2+4*x^2*((q)/(x-1)^3)^(2/3)+765892)/(x-1)^2)^(1/2))/(24*((q)/(x-1)^3)^(1/6))+(3^(1/2)*((27*(158094956*x+214846338*x^2+140772972*x^3+8107*x^4-1404501)*((2304000*x)/(x-1)-(6725856*x)/(x-1)^2+(2142993*x)/(4*(x-1)^3)-(409092273*x)/(64*(x-1)^4)-26592/(x-1)-378288/(x-1)^2-11145651/(8*(x-1)^3)-228705129/(256*(x-1)^4)+(112614*x+1667*x^2+3331)^2/(256*(x-1)^4)-(10820400*x^2)/(x-1)^2+(122316621*x^2)/(8*(x-1)^3)-(2195269803*x^2)/(128*(x-1)^4)+(17661255*x^3)/(x-1)^3-(1308916737*x^3)/(64*(x-1)^4)-(2341301769*x^4)/(256*(x-1)^4)+((q)/(x-1)^3)^(2/3)+(((q)/(x-1)^3)^(1/3)*(112614*x+1667*x^2+3331))/(8*(x-1)^2))^(1/2))/(128*(x-1)^4)-((112614*x+1667*x^2+3331)^2*((2304000*x)/(x-1)-(6725856*x)/(x-1)^2+(2142993*x)/(4*(x-1)^3)-(409092273*x)/(64*(x-1)^4)-26592/(x-1)-378288/(x-1)^2-11145651/(8*(x-1)^3)-228705129/(256*(x-1)^4)+(112614*x+1667*x^2+3331)^2/(256*(x-1)^4)-(10820400*x^2)/(x-1)^2+(122316621*x^2)/(8*(x-1)^3)-(2195269803*x^2)/(128*(x-1)^4)+(17661255*x^3)/(x-1)^3-(1308916737*x^3)/(64*(x-1)^4)-(2341301769*x^4)/(256*(x-1)^4)+((q)/(x-1)^3)^(2/3)+(((q)/(x-1)^3)^(1/3)*(112614*x+1667*x^2+3331))/(8*(x-1)^2))^(1/2))/(128*(x-1)^4)-((56307*x*((q)/(x-1)^3)^(1/3)-53442112*x-8*x*((q)/(x-1)^3)^(2/3)+(3331*((q)/(x-1)^3)^(1/3))/2+4*((q)/(x-1)^3)^(2/3)+40000*x^2+(1667*x^2*((q)/(x-1)^3)^(1/3))/2+4*x^2*((q)/(x-1)^3)^(2/3)+765892)/(x-1)^2)^(1/2)*((q)/(x-1)^3)^(2/3)+(((q)/(x-1)^3)^(1/3)*(112614*x+1667*x^2+3331)*((2304000*x)/(x-1)-(6725856*x)/(x-1)^2+(2142993*x)/(4*(x-1)^3)-(409092273*x)/(64*(x-1)^4)-26592/(x-1)-378288/(x-1)^2-11145651/(8*(x-1)^3)-228705129/(256*(x-1)^4)+(112614*x+1667*x^2+3331)^2/(256*(x-1)^4)-(10820400*x^2)/(x-1)^2+(122316621*x^2)/(8*(x-1)^3)-(2195269803*x^2)/(128*(x-1)^4)+(17661255*x^3)/(x-1)^3-(1308916737*x^3)/(64*(x-1)^4)-(2341301769*x^4)/(256*(x-1)^4)+((q)/(x-1)^3)^(2/3)+(((q)/(x-1)^3)^(1/3)*(112614*x+1667*x^2+3331))/(8*(x-1)^2))^(1/2))/(2*(x-1)^2)+(9*6^(1/2)*((q)/(x-1)^3)^(1/2)*(1348047*x+1243887*x^2-3179*x^3-1291))/(8*(x-1)^3))^(1/2))/(12*((q)/(x-1)^3)^(1/6)*((2304000*x)/(x-1)-(6725856*x)/(x-1)^2+(2142993*x)/(4*(x-1)^3)-(409092273*x)/(64*(x-1)^4)-26592/(x-1)-378288/(x-1)^2-11145651/(8*(x-1)^3)-228705129/(256*(x-1)^4)+(112614*x+1667*x^2+3331)^2/(256*(x-1)^4)-(10820400*x^2)/(x-1)^2+(122316621*x^2)/(8*(x-1)^3)-(2195269803*x^2)/(128*(x-1)^4)+(17661255*x^3)/(x-1)^3-(1308916737*x^3)/(64*(x-1)^4)-(2341301769*x^4)/(256*(x-1)^4)+((q)/(x-1)^3)^(2/3)+(((q)/(x-1)^3)^(1/3)*(112614*x+1667*x^2+3331))/(8*(x-1)^2))^(1/4)) R(z) = (3^(1/2)*((27*(158094956*x+214846338*x^2+140772972*x^3+8107*x^4-1404501)*((1197504*x)/(x-1)-(6623100*x)/(x-1)^2-(2509515*x)/(x-1)^3-(657844497*x)/(64*(x-1)^4)-196992/(x-1)-1472040/(x-1)^2-27498177/(8*(x-1)^3)-515607849/(256*(x-1)^4)+(112614*x+1667*x^2+3331)^2/(256*(x-1)^4)-(6053940*x^2)/(x-1) ^2+(83057859*x^2)/(8*(x-1)^3)-(2517956523*x^2)/(128*(x-1)^4)+(40770189*x^3)/(4*(x-1)^3)-(1070855073*x^3)/(64*(x-1)^4)-(1366263369*x^4)/(256*(x-1)^4)+((q)/(x-1)^3)^(2/3)+(((q)/(x-1)^3)^(1/3)*(112614*x+1667*x^2+3331))/(8*(x-1)^2))^(1/2))/(128*(x-1)^4)-((112614*x+1667*x^2+3331)^2*((1197504*x)/(x-1)-(6623100*x)/(x-1)^2-(2509515*x)/(x-1)^3-(657844497*x)/(64*(x-1)^4)-196992/(x-1)-1472040/(x-1)^2-27498177/(8*(x-1)^3)-515607849/(256*(x-1)^4)+(112614*x+1667*x^2+3331)^2/(256*(x-1)^4)-(6053940*x^2)/(x-1)^2+(83057859*x^2)/(8*(x-1)^3)-(2517956523*x^2)/(128*(x-1)^4)+(40770189*x^3)/(4*(x-1)^3)-(1070855073*x^3)/(64*(x-1)^4)-(1366263369*x^4)/(256*(x-1)^4)+((q)/(x-1)^3)^(2/3+(((q)/(x-1)^3)^(1/3)*(112614*x+1667*x^2+3331))/(8*(x-1)^2))^(1/2))/(128*(x-1)^4)-((56307*x*((q)/(x-1)^3)^(1/3)-53442112*x-8*x*((q)/(x-1)^3)^(2/3)+(3331*((q)/(x-1)^3)^(1/3))/2+4*((q)/(x-1)^3)^(2/3)+40000*x^2+(1667*x^2*((q)/(x-1)^3)^(1/3))/2+4*x^2*((q)/(x-1)^3)^(2/3)+765892)/(x-1)^2)^(1/2)*((q)/(x-1)^3)^(2/3)+(((q)/(x-1)^3)^(1/3)*(112614*x+1667*x^2+3331)*((1197504*x)/(x-1)-(6623100*x)/(x-1)^2-(2509515*x)/(x-1)^3-(657844497*x)/(64*(x-1)^4)-196992/(x-1)-1472040/(x-1)^2-27498177/(8*(x-1)^3)-515607849/(256*(x-1)^4)+(112614*x+1667*x^2+3331)^2/(256*(x-1)^4)-(6053940*x^2)/(x-1)^2+(83057859*x^2)/(8*(x-1)^3)-(2517956523*x^2)/(128*(x-1)^4)+(40770189*x^3)/(4*(x-1)^3)-(1070855073*x^3)/(64*(x1)^4)-(1366263369*x^4)/(256*(x-1)^4)+((q)/(x-1)^3)^(2/3)+(((q)/(x-1)^3)^(1/3)*(112614*x+1667*x^2+3331))/(8*(x-1)^2))^(1/2))/(2*(x-1)^2)+(9*6^(1/2)*((q)/(x-1)^3)^(1/2)*(1348047*x+1243887*x^2-3179*x^3-1291))/(8*(x-1).^3))^(1/2))/(12*((q)/(x-1)^3)^(1/6)*((1197504*x)/(x-1)-(6623100*x)/(x-1)^2-(2509515*x)/(x-1)^3-(657844497*x)/(64*(x-1)^4)-196992/(x-1)-1472040/(x-1)^2-27498177/(8*(x-1)^3)-515607849/(256*(x-1)^4)+(112614*x+1667*x^2+3331)^2/(256*(x-1)^4)-(6053940*x^2)/(x-1)^2+(83057859*x^2)/(8*(x-1)^3)-(2517956523*x^2)/(128*(x-1)^4)+(40770189*x^3)/(4*(x-1)^3)-(1070855073*x^3)/(64*(x-1)^4)-(1366263369*x^4)/(256*(x-1)^4)+((q)/(x-1)^3)^(2/3)+(((q)/(x-1)^3)^(1/3)*(112614*x+

1667*x^2+3331))/(8*(x-1)^2))^(1/4))-(6^(1/2)*((56307*x*((q)/(x-1)^3)^(1/3)-53442112*x-8*x*((q)/(x-1)^3)^(2/3)+(3331*((q)/(x-1)^3)^(1/3))/2+4*((q)/(x-1)^3)^(2/3)+40000*x^2+(1667*x^2*((q)/(x-1)^3)^(1/3))/2+4*x^2*((q)/(x-1)^3)^(2/3)+765892)/(x-1)^2)^(1/2))/(24*((q)/(x-1)^3)^(1/6))-(111*x+87)/(8*(x-1)) where, p = 26886621869226*x+2258332196128581 69*x^2-20985004384090351956*x^3-37391357266996683*x^4+88103673000000*x^5–163394739144, q =8775680556*x-6*3^(1/2)*(-(p)/(x-1)^6)^(1/2)-6761677542*x^2+1000000*x^3+18*3^(1/2)*x*(-(p)/(x-1)^6)^(1/2)-18*3^(1/2)*x^2*(-(p)/(x-1)^6)^(1/2)+6*3^(1/2)*x^3*(-(p)/(x-1)^6)^(1/2)-83889287

The overall shape turns out as expected. The membership functions L(z) and R(z) have complex values with their imaginary parts approaching zero when $601/28160 \le z \le 7/60$ for L(z) and $7/60 \le z \le 1615/3528$ for R(z). The imaginary parts of these two functions can be ignored, since, they do not have influence on the computational parts Fig. 1 and 2. Crisp intervals for fuzzy expected number of customers in priority queue at different possiblistic α-levels are presented in Table 1 and 2. The support of $\tilde{L}_{q1}$ ranges from 0.0115-0.1645, this indicates that the expected number of customers in priority queue is fuzzy; It is not possible for its values to fall below 0.0115 or exceed 0.1645. The α-cut at α = 1 is 0.0525 which is the most possible value for the fuzzy expected number of customers in priority queue. The α-cuts of $\tilde{L}_{q_2}$:

$$\left(L_{q_2}\right)_\alpha^L = \frac{4432+3708\alpha+944\alpha^2+77\alpha^3+2\alpha^4}{73800-35430\alpha+4207\alpha^2-91\alpha^3-2\alpha^4}$$

$$\left(L_{q_2}\right)_\alpha^U = \frac{16416-8544\alpha+1454\alpha^2-93\alpha^3+2\alpha^4}{19008+19758\alpha+3613\alpha^2+107\alpha^3-2\alpha^4}$$

With the help of MATLAB 8.6, the inverse functions of $\left(\tilde{L}_{q_2}\right)_\alpha^L$ and $\left(L_{q_2}\right)_\alpha^U$ exist, yield the:

$$\text{Membership function}_{L_{q2}}(z) = \begin{cases} L(z); \frac{554}{9225} \le z \le \frac{9235}{42484} \\ R(z); \frac{9235}{42484} \le z \le \frac{19}{22} \end{cases}$$

where, L(z)=(3^(1/2)*((3*(1753671540*x+5794079666*x^2+6329149300*x^3+2297065907*x^4-8209485)*(106368/(x+1)-(9216270*x)/(x+1)^2-(35142261*x)/(8*(x+1)^3)-(373900527*x)/(64*(x+1)^4)-(1771200*x)/(x+1)-873180/(x+1)^2+2098866/(x+1)^3-316377369/(256*(x+1)^4)+(94250*x+92155*x^2+2683)^2/(256*(x+1)^4)+((q)/(x+1)^3)^(2/3)-(9672390*x^2)/(x+1)^2-(76709451*x^2)/(4*(x+1)^3)-

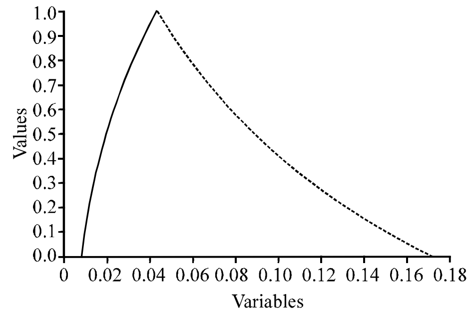

Fig. 1: The membership function for fuzzy expected number of customers in low priority queue

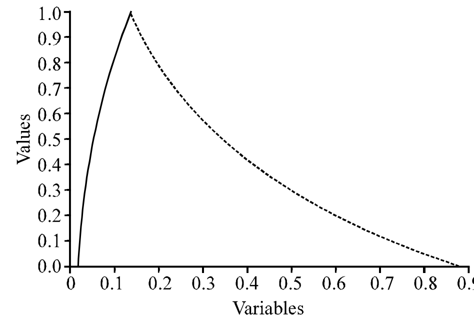

Fig. 2: The membership function for fuzzy expected number of customers in high priority queue

(1325647323*x^2)/(128*(x+1)^4)-(104514501*x^3)/(8*(x+1)^3)-(522224703*x^3)/(64*(x+1)^4)-(617174649*x^4)/(256*(x+1)^4)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683))/(8*(x+1)^2))^(1/2))/(128*(x+1)^4)-((q)/(x+1)^3)^(2/3)*((47125*x*((q)/(x+1)^3)^(1/3)-75295672*x+8*x*((q)/(x+1)^3)^(2/3)+(2683*((q)/(x+1)^3)^(1/3))/2+4*((q)/(x+1)^3)^(2/3)+(92155*x^2*((q)/(x+1)^3)^(1/3))/2+4*x^2*((q)/(x+1)^3)^(2/3)+25021036*x^2+497296)/(x+1)^2)^(1/2)-((94250*x+92155*x^2+2683)^2*(106368/(x+1)-(9216270*x)/(x+1)^2-(35142261*x)/(8*(x+1)^3)-(373900527*x)/(64*(x+1)^4)-(1771200*x)/(x+1)-873180/(x+1)^2+2098866/(x+1)^3-316377369/(256*(x+1)^4)+(94250*x+92155*x^2+2683)^2/(256*(x+1)^4)+((q)/(x+1)^3)^(2/3)-(9672390*x^2)/(x+1)^2-(76709451*x^2)/(4*(x+1)^3)-(1325647323*x^2)/(128*(x+1)^4)-(104514501*x^3)/(8*(x+1)^3)-(522224703*x^3)/(64*(x+1)^4)-(617174649*x^4)/(256*(x+1)^4)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683))/(8*(x+1)^2))^(1/2))/(128*(x+1)^4)+(3*6^(1/2)*((q)/(x+1)^3)^(1/2)*(4317073*x+9268367*x^2+4950027*x^3-4011))/(8*(x+1)^3)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683)*(106368/(x+1)-(9216270*x)/(x+1)^2-(35142261*x)/(8*(x+1)^3)-(373900527

Table 1: The α-cuts of arrival and service rates and fuzzy expected number of customers in low priority queue

α	$x_{1\alpha}^{L}$	$x_{1\alpha}^{U}$	$x_{2\alpha}^{L}$	$x_{2\alpha}^{U}$	$y_{1\alpha}^{L}$	$y_{1\alpha}^{U}$	$y_{2\alpha}^{L}$	$y_{2\alpha}^{U}$	$(L_{q1)})^{L}_{\alpha}$	$(L_{q1)})^{U}_{\alpha}$
0.0	1.0	3.0	4.0	6.0	14	16	18	20	0.0115	0.1645
0.1	1.1	2.9	4.1	5.9	14.1	15.9	18.1	19.9	0.0139	0.1479
0.2	1.2	2.8	4.2	5.8	14.2	15.8	18.2	19.8	0.0165	0.1329
0.3	1.3	2.7	4.3	5.7	14.3	15.7	18.3	19.7	0.0195	0.1192
0.4	1.4	2.6	4.4	5.6	14.4	15.6	18.4	19.6	0.0228	0.1067
0.5	1.5	2.5	4.5	5.5	14.5	15.5	18.5	19.5	0.0265	0.0954
0.6	1.6	2.4	4.6	5.4	14.6	15.4	18.6	19.4	0.0307	0.0851
0.7	1.7	2.3	4.7	5.3	14.7	15.3	18.7	19.3	0.0353	0.0757
0.8	1.8	2.2	4.8	5.2	14.8	15.2	18.8	19.2	0.0405	0.0672
0.9	1.9	2.1	4.9	5.1	14.9	15.1	18.9	19.1	0.0462	0.0595
1.0	2.0	2.0	5.0	5.0	15	15	19	19	0.0525	0.0525

Table 2: The α-cuts of arrival and service rates and fuzzy expected number of customers in high priority queue

α	$x_{1\alpha}^{L}$	$x_{1\alpha}^{U}$	$x_{2\alpha}^{L}$	$x_{2\alpha}^{U}$	$y_{1\alpha}^{L}$	$y_{1\alpha}^{U}$	$y_{2\alpha}^{L}$	$y_{2\alpha}^{U}$	$(L_{q1)})^{L}_{\alpha}$	$(L_{q1)})^{U}_{\alpha}$
0.0	1.0	3.0	4.0	6.0	14	16	18	20	0.0600	0.8636
0.1	1.1	2.9	4.1	5.9	14.1	15.9	18.1	19.9	0.0686	0.7410
0.2	1.2	2.8	4.2	5.8	14.2	15.8	18.2	19.8	0.0781	0.6390
0.3	1.3	2.7	4.3	5.7	14.3	15.7	18.3	19.7	0.0889	0.5534
0.4	1.4	2.6	4.4	5.6	14.4	15.6	18.4	19.6	0.1012	0.4809
0.5	1.5	2.5	4.5	5.5	14.5	15.5	18.5	19.5	0.1149	0.4193
0.6	1.6	2.4	4.6	5.4	14.6	15.4	18.6	19.4	0.1306	0.3664
0.7	1.7	2.3	4.7	5.3	14.7	15.3	18.7	19.3	0.1483	0.3209
0.8	1.8	2.2	4.8	5.2	14.8	15.2	18.8	19.2	0.1684	0.2815
0.9	1.9	2.1	4.9	5.1	14.9	15.1	18.9	19.1	0.1913	0.2472
1.0	2.0	2.0	5.0	5.0	15	15	19	19	0.2174	0.2174

x)/(64(x+1)^4)-(1771200*x)/(x+1)-873180/(x+1)^2+2098866/(x+1)^3-316377369/(256*(x+1)^4)+(94250*x+92155*x^2+2683)^2/(256*(x+1)^4)+((p)/(x+1)^3)^(2/3)-(9672390*x^2)/(x+1)^2-(76709451*x^2)/(4*(x+1)^3)-(1325647323*x^2)/(128*(x+1)^4)-(104514501*x^3)/(8*(x+1)^3)-(522224703*x^3)/(64*(x+1)^4)-(617174649*x^4)/(256*(x+1)^4)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683))/(8*(x+1)^2))^(1/2))/(2*(x+1)^2))^(1/2))/(12*((q)/(x+1)^3)^(1/6)*(106368/(x+1)-(9216270*x)/(x+1)^2-(35142261*x)/(8*(x+1)^3)-(373900527*x)/(64*(x+1)^4)-(1771200*x)/(x+1)-873180/(x+1)^2+2098866/(x+1)^3-316377369/(256*(x+1)^4)+(94250*x+92155*x^2+2683)^2/(256*(x+1)^4)+((q)/(x+1)^3)^(2/3)-(9672390*x^2)/(x+1)^2-(76709451*x^2)/(4*(x+1)^3)-(1325647323*x^2)/(128*(x+1)^4)-(104514501*x^3)/(8*(x+1)^3)-(522224703*x^3)/(64*(x+1)^4)-(617174649*x^4)/(256*(x+1)^4)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683))/(8*(x+1)^2))^(1/4))-(6^(1/2)*((47125*x*((q)/(x+1)^3)^(1/3)-75295672*x+8*x*((q)/(x+1)^3)^(2/3)+(2683*((q)/(x+1)^3)^(1/3))/2+4*((q)/(x+1)^3)^(2/3)+(92155*x^2*((q)/(x+1)^3)^(1/3))/2+4*x^2*((q)/(x+1)^3)^(2/3)+25021036*x^2+497296)/(x+1)^2)^(1/2))/(24*((q)/(x+1)^3)^(1/6))-(91*x+77)/(8*(x+1))

R(z)=(107*x+93)/(8*(x+1))+(6^(1/2)*((47125*x*((q)/(x+1)^3)^(1/3)-75295672*x+8*x*((q)/(x+1)^3)^(2/3)+(2683*((q)/(x+1)^3)^(1/3))/2+4*((q)/(x+1)^3)^(2/3)+(92155*x^2*((q)/(x+1)^3)^(1/3))/2+4*x^2*((q)/(x+1)^3)^(2/3)+25021036*x^2+497296)/(x+1)^2)^(1/2))/(24*((q)/(x+1)^3)^(1/6))+(3^(1/2)*((3*(1753671540*x+5794079666*x^2+6329149300*x^3+2297065907*x^4-8209485)*(393984/(x+1)-(8255106*x)/(x+1)^2-(6933987*x)/(8*(x+1)^3)-(774595791*x)/(64*(x+1)^4)-(456192*x)/(x+1)-2383776/(x+1)^2+18863469/(4*(x+1)^3)-673246809/(256*(x+1)^4)+(94250*x+92155*x^2+2683)^2/(256*(x+1)^4)+((q)/(x+1)^3)^(2/3)-(6342318*x^2)/(x+1)^2-(20722155*x^2)/(x+1)^3-(2673604827*x^2)/(128*(x+1)^4-(124095711*x^3)/(8*(x+1)^3)-(1025360991*x^3)/(64*(x+1)^4)-(1179716409*x^4)/(256*(x+1)^4)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683))/(8*(x+1)^2))^(1/2))/(128*(x+1)^4)-((q)/(x+1)^3)^(2/3)*((47125*x*((q)/(x+1)^3)^(1/3)-75295672*x+8*x*((q)/(x+1)^3)^(2/3)+(2683*((q)/(x+1)^3)^(1/3))/2+4*((q)/(x+1)^3)^(2/3)+(92155*x^2*((q)/(x+1)^3)^(1/3))/2+4*x^2*((q)/(x+1)^3)^(2/3)+25021036*x^2+497296)/(x+1)^2)^(1/2)-((94250*x+92155*x^2+2683)^2*(393984/(x+1)-(8255106*x)/(x+1)^2-(6933987*x)/(8*(x+1)^3)-(774595791*x)/(64*(x+1)^4)-(456192*x)/(x+1)-2383776/(x+1)^2+18863469/(4*(x+1)^3)-673246809/(256*(x+1)^4)+(94250*x+92155*x^2+2683)^2/(256*(x+1)^4)+((q)/(x+1)^3)^(2/3)-(6342318*x^2)/(x+1)^2(20722155*x^2)/(x+1)^3-(2673604827*x^2)/(128*(x+1)^4)-(124095711*x^3)/(8*(x+1)^3)-(1025360991*x^3)/(64*(x+1)^4)-(1179716409*x^4)/(256*(x+1)^4)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683))/(8*(x+1)^2))^(1/2))/(128*(x+1)^4)+(3*6^(1/2)*((q)/(x+1)^3)^(1/2)*(4317073*x+9268367*x^2+4950027*x^3-4011))/(8*(x+1)^3)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683)*(393984/(x+1)-(8255106*x)/(x+1)^2-(6933987*x)/(8*(x+1)^3)-(774595791*x)/(64*(x+1)^4)-(456192*x)/(x+1)-2383776/(x+1)^2+18863469/(4*(x+1)^3)-673246809/(256*(x+1)^4)+(94250*x+92155*x^2+2683)^2/(256*(x+1)^4)+((q)/(x+1)^3)^(2/3)-(6342318*x^2)/(x+1)^2-

(20722155*x^2)/(x+1)^3-(2673604827*x^2)/(128*(x+1)^4)-(124095711*x^3)/(8*(x+1)^3)-(1025360991*x^3)/(64*(x+1)^4)-(1179716409*x^4)/(256*(x+1)^4)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683))/(8*(x+1)^2))^(1/2))/(2*(x+1)^2))^(1/2))/(12*(((q)/(x+1)^3)^(1/6)*(393984/(x+1)-(8255106*x)/(x+1)^2-(6933987*x)/(8*(x+1)^3)-(774595791*x)/(64*(x+1)^4)-(456192*x)/(x+1)-2383776/(x+1)^2+18863469/(4*(x+1)^3)-673246809/(256*(x+1)^4)+(94250*x+92155*x^2+2683)^2/(256*(x+1)^4)+((q)/(x+1)^3)^(2/3)-(6342318*x^2)/(x+1)^2-(20722155*x^2)/(x+1)^3-(2673604827*x^2)/(128*(x+1)^4)-(124095711*x^3)/(8*(x+1)^3)-(1025360991*x^3)/(64*(x+1)^4)-(1179716409*x^4)/(256*(x+1)^4)+(((q)/(x+1)^3)^(1/3)*(94250*x+92155*x^2+2683))/(8*(x+1)^2))^(1/4)) where, p = 1701354322373544806O4*x^3-7196941252574 75844*x^2-933319780755264*x +1789263907197161701 95*x^4+2085921370223266878*x^5-5261894588703785625 *x^6+879096992256 where, q = 3*3^(1/2)*((p)/(x+1)^6)^(1/2)-10180607286*x+106248486183*x^2-10133423848*x^3+9*3^(1/2)*x*((p)/(x+1)^6)^(1/2)+9*3^(1/2)*x^2*((p)/(x1)^6)^(1/2)+3*3^(1/2)*x ^3*((p)/(x+1)^6)^(1/2)+44106056.

RESULTS AND DISCUSSION

Since, the performance measures are described by the membership function, the values conserve completely all of fuzziness of arrival rate and service rate. If we prefer a suitable single crisp value for the system characteristics, we defuzzify the fuzzy values using Yager's ranking index method. The system characteristics are calculated by:

$$O(\tilde{\lambda}) = \int_0^1 \frac{\wedge_\alpha^L + \wedge_\alpha^U}{2} d\alpha$$

Based on the example the suitable number of customers in low priority queue is:

$$O(\tilde{L}_{q_1}) = \int_0^1 \frac{1}{2}\left[\frac{1108+1776\alpha+737\alpha^2+71\alpha^3+2\alpha^4}{96000-28400\alpha+2920\alpha^2-127\alpha^3+2\alpha^4} + \frac{8208-5640\alpha+1211\alpha^2-87\alpha^3+2\alpha^4}{49896+18180\alpha+2206\alpha^2+111\alpha^3+2\alpha^4}\right] d\alpha = 0.0641$$

The suitable number of customers in high priority queue is:

$$O(\tilde{L}_{q_2}) = \int_0^1 \frac{1}{2}\left[\frac{4432+3708\alpha+944\alpha^2+77\alpha^3+2\alpha^4}{73800-35430\alpha+4207\alpha^2-91\alpha^3-2\alpha^4} + \frac{16416-8544\alpha+1454\alpha^2-93\alpha^3+2\alpha^4}{19008+19758\alpha+3613\alpha^2+107\alpha^3-2\alpha^4}\right] d\alpha = 0.29095$$

CONCLUSION

Fuzzy sets theory has been applied to classical queuing systems for wider applications. When the arrival time and service time are fuzzy variables, the performance measures will be fuzzy as well. With the α-cuts and Zadeh's extension principle the membership function of performance measures is derived. Four pairs of mixed integer non linear program with 0-1 variable are described to transform fuzzy queue to a family of crisp queues. In numerical example the arrival rate and service rate are considered as triangular fuzzy numbers. The membership function of waiting time in the system and queue can be derived in a similar manner. The above method can be extended to other queuing models.

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