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ON α-g-closed Sets with Respect to an Ideal

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Abstract: This research informs new types of weakly α-open sets via. ideals which generalize a usual notion for α-g-open sets which is finer than α-open set. The relationship between this notion and previously defined concepts was studied. Some properties are studied like compactness by using this notion. Some examples have been shown that the opposite is not true for many propositions and various types of ideals were used in these examples. The relationship between different types of compactness has been clarified in a flowchart. As a result, we obtained many propositions and remarks of knowledge sets, including that the existence of two different ideals on the same set such that one of them subset of the other does not necessarily indicate that there is a relationships between the two sets defined on these ideals. It is also by using different types of topologies and ideals to know the form of new knowledge sets.

Key words: α_I-g-O set, α_I-g-C set, α_I-g-compact, α_I-gc-compact, ideals, α-g-open

INTRODUCTION

Throughout this research T.S. indicates to Topological space and I.T.S. indicate to Ideal Topological Space. I(A) and C(A) will indicate the interior and closure of A, respectively. The nonempty family $I\subseteq P(X)$ known as ideal on X, if it hold the heredity and finite additive property (Kuratowski, 1969; Nasef *et al*., 2015).

For any T.S. (X, T), the aggregation {5-l: 5∈T and l∈I} is a base for T^* where T^* is a topology, finer than T, constructed by Vaidyanathaswamy (1945). Each element in T* is called T*-open sets (T*-O). I = {∅} implies T = T* (Dontchev *et al*., 1999; Jankovic and Hamlett, 1990). Any set A of a T.S. (X, T) is α-open (α-O) (Njastad, 1965), if A⊆I(C(I(A))) and then X-A is α-closed (α-C) (Nasef *et al*., 2015). $I_\alpha(A)$ and $C_\alpha(A)$ will indicate the interior and closure of A in (X, T_α), resp.

A subset A of a T.S. (X, T) is said to be c-compact (Viglion, 1969) where each closed subset F and each open cover of F, there is a finite subfamily W that hold {C (V): V∈W} covers F. Every compact space is c-compact (Esmaeel, 2009; Viglion, 1969).

MATERIALS AND METHODS

α-g-closed sets via. ideal

Definition 2.1: Any set A of a I.T.S. (X, T, I) is α-g-closed set via. I (α_I -g-C) iff C_α (A)/5∈I, whenever A\5∈I and 5 is an α-O. So, X- A is α_I-g-open set (α_I -g-O). The collection of all α_I-g-C sets is indicated by α_I-g-C (X).

Remark 2.2: For any I.T.S. (X, T, I)

- α_I-g-C(X) = P(X) where I = P (X)
- If F∈I for each α-C set F, then α_I-g-C(X) = P(X)
- Every α-C set is α_I-g-C
- All closed sets are α_I-g-C

The convers of the implication in 2.2, need not be true.

Example 2.3: For the I.T.S. (X, T, I) when X = {e_1, e_2, e_3}, T = P (X) and I = {∅} it's clear that α_I-g-C (X) = P (X).

Example 2.4: For the I.T.S. (X, T, I) where X = {e_1, e_2, e_3}, T = {X, ∅, {e_1}, {e_2, e_3}} and I = {∅, {e_3}}. A subset A = {e_1, e_2} of X is α_I-g-C which isn't closed (resp., α-C).

Proposition 2.5: Let (X, T, I) be a I.T.S., A and B any two sets in X, A∪B is α_I-g-C, whenever, both A and B are α_I-g-C sets.

Proof: Let (A∪B)\5∈I whenever 5 is α-O set then we get that A\5∈I and B\5∈I. Because of, A and B are both α_I-g-C sets, that's lead C_α(A)\5∈I and C_α (B)\5∈I. So (C_α(A)\5)∪ (C_α(B)\5)∈I, implies (C_α(A)∪C_α(B))\5∈I, so, C_α (A∪B)\5∈I. Therefore, QA∪B is α_I-g-C.

Corollary 2.6: Let (X, T, I) be a I.T.S., A and B any 2 sets in X, If A and B are both α_I-g-O sets then, so is their intersection.

Remark 2.7: The arbitrary union of α_I-g-C sets in an I.T.S. (X, T, I) may be not α_I-g-C in general.

Example 2.8: Consider the I.T.S. (N, T_{co}, I), where $T_{co} = \{5 \subseteq N; (N\text{-}5)$ is finite set or $5 = \emptyset\}$ and $I = P(E)$ is an ideal on X "where E is set of all even natural numbers". $\{e\}$ is α_I-g-C set for all $e \in E$ but $\cup\{\{e\}: e \in E\} = E$ is not.

Proposition 2.9: A subset A of a I.T.S. (X, T, I) is α_I-g-O set iff $F\backslash I_\alpha(A) \in I$ where $F\backslash A \in I$ and F is an α-C set.

Proof: For a α_I-g-O set A and $F\backslash A \in I$ when F is an α-C set, $F\backslash A = (X\backslash A)\backslash(X\backslash F)$ and $(X\backslash A)$ is an α_I-g-C set, so, $C_\alpha(X\backslash A)\backslash(X\backslash F) \in I$. Hence, $f\backslash I_\alpha(A) \in I$. Sufficient by the same idea we can show that if $F\backslash I_\alpha(A) \in I$, when $F\backslash A \in I$ with F is an α-C set then A is an α_I-g-O set.

Proposition 2.10: Let (X, T, I) be an I.T.S. such that $T = \{X, \emptyset, \{x\}\}$ for any $x \in X$, if $I = \{5 \subseteq X; \{x\} \in 5\}$ then α_I-g-C(X) = P(X). If $I = \{\emptyset\}$ then, α_I-g-C (X) = $I \cup X$.

Proposition 2.11: Let (X, T) be a T.S. if $I = I_n = \{A \subseteq X: I(C(A)) = \emptyset\}$ then α_I-g-C (X) = P (X).

Proof: For any A in X, if $A\backslash 5 \in I_n$ when $5 \in T_\alpha$, implies $I(C(A\backslash 5)) = \emptyset$. Now, $I(C(C_\alpha(A)\backslash 5)) \subseteq I(C(C(A)\backslash 5)) = I(C(C(A) \cap (X\backslash 5))) \subseteq I(C(C(A)) \cap C(X\backslash 5)) = I(C(A) \cap C(X\backslash 5))$. We claim that, $I(C(A) \cap C(X\backslash 5)) = \emptyset$. Let $x \in I(C(A) \cap C(X\backslash 5))$, implies that there is $V \in T$ such that $x \in V \subseteq C(A) \cap C(X\backslash 5)$. So, $V \cap X\backslash(C(A) \cap C(X/5)) = V \cap ((X\backslash C(A)) \cup (X\backslash C(X\backslash 5))) = V n (I(X\backslash A) \cup I(5)) = \emptyset$.

That's lead, $V \cap (I(X\backslash A)) = \emptyset$ and $V \cap (I(5)) = \emptyset$, this means $V \subseteq (C(A) \cup C(X\backslash 5)) = C(A \cup (X\backslash 5))$. Then, $x \in I(C(A\backslash 5)) = \emptyset$ which a contradiction. Hence, $I(C(A) \cap C(X\backslash 5)) = \emptyset$. Therefore, $(C_\alpha(A)\backslash 5) \in I_n$ and A is an α_I-g-C set.

Proposition 2.12: Let J and I are 2 ideals in T.S. (X, T) then, the condition $I \subseteq J$ is not sufficient to get every α_I-g-C to be α_J-g-C set and not conversely.

Example 2.13: Let (X, T) be a T.S. where $X = \{e_1, e_2, e_3\}$, $T = \{X, \emptyset, \{e_1\}\}$, $I = \{\emptyset, \{e_1\}\}$, $J = \{\emptyset, \{e_1\}, \{e_3\}, \{e_1, e_3\}\}$ and $A = \{e_3\}$. Clear that A is α_I-g-C which isn't α_J-g-C since, $\{e_3\}\text{-}\{\emptyset\} \in J$ but $C_\alpha(\{e_3\})\text{-}\{\emptyset\} = \{e_2, e_3\}$ EJ.

Example 2.14: Let (X, T) be a T.S. where $X = \{e_1, e_2, e_3\}$, $T = \{X, \emptyset, \{e_1\}\}$, $I = \{\emptyset, \{e_1\}\}$, J = P (X) and $A = \{e_1, e_3\}$. Clear that A is α_J-g-C which isn't αJ-g-C, since, $\{e_1, e_3\}\text{-}\{e_1, e_3\} \in I$ but $C_\alpha(\{e_1, e_3\})\text{-}\{e_1, e_3\} = \{e_2\} \in I$.

RESULTS AND DISCUSSION

α_I-g-compactness

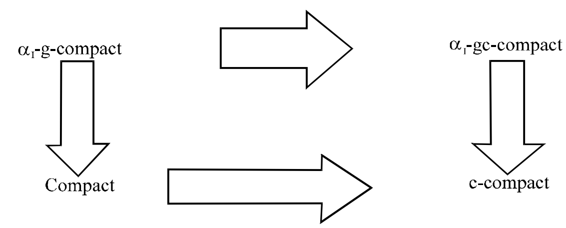
Definition 3.1: An I.T.S. (X, T, I) known as α_I-g-compact if any α_I-g-O cover has a finite subcover.

Definition 3.2: An I.T.S. (X, T, I) is known as α_A-gc-compact if for each α_I-g-C set $A \subseteq X$ each family of α_I-g-O subset of X which covers A has a finite subfamily whose α-closures in X covers A.

Proposition 3.3: For I.T.S. (X, T, I)

- Every α_I-g-compact space is compact
- Every α_I-g-compact space is α_I-gc-compact
- Every α_I-gc-compact space is c-compact

The relations among different types of compactness in 3.3, explain in following flowchart



The convers of flowchart may be not hold.

Example 3.4: For a I.T.S. (R, T, I) when R is real numbers, $T = \{R, \emptyset\}$ with I = P (X). (R, T, I) is compact and α_I-gc-compact but the cover $\{\{x\}: x \in R\}$ shows (R, T, I) isn't α_I-g-compact.

Example 3.5: Consider the I.T.S. (N, T_{co}, I_f) where $T_{co} = \{5 \subseteq N; (N\text{-}5)$ is finite set or $5 = \emptyset\}$ and $I_f = \{5 \subseteq N; 5 = \{e_1, e_2, e_3, ..., e_n\}\}$. Clear that (N, T_{co}, I_f) is a c-compact but the cover $\{\{x\}: x \in N\}$ shows (N, T_{co}, I_f) isn't α_I-gc-compact.

CONCLUSION

This research informs new types of weakly α-open sets via. ideals namely α-g-closed set via. I (α_I-g-C) we obtained many remarks and propositions of knowledge sets, some examples have been shown that the opposite is not true for many propositions and some properties are studied like α_I-g-Compactness. The relationship between some types of compactness and α_I-g-compactness has been clarified in a flowchart.

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