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A Proposed Algorithm to Find Efficient Solutions for Multicriteria Problem

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Abstract: This study proposes an algorithm (CLV) to find efficient solutions for multicriteria scheduling (P) problem of total completion time with maximum late work and maximum lateness on a single machine. Based on results of computational experiments, conclusions are formulated on the efficiency of the (CLV) algorithm.

Key words: Single machine, efficient solutions, multicriteria scheduling, lateness, computationa, completion

INTRODUCTION

Scheduling concerns the allocation of limited resources to tasks over time. It is a decision-making process that has as a goal the optimization of one or more objectives (Pinedo, 2008).

The basic scheduling problem can be described as finding for each of the tasks which are also called jobs an execution interval on one of the machines that are able to execute it such that all side constraints are met obviously, this should be done in such a way that the resulting solution which is called a schedule is best possible that is it minimizes the given objective function (Hoogeveen, 2005).

In this study, the single machine case is considered. The jobs j (j = 1, 2 , …, n) require processing times (p_j) due date (d_j), define completion times ($C_j = \sum_{i=1}^{j} p_i$) for particular schedule of jobs. The late work criterion is defined as $V_j = \min\{\max\{0, C_j, d_j\}, p_j\}$ and the lateness for job j is $L_j = C_j - d_j$.

In the simultaneous multicriteria problems approach, two or more criteria are considered simultaneously. This approach typically generates all efficient schedules and selects the one that yields the best composite objective function value of the criteria. Most multicriteria scheduling problems are NP-hard in nature (Akande *et al*., 2014). Eren (2007) gave a heuristic method for multicriteria scheduling problem with sequencing dependent setup time for minimizing the weighted sum of total completion time, maximum tardiness and maximum earliness by integer programming model.

Manufacturing facilities are complex, dynamic and stochastic systems. From the beginning of organized manufacturing, workers, supervisors, engineers and managers have developed many clever and practical methods for controlling production activities (Simeonovova *et al*., 2015). Many manufacturing organizations generate and update production schedules which are plans that state when certain controllable activities (e.g., processing of jobs by resources) should take place. Production schedules coordinate activities to increase productivity and minimize operating costs. A production schedule can identify resource conflicts, control the release of jobs to the shop, ensure that required raw materials are ordered in time, determine whether delivery promises can be met and identify time periods available for preventive maintenance (Hermann, 2007).

Angelidis *et al*. (2013) presents a specific custom-built simulator designed to support solution approaches for scheduling problems in complex assembly lines found in industrial environments. In recent years as a powerful optimization tool (Eiben and Smith, 2015; Du *et al*., 2017), Evolutionary Algorithms (EAs) have been introduced to solve the order scheduling problems.

MATERIALS AND METHODS

Definition (Hoogeveen, 2005): A feasible solution (schedule) σ is efficient (Pareto optimal or non-dominated) with respect to the performance criteria f and g if there is no feasible solution (schedule) π such that both $f(\pi) \leq f(\sigma)$ and $g(\pi) \leq g(\sigma)$ where at least one of the inequalities is strict.

Lawler's Algorithm (LA) which solves the $1/prec/f_{max}$ problem or $1//f_{max}$ problem where, $f_{max} \in (C_{max}, L_{max}, T_{max}, V_{max})$ (Pinedo, 2008) to find minimum f_{max}. Lawler's Algorithm (LA) is described by the following steps (algorithm 1):

Algorithm 1; Lawler's algorithm:
Step (1): Let N = {1, 2, ..., n} F is the set of all jobs with no successors and $\pi = \phi$

Step (2): Let j* be a job such that $f_{j*}(\sum_{i\in N} p_i) = \min_{i\in F}\{f_j(\sum_{i\in N} p_i)\}$

Step (3): Set N = N-{j*} and sequence job j* in last position of π, i.e., $\pi = (\pi, j^*)$.

Step (4): Modify F with respect to the new set of schedulable jobs

Step (5): If N = ϕ stop, otherwise go to step (2)

Formulation of the simultaneous multicriteria (P) problem: The simultaneous multicriteria scheduling (P) problem of total completion time with maximum late work and maximum lateness is formulated as follows:

$$\left.\begin{array}{l} \underset{\text{Min}}{\sum_{j=1}^{n} C_j \{V_{max}\}} \\ L_{max} \\ \text{Subjet to(P)} \\ C_j = \sum_{i=1}^{j} p_i,\ j = 1, 2, ..., n \\ V_j = \min\{\max\{0, C_j\text{-}d_j\},\ p_j\}, j = 1,2, ..., n \\ L_j = C_j\text{-}d_j,\ j = 1, 2, ..., n \end{array}\right\}$$

An algorithm (CLV) to find efficient solutions of the (P) problem: This algorithm is implemented without reset the Upper Bound (UB) in the Branch and Bound (BAB) algorithm as follows:

Algorithm 2; CLV algorithm:

Step (1): The first Upper Bound (UB_1) is found by the (SPT) rule that is sequencing the jobs in non-decreasing order σ of their processing times p_j, j = 1, 2 ,…, n for this order σ compute $\sum_{j=1}^{n} C_j(\sigma), V_{max}(\sigma), L_{max}(\sigma)$ and put $UB_1 = \sum_{j=1}^{n} C_j(\sigma) + V_{max}(\sigma) + L_{max}(\sigma)$

Step (2): The second Upper Bound (UB_2) is obtained by Lawler's Algorithm (LA) for the sequence σ_1 of (LA) compute $\sum_{j=1}^{n} C_j(\sigma_1), V_{max}(\sigma_1), L_{max}(\sigma_1)$ and put $UB_2 = \sum_{i=1}^{n} C_j(\sigma_1) + V_{max}(\sigma_1) + L_{max}(\sigma_1)$

Step (3): The third Upper Bound (UB_3) is obtained by (EDD) rule that is sequencing the jobs in non-decreasing order of their due dates d_j for this order $\sigma 2$ compute $\sum_{j=1}^{n} C_j(\sigma_2), V_{max}(\sigma_2), L_{max}(\sigma_2)$ and put $UB_3 = \sum_{i=1}^{n} C_j(\sigma_2) + V_{max}(\sigma_2) + L_{max}(\sigma_2)$

Step (4): Set the upper bound UB = min {UB_1, UB_2, UB_3}at the parent node of the search tree

Step (5): For each node (IN) in the search tree, compute the lower bound LB(IN) = cost of sequencing jobs +cost of unsequencing jobs where the cost of unsequencing jobs is found by SPT rule for $\sum_{i=1}^{n} C_i$, LA for V_{max} and EDD rule for L_{max}

Step (6): Branch each node IN with

Step (7): At the last level of the search tree of the (BAB) algorithm we get a set of solutions for this set eliminate the dominated solutions and the remaining solutions are the efficient solutions

Step (8): Stop

Analysis of number of efficient solutions: As our aim is to identify the set of all efficient solutions, we should try to hold the entire set. It is clear that if the objectives can be optimized individually, we can deduce that the set of efficient solutions has no more elements only one with extreme values of the individual objective functions. Because we are using (CLV) algorithm which depend on BAB algorithm, we can be sure that a solution is truly an efficient solution. However, we can determine if some solutions of the (CLV) algorithm is determined by other solutions. It should be noted that the SPT schedule is one of the efficient solutions for the problem (P).

Table 1: Average computation time in seconds and average number of efficient points

No. of jobs (n)	Average computation time	Average number of efficient points
5	0.1074	3
10	45.2353	6
15	430.6968	10
20	638.7972	12

Test problems with computational experiments: The CLV algorithm is tested on (P) problem for generating efficient solutions by coding it in MATLAB R2009b and running on a personal computer hp with Ram 2.50 GB. Test problems are generated as follows for each job jan integer processing time p_j is generated from the discrete uniform distribution (Akande *et al*., 2014). Also, for each job j an integer due date d_j is generated from the discrete uniform distribution [P(1-TF-RDD/2), P(1-TF+RDD/2)] where, $P = \sum_{i=1}^{n} p_i$ depending on the relative range of due date (RDD) and on the average Tardiness Factor (TF). For both parameters, the values 0.2, 0.4, 0.6, 0.8, 1.0 are considered. For each selected value of n, two problems are generated for each of the five values of parameters producing 10 problems for each value of n where the number of jobs n = 5, 10, 15, 20. For the (P) problem average computation times in seconds and average number of efficient points are given in Table 1.

From the results we can conclude that the average number of efficient points is very small when compared to the number of permutation schedules and the average computation times rapidly increase with the problem size n≥15. The objective of the experimental work reported here was to obtain some idea of the computational performance of the (CLV) algorithm. Also, we solved the problem (P) by complete enumeration method to find exact efficient solutions set and programmed in MATLAB R2009b and implemented on the same above personal computer and we get the same results when compared the results with (CLV) algorithm with size number n = 4-7 jobs. But this method is not practically, since, the scheduling problem is defined on finite set of candidate schedules.

This set is usually, so, large such that finding the efficient schedules by complete enumeration within a reasonable time is not possible.

CONCLUSION

In this study, an algorithm (CLV) is presented to multicriteria optimization and investigated its performance on a specific single machine multicriteria scheduling problem (P). Since, we are using (CLV) algorithm which is depend on a BAB algorithm, we can be sure that a solution for problem (P) is truly an efficient solution. Hence, the algorithm (CLV) is a general one and can be used for many multicriteria scheduling problems to find the set of efficient solutions. As a result of our experiments, we conclude that the (CLV) algorithm performs quite well for the multicriteria Problem (P). The research presented, here, contributes to the multi-objective scheduling literature by adapting (CLV) algorithm to multi-objective problems. For future research, we recommend the topic that would involve experimentation with the following machine scheduling problems.

Notation and some fundamental concepts of multicriteria scheduling:

N = Set of jobs
n = Number of jobs
p_j = Processing time for job j
d_j = Due date for job j
C_j = Completion time for job j
C_j = Total completion time
L_j = Lateness for job j
L_{max} = Maximum lateness
V_j = Late work for job j
V_{max} = Maximum late work

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