Multiobjective Mechanical Buckling Optimization of Variable Thickness FG Cylindrical Shell with Initial Imperfection

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Abstract: In this study, we evaluate the buckling load with energy approach with finite element models for mechanical buckling analysis of Functionally Graded Materials (FGMs) variable thickness cylindrical shell with initial imperfection. By defining the shell, mathematical equations are obtained which are dominating on the displacement of the shell, using third order shear deformation theory and Von Karman-type kinematic nonlinearity relationships. Considering cylindrical shell properties, so that, the function of properties change is considered as exponential function and variable properties are in the line with the thickness and variable length, the research continues and by applying a mechanical force in the longitudinal direction of the shell, strain-displacement relations and external load research in to the shell and bearing in mind the lasting relationship, the nonlinear finite element model of the relationship of the cylindrical shell of FGM variable thickness with initial imperfection will be defined. Using picard numerical iterative method, nonlinear finite element model of the problem will be solved and then using the Budiansky's criterion, critical buckling load is achieved. Simultaneous with the critical buckling load, another objective should be minimized like weight and cost of manufacturing cylindrical shell. Therefore, multi-objective optimization problem is defined in which using the genetic algorithm method such goals can be achieved, namely, maximum critical buckling load and minimum weight of the shell and are shown in Pareto Front diagram.

Key words: FGM variable thickness, critical buckling load, initial geometric imperfections, optimization, multi-objective Genetic algorithm, cylindrical

INTRODUCTION

Nowadays, along with incredible scientific advances and massive industrial developments and due to the tendency of big business holders to use appropriate methods to improve industrial technologies and achieve superior technology, addressing new research topics always was on the agenda of related industries and researchers. In this regard, one of the most important issues which are always taken into consideration is to discuss about the use of new technologies toward new and efficient materials with specific capabilities in order to achieve better performance as well as competitive facilities in scientific, industrial and economical arenas.

Functionally Graded Materials (FGM) cylindrical shells are very widely used in various industries and with some changes in properties, it can be reached to structures with higher quality. One of the methods in this regard is the changes in shell thickness which makes beneficial changes in certain behaviors such as buckling and vibration of structures. Yang *et al.* (2013) evaluated at an article entitled, " Buckling of cylindrical shells with

general axisymmetric thickness imperfections under external pressure" using linear regression equations, partial layouts complete with variable thickness and analysis this numerical scales. In this study, three cases of defects buckling of cylindrical shell thickness have been assessed under ambient pressure. Shariat and Asghari (2013) studies an article entitled "Nonlinear thermal buckling and postbuckling analyses of imperfect variable thickness temperature-dependent bidirectional functionally graded cylindrical shells" using numerical methods of limited elements. Characterization of materials used in the shell in both radial and longitudinal directions is variable and material properties may be dependent on temperature. In this study, critical buckling loads are evaluated using Budiansky's criterion. Yang *et al.* (2014) have presented an article titled, "An analytical method for the buckling analysis of cylindrical shells with non-axisymmetric thickness variations under external pressure" in this study, evaluated the analytical method using fourier expansion series and a turmoil method was used to analyze the buckling of cylindrical shells with variable asymmetric thickness under external pressure.

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The foregoing review reveals that buckling analysis of functionally graded cylindrical shell with variable thickness have not been considered so far, especially for cylindrical shells with tow directional heterogeneity. In the present study, nonlinear mechanical buckling analysis of imperfect variable thickness cylindrical shells made of two-directional functionally graded materials is presented, employ the third-order shear-deformation theory and using picard numerical iterative method, nonlinear finite element model of the problem will be solved and then using the Budiansky's criterion, critical buckling load is achieved. Simultaneous with the critical buckling load, another objective should be minimized like weight and cost of manufacturing cylindrical shell. Therefore, multi-objective optimization problem is defined in which using the genetic algorithm method such goals can be achieved, namely, maximum critical buckling load and minimum weight of the shell and are shown in Pareto Front diagram.

Description of problem

The governing equations of FGM shell variable thickness: Geometric parameters as well as the coordinate system of the considered FGM cylindrical shell are shown in Fig. 1. The material properties used in the shell in both radial and longitudinal directions are different and varied. In Eq. 1, h is the shell variable thickness, R is the Radius of the shell to the middle plate and l is the length of the cylindrical shell and P is the Properties of materials, P_i is the innermost layer of the shell and P_0 is the outermost layer of the shell. $Z = r-R$ is also established:

$$P(x, z) = \left\{ P_i + (P_0 - P_i) \left(0.5 + \frac{z}{h} \right)^n \right\} e^{\frac{\mu x}{l}} \qquad (1)$$

In Eq. 1, μ is the representative function view of material properties and changing the rate of change of material properties in the longitudinal direction (X-axis) of
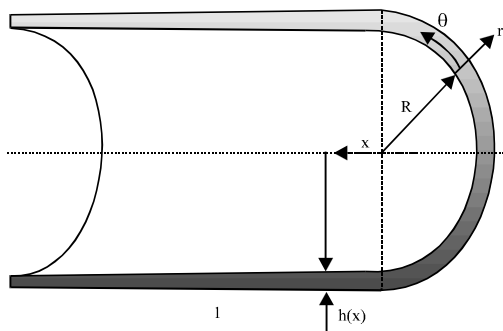

Fig. 1: The coordinate system and geometric parameters of the bidirectional FGM cylindrical shell

cylindrical shell and n represents exponential function of material properties and changing the rate of change in the material properties along with the thickness of the cylindrical shell. Elastic coefficient matrix D is defined as Eq. 2:

$$D(x, z) = \frac{E(x, z)}{1-\theta^2} \begin{bmatrix} 1 & \vartheta & 0 & 0 & 0 \\ \vartheta & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\vartheta}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\vartheta}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\vartheta}{2} \end{bmatrix} \qquad (2)$$

Where:
E = The Young's modulus
ϑ = Poisson ratio

The governing equations of shell: Displacement field of the cylindrical shell is defined based on third order shear deformation theory:

$$u = u_0 + Z\psi_x - \frac{4z^3}{ah^2(x)} (\psi_x + w_{0,x})$$

$$v = v_0 + Z\psi_\theta - \frac{4z^3}{ah^2(x)} (\psi_\theta + \frac{1}{R} w_0, \theta) \qquad (3)$$

$$w = w_0 (x, \theta)$$

In this relationship, w_0, v_0, u_0 are moved through the crust middle plate and in this study, strain-displacement relations are used in the following form:

$$\varepsilon_x = u_x + \frac{1}{2} (w_x)^2 + w_x \bar{w}_x$$

$$\varepsilon_\theta = \frac{1}{R} (\vartheta_\theta + w) + \frac{1}{2R^2} (w^2\theta + 2w_\theta \bar{w}_\theta)$$

$$\gamma_{xz} = \psi_x + w_x \qquad (4)$$

$$\gamma_{\theta z} = \psi_\theta + \frac{w_\theta}{R}$$

$$\gamma_{x\theta} = v_x + \frac{1}{R} \left[\mu_\theta + (w_x + \bar{w}_x) w_\theta + w_x \bar{w}_\theta \right]$$

In Eq. 4, $\bar{w}$ represents cylindrical shell initial function geometric imperfections. Cylindrical shell initial geometric imperfections function is defined as Eq. 5:

$$\bar{W} = \frac{w^*}{h} \sin \left(\frac{5\pi x}{l} \right) \sin (3\theta) \qquad (5)$$

Where:

w^* = The cylindrical shell initial geometric imperfections
h = Variable thickness
l = The length of the shell

The relationship of cylindrical shell variable thickness is defined as the form of Eq. 6 and h_0 is the thickness of shell at $x = 0$:

$$h(x) = h_0 e^{\beta x/l} \qquad (6)$$

In Eq. 6, β is a view of change function of the thickness of the cylindrical shell and this variable represents the rate of changes toward the longitudinal axis of the cylinder. Strain-displacement relation is derived from the displacement and we express them in the form of a matrix. Derivation of different operators is as follow:

$$D^0 = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 & \dfrac{1}{2}(w_0 + 2\bar{w})x\dfrac{\partial}{\partial x} & 0 & 0 \\[2ex] 0 & \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & \dfrac{1}{R} + \dfrac{1}{2R^2}\left[(w_0 + 2\bar{w})\theta\dfrac{\partial}{\partial \theta}\right] & 0 & 0 \\[2ex] 0 & 0 & \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & 0 & 1 \\[2ex] 0 & 0 & \dfrac{\partial}{\partial x} & 1 & 0 \\[2ex] \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial x} & \dfrac{1}{R}\left[(w_0 + \bar{w}_x)x\dfrac{\partial}{\partial \theta} + \bar{w}_\theta\dfrac{\partial}{\partial x}\right] & 0 & 0 \end{bmatrix}$$

$$(7)$$

$$D^1 = \begin{bmatrix} 0 & 0 & 0 & \dfrac{\partial}{\partial x} & 0 \\[2ex] 0 & 0 & 0 & 0 & \dfrac{1}{R}\dfrac{\partial}{\partial \theta} \\[2ex] 0 & 0 & 0 & 0 & 0 \\[2ex] 0 & 0 & 0 & 0 & 0 \\[2ex] 0 & 0 & 0 & \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial x} \end{bmatrix} \qquad (8)$$

$$D^2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\[2ex] 0 & 0 & 0 & 0 & 0 \\[2ex] 0 & 0 & -\dfrac{4}{Rh^2}\dfrac{\partial}{\partial \theta} & 0 & -\dfrac{4}{h^2} \\[2ex] 0 & 0 & -\dfrac{4}{h^2}\dfrac{\partial}{\partial x} & -\dfrac{4}{h^2} & 0 \\[2ex] 0 & 0 & 0 & \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial x} \end{bmatrix} \qquad (9)$$

$$D^3 = \begin{bmatrix} 0 & 0 & \dfrac{4}{3h}\left(\dfrac{2h_x}{h^2}\dfrac{\partial}{\partial x} - \dfrac{\partial^2}{h\partial x^2}\right) & \dfrac{4}{3h}\left(\dfrac{2h}{h^2} - \dfrac{\partial}{h\partial x}\right) & 0 \\[2ex] 0 & 0 & -\dfrac{4}{3R^2h^2}\dfrac{\partial^2}{\partial \theta^2} & 0 & -\dfrac{4}{3Rh^2}\dfrac{\partial}{\partial \theta} \\[2ex] 0 & 0 & 0 & 0 & 0 \\[2ex] 0 & 0 & 0 & 0 & 0 \\[2ex] 0 & 0 & \dfrac{4}{3Rh}\left(\dfrac{2h_x}{h^2}\dfrac{\partial}{\partial \theta} - \dfrac{\partial^2}{h\partial x\partial \theta}\right) & -\dfrac{4}{3Rh^2}\dfrac{\partial}{\partial \theta} & \dfrac{4}{3h}\left(\dfrac{2h_x}{h^2} - \dfrac{\partial}{h\partial x}\right) \end{bmatrix}$$

$$(10)$$

Based on the displacement and displacement-strain relationships, these relationships can be short and be written as the following form:

$$\varepsilon = \left(D^0 + ZD^1 + Z^2 D^2 + Z^3 D^3\right)U = BU \qquad (11)$$

In Eq. 11, U is the position vector which is defined as follows:

$$U^T = \langle u_0 \ v_0 \ w_0 \ \psi_x \ \psi_\theta \rangle \qquad (12)$$

According to strain-displacement relations, we can represent relation changes as follows:

$$\delta\varepsilon_x = \delta u_x + (w_x + \bar{w})\,\delta w_x$$
$$\delta\varepsilon_\theta = \dfrac{1}{R}(\delta\vartheta_\theta + \delta w) + \dfrac{1}{R^2}(w_\theta + \bar{w}_\theta)\,\delta w_\theta$$
$$\delta\gamma_{xz} = \delta\psi_x + \delta w_x \qquad (13)$$
$$\delta\gamma_{\theta z} = \delta\psi_\theta + \dfrac{\delta w_\theta}{R}$$
$$\delta\gamma_{x\theta} = \delta v_x + \dfrac{1}{R}\left[\delta u_\theta + (w_x + \bar{w}_x)\,\delta w_\theta + (w_\theta + \bar{w}_\theta)w_x\right]$$

Equation 13 as a matrix could be written as follows:

$$\hat{D}^0 = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 & (w_0 + \bar{w})_x\dfrac{\partial}{\partial x} & 0 & 0 \\[2ex] 0 & \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & \dfrac{1}{R} + \dfrac{1}{R^2}\left[(w_0 + \bar{w})\theta\dfrac{\partial}{\partial \theta}\right] & 0 & 0 \\[2ex] 0 & 0 & \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & 0 & 1 \\[2ex] 0 & 0 & \dfrac{\partial}{\partial x} & 1 & 0 \\[2ex] \dfrac{1}{R}\dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial x} & \dfrac{1}{R}\left[(w_0 + \bar{w})x\dfrac{\partial}{\partial x} + (w_0 + \bar{w})_\theta\dfrac{\partial}{\partial x}\right] & 0 & 0 \end{bmatrix}$$

$$(14)$$

$$\partial\varepsilon = (\hat{D}^0 + ZD^1 + Z^2D^2 + Z^3D^3)\,\delta U = B^* \times \delta U \qquad (15)$$

According to the definition of potential energy which was explained in the previous study, we write potential energy relationship as Eq. 16 and we take the changes and put it equal to zero:

$$\delta\Pi = \int_v \delta U^{(e)T} (B^{*T}\,DB)\,U^{(e)}\,dV - \int_\theta \delta U^{(e)T}\,N^T\,F_x\,d\theta = 0 \qquad (16)$$

As for as Eq. 11 and Eq. 15 and using element form functions we'll have:

$$\hat{B} = BN = (\hat{D}^0 + ZD^1 + Z^2D^2 + Z^3D^3)N \qquad (17)$$

$$\hat{B}^* = B^*N = (\hat{D}^0 + ZD^1 + Z^2D^2 + Z^3D^3)N \qquad (18)$$

According to this relation, Eq. 19 is attained as follows: Π = Total strain energy mechanical force work:

$$\delta\Pi = \int_v \delta U^{(e)T} (\hat{B}^{*T}\,D\hat{B})\,U^{(e)}\,dV - \int_\theta \delta U^{(e)T}\,N^T\,F_x d\theta = 0 \qquad (19)$$

In which Eq. 19, we have:

$$\hat{K}\,(U^{(e)}) = \int_v \hat{B}^{*T}\,(U^{(e)})\,D\,\hat{B}(U^{(e)})\,dV \qquad (20)$$

$$\hat{F}_M = \int_v \delta U^{(e)T}\,N^T F_x\,d\theta \qquad (21)$$

Equation 20 is the strain energy and Eq. 21 is forces entered the border of cylindrical shell. So, in summary these relationships can be written as Eq. 22:

$$\delta\Pi = \delta U^{(e)T}\,\hat{K}U^{(e)} - \hat{F}_M = 0 \qquad (22)$$

Where:
$\hat{K}$ = Element stiffness matrix
$\hat{F}_M$ = Mechanical load into the edge of the cylindrical shell

MATERIALS AND METHODS

The governing equations of finite element model: Shape functions for eight-node quadrilateral element is as follows:

$$N^T = \begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \\ N_8 \end{Bmatrix} = \frac{1}{4} \begin{Bmatrix} (1-\xi)\,(1-\eta)\,(-\xi-\eta-1) \\ (1+\xi)\,(1-\eta)\,(\xi-\eta-1) \\ (1+\xi)\,(1+\eta)\,(\xi+\eta-1) \\ (1-\xi)\,(1+\eta)\,(-\xi+\eta-1) \\ 2(1-\xi^2)\,(1-\eta) \\ 2(1+\xi)\,(1-\eta^2) \\ 2(1-\xi^2)\,(1+\eta) \\ 2(1-\xi)\,(1-\eta^2) \end{Bmatrix} \qquad (23)$$

where, ξ and η are natural coordinate axes of eight-node quadrilateral element and we have $\xi(-1 \le \xi \le 1)$ and $\eta(-1 \le \eta \le 1)$. Thus, for each node of elements we will have a form of function which are from N_1 to N_8 and elements natural coordinate axes, according to the following equation will be concerned with the axis of the shell:

$$x = [i+(\xi-1)]\Delta x, \ \theta = [j+(\eta-1)]\Delta\theta \qquad (24)$$

In Eq. 24, i and j are counters elements in the longitudinal direction and shell environment, respectively Δx and $\Delta\theta$ are longitudinal and perimeter sizes of the elements, respectively (Fig. 2).

In the rectangular element of eight nodes, each element node is defined by 5° of freedom and therefore the relationship between the position vector and functions is defined as Eq. 25:

$$U = NU^{(e)} \qquad (25)$$

In Eq. 25, $U^{(e)}$ is defined as follows:

$$U^{(e)T} = (u_1\ v_1\ w_1\ \psi_{x1}\ \psi_{\theta 1}, ..., u_8\ v_8\ w_8\ \psi_{x8}\ \psi_{\theta 8}) \qquad (26)$$

N as matrix is defined as follows:

$$N = \begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & ,..., & N_8 & 0 & 0 & 0 & 0 \\ 0 & N_1 & 0 & 0 & 0 & ,..., & 0 & N_8 & 0 & 0 & 0 \\ 0 & 0 & N_1 & 0 & 0 & ,..., & 0 & 0 & N_8 & 0 & 0 \\ 0 & 0 & 0 & N_1 & 0 & ,..., & 0 & 0 & 0 & N_8 & 0 \\ 0 & 0 & 0 & 0 & N_1 & ,..., & 0 & 0 & 0 & 0 & N_8 \end{bmatrix}$$

(27

After assembling matrix element, we will get to form of finite element equation of the shell which is in the form of Eq. 28:

$$\hat{K}(\hat{U})\hat{U} = \hat{F} \qquad (28)$$

In relation Eq. 28 $\hat{u}, \hat{\kappa}, \hat{f}$ are the vector of inserted force, stiffness matrix and nodes position vector of the cylindrical shell.
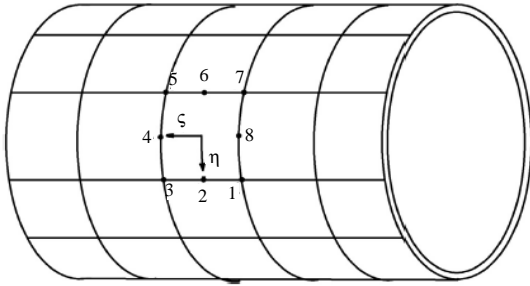
Fig. 2: Discretization of the cylindrical shell by eight-node serendipity elements

Commentary

Numerical method: To be able to base its application, it is required at first to consider an algorithm to solve the problem. Finite element form of nonlinear systems, ruling on total shells which were obtained in the previous section, we use from Picard's method to solve nonlinear equations with successive iterations and get the answer by setting error. The main solutions process is as follows:

Segmentation mechanical load into the cylindrical shell in the axial direction to small parts. Selecting a guess vector for displacement values which are generally considered to start guessing zero vector solution and in the next iterations to increase the speed, the vectors values displacement of previous step are used.

Put these values in the stiffness matrix and create new stiffness matrix and at every step, updating the stiffness matrix in which it's mathematical form would be as following:

$$K(U^{(k-1)}) U^{(K)} = F \qquad (29)$$

$$U^{(K)} = \left[K \left(U^{(K-1)} \right) \right]^{-1} F^{(K-1)} \qquad (30)$$

K is the number of repeat number in each stage. Solving nonlinear equations using numerical iterative method with the use of picard method and repeating each step to get the answer. Achieving a desired response is checked on the basis of the following criteria:

$$\left| U_i^{k-1} \right| \leq 0.001 \qquad (31)$$

After reaching the answer in step, a small amount is added to power and solution process repeated again. Displacement force curve is plotted and using the Budiansky's criterion which has cylinder shell buckling critical FGM, variable thickness will be obtained of the initial imperfection.

Table 1: Genetic algorithm parameters in optimization

Genetic algorithm parameters	Supposed numbers
Number of populations	70
Numner of generations	500

Boundary conditions: Boundary relations conditions can be defined as follows:

$$X = 0: U, V, W, \psi_\theta = 0 \qquad (32)$$

$$X = 1: V, W, \psi_x, \psi_\theta = 0 \qquad (33)$$

This boundary conditions state that in x = 0, cylindrical shell could rotate only in the direction of ψ_x and it is inhibited at all angles around the shell as well as displacement radial cylindrical shell is zero and in x = 1 shell cylindrical can be displacement just in the longitudinal axis and it is harnessed in all angles around it and also it is zero the displacement radial cylindrical shell. Natural boundary conditions cylindrical shell edge is defined mathematically as:

$$M_x = \frac{\partial^2 w}{\partial x^2} = 0 \qquad (34)$$

The boundary conditions would be added to the equations governing the cylindrical shell.

Assumptions of problem optimization: Multi-objective optimization is performed using multi-objective genetic algorithms. Genetic algorithm parameters and values are shown in Table 1 and the other parameters are as default MATLAB Software.

RESULTS AND DISCUSSTION

The analysis of results: In the present model the material of metal and ceramics are used in which the properties of these materials are shown in Table 2 and geometrical parameters are shown in Table 3. Also, for the inside of the shell, ceramics is intended and for its exterior metal is intended. The model dimensionless geometrical parameters are assumed as follows:

$$\frac{l^2}{Rh} = 300, \quad \frac{R}{h} = 400$$

In this study, three design variables are used in which n is the function of material properties, μ is the view of the characteristics of the materials used in the cylindrical shell as well as β is the view of function change of the thickness of the shell cylindrical as in Table 4.

Table 2: Material properties use in FGM cylindrical shell

Materials	Elastic modulus (GPa)	Density (kg/m^2)
Si3 N4	348.43	2370
SUS304	201.04	8166

Table 3: Geometric parameters of the FGM shell

Geometric parameter	Sparameter's numbers
Amplitude of the initial	0.0001 (m)
Geometric imperfection	0.04 (w*)
Poisson's ratio (ϑ)	0.28

Table 4: Boundary of coefficient design

Coefficients design	Lower bound	Upper bound
n	0	3
β	-0.2	+0.2
μ	-0.5	+0.5

Table 5: A comparison between present results and results reported by Shen and Noda (2005)

σ_{cr} [MPa]	n = 0.2	n = 1	n = 2
Results reported by Shen and Noda (2005)	400.462	461.427	489.047
Results reported by Shariyat (2007, 2008)	382.434	438.340	463.736
Present results	351.063	404.255	425.531

In this part, the numerical method of critical buckling load for constant thickness of cylindrical shell under evenly axial load and without initial imperfections and homogeneous are examined and the results are compared with existing references. Assuming a homogeneous cylindrical shell with a fixed radius of R as well as shell thickness is considered constant, h_0.

Since, the shell is considered to be homogeneous, so, it is enough that the parameter n is equal to zero in which case the shell will be homogeneous in the thickness direction also, parameter view of material properties, μ is set equal to zero in which case, a pure and homogeneous metal shell is obtained in both axial direction and thickness. To check the accuracy of proposed finite element model, the results will be compared with results available in the literature. For this purpose, references in this particular case as explained above, would be applied on research and result is displayed in Table 5.

Table 5 the critical layouts for three different values of n are shown. The force-displacement diagram for coefficient design n = 0, β = -0.2, μ = -0.5 is shown in Fig. 3. Comparing the critical buckling load for cylindrical shell with variable thickness is accompanied with increasing the n parameter, i.e. by increasing the function of material properties which alters the properties materials in order to thick cylindrical shell, two parameters of μ and β are considered to be constant and by increasing the variable of n, the critical buckling load will be increases.

Also n = 0 which is an expression of a homogeneous cylindrical shell as is clear that in the diagram, the lowest critical buckling load will result. The comparison is shown in Fig. 4.

Table 6: The value of design variable in Pareto Front

Design variable	n	β	μ
Upper point	2.9704	-0.1337	-0.4846
Utopia point	2.7240	0.1942	-0.3850
Lower point	2.1633	0.1970	0.1960

Table 7: The value of objectives functions in Pareto Front

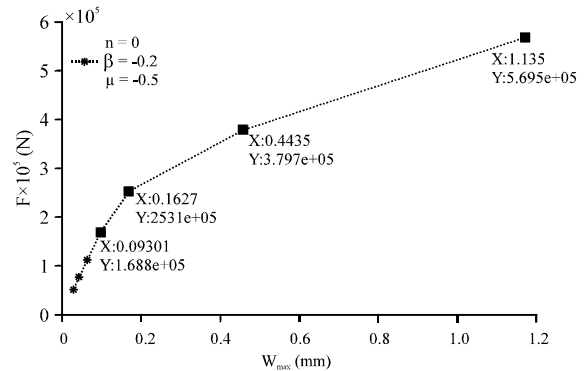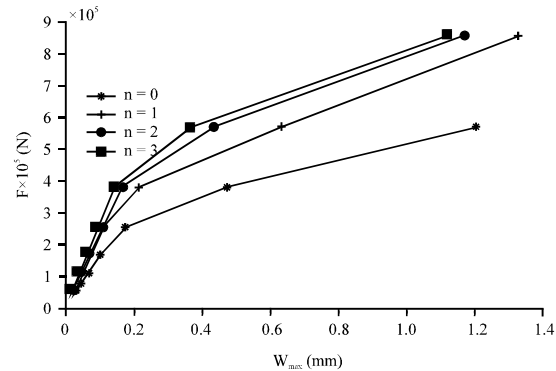| value of objectives functions	Lower point (N)	Upper point (N)	Utopia point (N)
Wight	38.0768	21.1272	26.4412
Buckling×10^5	7.9419	5.1226	7.4256

Fig. 3: Force-displacement

Fig. 4: Influence of power-law index on the force-displacement

With the view that on the impact of design variables in the process of solving the optimization problem will be defined in which design variables are included power and exponential function to change the properties of materials, facade function of variable thickness at the same time and in accordance with Table 4 which specifies the scope of design variables, the answer must be sought in this area to eliminate all possible states, finally, the lowest value obtained, at the same time for each two objectives, including the critical buckling load and weight of the shell. The Pareto Front as shown in Fig. 5 is considered cylindrical shell which bears the greatest burden of the crisis and at the same time have the least weight (Table 6 and 7).
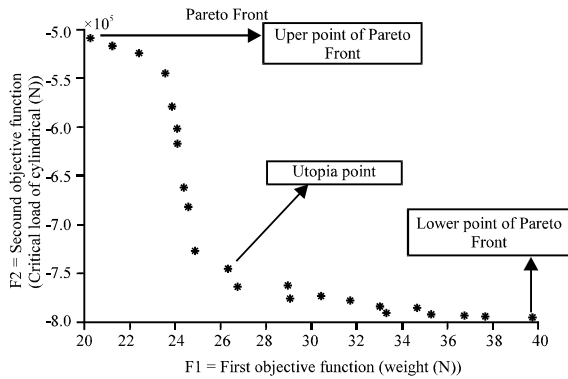
Fig. 5: F1 first objective function (weight (N))

CONCLUSION

In this study, for the first time, a two-objective optimization including the critical buckling load and FGM variable thickness cylindrical shell weight, bi-directional with initial imperfection, under mechanical loading axis with three design variables including power and exponential function of the material properties and view change function of thickness and the impact of various parameters on the critical buckling load, weight and cylindrical shell to obtain the optimal goal. Using two-objective optimization, the results in this study can be used in design and choosing the parameters achieved in such a way that the critical buckling load maximum and minimum weight of the shell. The objective function value has been obtained with respect to the design variables in the lower, upper and compromise of the Pareto Front.

According to the results in general to enhance the critical buckling load capacity and lower weight of the shell the following notes should be considered.

Thickness always increases the critical load and the weight of the cylindrical shell therefore, finding the minimum thickness is considered to optimize the weight in which the optimal parameters for cylindrical shell which is discussed in this issue is shown in Pareto diagram.

According to the results with an increase of n parameter (exponential function of material properties) the critical buckling load cylinder will be increased.

With the increases of β, the view change function of thickness (critical buckling of cylindrical shell) would be increased. Shell weight also, changes according to the design parameters and its optimized value is shown in the diagram of Pareto Front.

NOMENCLATURE

B, B^*	=	The total differentiation operators of the strain and strain increment vectors
$\hat{B}, \hat{B}^*$	=	The total differentiation operators of the shape functions associated with the strain and strain increment vectors
D	=	Elastic coefficients matrix
D^0, D^1, D^2, D^3	=	Matrices of the differentiation operators
$\hat{D}^0$	=	First matrix of the matrices of the differentiation operators associated with the strain vector increment
E	=	Young's modulus
h, h_0	=	Variable thickness, the reference thickness
$K, \hat{K}, \tilde{K}$	=	Stiffness matrix, stiffness matrix of the element, stiffness matrix of the whole shell
Ω	=	Volume
n	=	Power-law material properties index
N, N	=	Shape function, shape functions matrix
P, P_1, P_0	=	A representative material property and its values at the inner and outer radii
σ_r, σ_θ	=	Radial and circumferential normal stresses
R	=	Radius of the mid-surface
v, v_0	=	Circumferential displacement component, circumferential displacement component of the mid-surface
$U, U, U^{(8)}, \hat{U}$	=	Nodal radial displacement, vector of the displacement parameters, vector of the nodal displacement parameters of the element, vector of the nodal displacement parameters of the whole shell
w, w_0	=	Lateral deflection, lateral deflection of the mid-surface
$\bar{w}, w^*$	=	Initial lateral geometric imperfection function, amplitude of the initial geometric imperfection

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