

## Mathematical Function and Algorithms Optimisation for Wireless Sensor Networks

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**Abstract:** Wireless Sensor Network Deployment or (WSND) is considered as an active research subject. Its goal is to plan the sensor network's configurations to achieve maximum coverage and lifetime while incurring minimum cost. Meta-heuristic searching optimisation was utilised to solve this problem. However, the complex optimisation surface and the multi-objective characteristic of this problem necessitate the development of customisable multi-objective meta-heuristic searching optimisation. For this study, Lagged Multi-Objective Jumping Particle Swarm Optimisation (LMOJPSO) was formulated to solve WSND. LMOJPSO is considered as a new multi-objective optimisation for WSND. Conduct of the optimisation search took place by utilising three kinds of Pareto front: iteration, global and local. Furthermore, the lag is incorporated in the algorithm for iteration of the Pareto front. From the MOO perspective this offers better Pareto solutions. Results of its comparison with state-of-the-art approach NSGA-II reveals that LMOJPSO is better compared to it.

**Key words:** Configurations, searching optimisation, WSND, (LMOJPSO), problem necessitate, NSGA-II

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### INTRODUCTION

One can define a wireless sensor network as a collection of wirelessly connected sensors used for data gathering in a specific field and then moving that data to a predefined node referred to as the sink. The sensor nodes are typically constructed with a sensor board, a radio, a processor and a battery. These components give the sensor the ability to conduct processing, sensing and communication tasks given a coverage radius. During operation of the network, data is collected by the sensors from the environment. The sensors then use multi-hop communication to distribute those data towards the sink node. Thus, from an application standpoint there is a need for such networks to maintain long operation periods without any failure and with enhanced efficiency in obtaining the sensed data.

Several works have demonstrated the importance of WSN's as well as their applications. They are known to be part of a class of sensor network applications that have massive potential benefits for both scientific communities and the entire society (More *et al.*, 2015). Structural health can offer the necessary data collection needed for quick structural assessment at the end of an event (Zhang *et al.*, 2014). Coal mines can also utilise WSN for monitoring coal mines remotely. Chen *et al.* (2013a-c) as well as testing for the presence of toxic organic compounds in the environment (Chen *et al.*, 2013a-c), etc.

However, it is considered challenging to deploy WSN. Because there is a need for the sensor nodes to be cheap and compact, they only possess limited energy storage and low communication and processing capabilities. The impossibility of replacing or recharging the node battery, particularly for networks that have been installed in difficult to access regions, poses a serious limitation for the designer. Therefore, it is highly desirable to have energy efficient topology architectures and communication protocols. There is also a need for more energy-efficient mechanisms, so that, the energy consumption of nodes can be reduced and the network lifetime can be maximised. This should all be achieved while satisfying the application requirements in terms of connectivity and coverage (Chen *et al.*, 2015; Lim and Bleakley, 2013). Balancing the network's energy consumption is a vital issue for conserving energy and extending the network lifetime. Topology control is another method of saving energy in WSN's. Adjusting the sensor's communication ranges is a common method that is in use in topology control (Younis *et al.*, 2014). In cases where sensors are randomly deployed there could be redundant sensor nodes that all sense the same area. This means that there is wastage of a vast amount of energy. Thus, the communication and sensing range of sensor nodes are two essential measures of energy consumption in WSN's (Lai *et al.*, 2014). Utilising maximum values for communication and sensing ranges

could result in numerous sensors being led to sense the same area. This kills the nodes quickly and reduces the network lifetime. Thus, there is a need to have effective optimisation methods of WSN. For heterogeneous WSN's, various sensing and communication ranges can be possessed by the sensor nodes. One can then efficiently adjust these ranges (Chen *et al.*, 2013a-c). Alternatively, reducing the communication and sensing range of sensors will influence the connectivity and coverage, respectively. Therefore, the amount of nodes, connectivity, coverage and energy conservation are considered conflicting matters in WSN's. For instance when the sensing and communication range of sensor nodes are reduced there will be a reduction in energy consumption. There is also a deterioration of coverage and connectivity. Thus, a multi-objective optimisation approach is needed to solve optimisation problems that have multiple conflicting objectives.

This study proposes adaptive multi-objective optimisation that has its basis on Jumping Particle Swarm Optimisation (JPSO) for connectivity, coverage and topology control. The technique of the proposed algorithm may simultaneously optimise the previously mentioned conflicting aspects to determine high-quality solutions. Moreover, the search conducted within the space not only takes into consideration the best global solution in order to attract other solutions. Instead, it develops the mobility rule for transporting the solutions using three factors as the basis: best local, best global solution and best iteration solutions. Furthermore, this method has the ability to dynamically adapt the mutation and crossover rates in the absence of any external control. This adaptation technique enhances the characteristics of the optimisation algorithm based on convergence and diversity.

**Literature review:** Coverage control is considered a fundamental issue in the field of WSNs, since, the degree of coverage quality has a direct effect on the lifetime of the network. Recently, tremendous progress has been achieved in this area. Mostafaei and Shojafar (2015) suggested an Imperialist Competitive Algorithm (ICA) based technique to determine the maximum set cover in used ICA approach and deployed network to prolong the network lifetime. The proposed algorithm took advantage of the ICA which helped in determining the sensor nodes that need to be chosen in various cover sets. As the presented algorithm progresses, generation of the cover sets takes place as a way to monitor all the deployed targets. On the other hand, Tretyakova *et al.* (2017) formulated two new scheduling heuristics that were used to address the problem of maximising a WSN's lifetime

given the constraint of coverage for a subset of fixed targets. For the first one, a stochastic greedy algorithm is utilised. The basis of the second one is the application of Simulated Annealing (SA). However, Jameii *et al.* (2016) suggested using adaptive multi-objective optimisation framework that has its basis on Learning Automata (LA) and non-dominated sorting Genetic algorithm-II for topology and coverage control in heterogeneous wireless sensor networks. For the proposed framework, the multi-objective optimisation approach is referred to as the MOOCTC (Multi-Objective Optimisation Coverage and Topology Control). It can also simultaneously optimise a number of conflicting issues like the coverage rate of the monitoring area, number of active sensor nodes and balanced energy consumption while network connectivity is maintained.

Other researchers did not take into consideration the complete coverage of sensing area given the desired connectivity, similar to what Lanza-Gutierrez and Gomez-Pulido (2015). The researchers implemented meta-heuristic approaches where the placements of relay nodes were described. There were three conflicting objectives: average sensitive area, average energy and cost and network reliability. Moreover, Chen *et al.* (2015) aims to maximise the network lifetime through the addition of some redundant nodes as well as the consideration of the connectivity and coverage without any restrictions in the transmission and sensing range. The researchers also developed an algorithm that can be used to maximise the disjoint sets of sensor nodes, so that, Coverage and Connectivity (MDS-MCC) problem can be maintained with evidence that it is an NP-complete problem. Gupta *et al.* (2016) formulated a relay node placement algorithm that uses GA as the basis. This algorithm mainly aims to situate minimum number relay nodes to the potential positions provided in such a way that one can connect all the sensor nodes (targets) to the relay nodes. Researchers only took into consideration the connectivity between relay nodes and sensor nodes. It did not consider the connectivity among the placed relay nodes. For WSN's the sensor deployment problem involves finding an optimal subset of locations, so that, the total network cost can be minimised. The problem of placing sensor nodes in WSN and taking into consideration the coverage with minimum energy consumption is examined (Abidin *et al.*, 2013). The MOO technique of TPSMA (MOTPSMA) implemented in this study utilises the maximum coverage and the minimum energy consumption as the objective functions. On the other hand, the single objective approach TPSMA only takes into consideration the maximum coverage. It ignores the other objective functions that the position of the sensor

nodes would affect. Researcher interesting study utilised the idea of multi-objective. Xu *et al.* (2018) suggested using the coverage optimisation problem in WSN. There were three objectives needed to achieve a balance between coverage and network life-time. These include maximising the coverage rate, minimising the energy consumption and maximising the equilibrium of energy consumption. Hybrid-MOEA-D-I and Hybrid-MOEA-D-II which are two improved hybrid multi-objective evolutionary algorithms were proposed. Hybrid-MOEA/D-I is based on the popular multi-objective evolutionary algorithm that is in turn based on decomposition (MOEA/D). Hybrid-MOEA/D-I is a hybrid of a differential evolutionary algorithm and a Genetic algorithm that aims to optimise sub-problems of the multi-objective optimisation problem in WSN effectively. It achieves this by integrating a discrete particle swarm algorithm.

All of the above-mentioned approaches did not concurrently consider all the objectives (lifetime, connectivity, coverage and cost). Thus, they did not enhance the exploration ability of the solutions. In this study, a multi-objective optimisation approach was proposed to simultaneously optimise the area coverage, network connectivity, network lifetime, transmission range adjustment of nodes, balanced energy consumption and cost in WSN. The limitation of the solution set is expanded by introducing three kinds of pareto fronts. This lends it the ability to interact with solutions from earlier iterations based on the defined lag. It also ensures saving the elitist solutions.

## MATERIALS AND METHODS

Development of the proposed multi objective optimisation framework to optimise the pre-defined issue of multiple objectives is presented in this study. Besides, the framework adopts the PSO-based algorithm which is a meta-heuristic algorithm that seeks iteratively for the best Pareto front.

**LMOJPS:** In this portion, lagged multi-objective jumping particle swarm optimisation is presented. This algorithm represents a multi-objective variant of PSO Fig. 1. This signifies that the output does not represent an optimum solution. Instead, it is a collection of non-dominated solutions referred to as Pareto. The algorithm bears similarity to classical PSO in terms of moving solutions towards the best solution within the searching space. However, when discussing multi-objective optimisation, no single optimal solution is present. Instead, there is a Pareto that all other solutions will be moving towards to. There are three Pareto in LMOJPOS: local, global and

iteration. Calculation of the global Pareto is done from the entire set of solutions for all iterations (or the iteration that takes place within a previous lag). Calculation of the local Pareto is done from the history of the actual solution. Lastly, the iteration pareto is computed using the current iteration. The mixture of the three Pareto represents the place where the solutions are attracted.

Table gives the pseudocode of LMOJPSO. It demonstrates that the solution moves to one of the three pareto based on the values assigned to the constant  $c_1$ ,  $c_2$ . Furthermore, since, the Pareto does not represent one solution, it performs a random selection, so that, one solution can be chosen from each Pareto such that the subject solution can move towards it. Its local Pareto is updated once there is an update in the solution. Then the entire solutions in the iteration need to be finished in order to perform an update on the iteration Pareto and the global Pareto.

**Solution interaction:** The mixture between two solutions which signifies the logic of transferring one subject solution toward the target solution is provided in Table 1 As observed, the solution approaches the target at a random velocity. The total behaviour includes the fact that  $c_1$ ,  $c_2$  represent the control parameters for the amount of solutions and the speed by which every solution approach each of the global, local and iteration Pareto (Algorithm 1).

### Algorithm 1; Target solution:

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Input
Solution
Target
Lowerlimit
Upperlimit
Output
New solution
Start
1. For index = 1 until solution dimension
2.   New solution (index) = solution(index)
   +Const*rand ()*(Upper limit (index)-
   Lower limit (index) )+LowerLimit (index)
3.   Mutation Chance = R and
4.   If (Mutation Chance>Mutation Rate)
5.     New solution (index) = solution (index)
   +Const*rand ()*(Upper Limit (index)-
   Lower limit (index))+Lower limit (index)
6.   End for
7. End
8. End
    
```

**Evaluation scenarios:** Multi-objective optimisation problem needs to use a collection of evaluation techniques for multi-objective optimisation.

Set coverage metric or C-metric is the first measure used. In this measure, the input is taken as two optimal sets. One can evaluate the set coverage metric as output using the following equation:

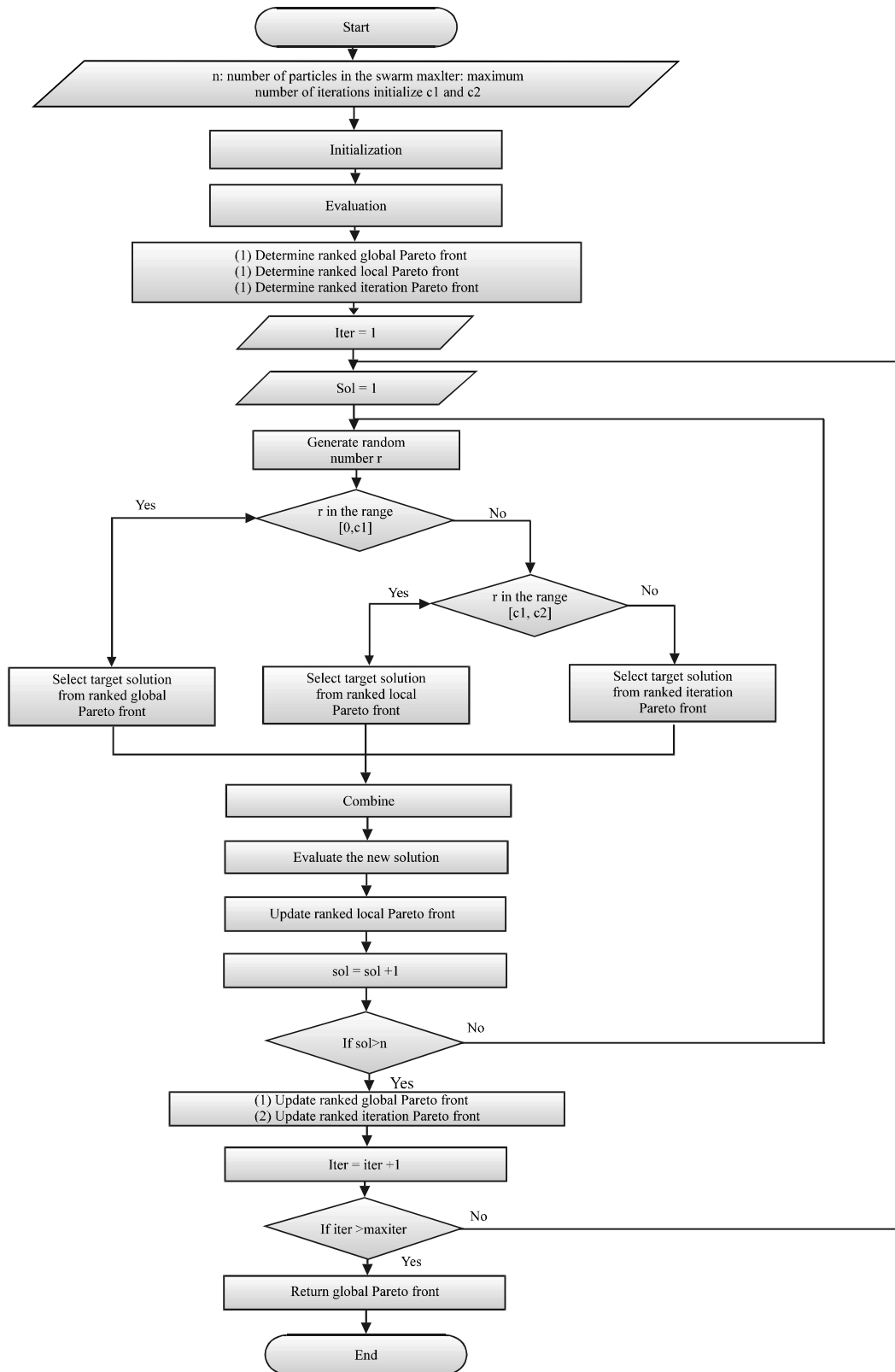


Fig. 1: Multi objective variant of PSO

Table 1: The logic of transferring one subject solution toward the target solution is provided

Problem	n	Variable bounds	Objective functions	Optimal solutions	Comments
FON	3	[-4,4]	$f_1(x) = 1 - \exp(-\sum_{i=1}^3 (x_i - \frac{1}{\sqrt{3}})^2)$ $f_2(x) = 1 - \exp(-\sum_{i=1}^3 (x_i + \frac{1}{\sqrt{3}})^2)$	$x_1 = x_2 = x_3$	Nonconvex
KUR	3	[-5,5]	$f_1(x) = \sum_{i=1}^{n-1} (-10 \exp(-0.2 \sqrt{x_i^2 + x_{i+1}^2}))$ $f_2(x) = \sum_{i=1}^n ( x_i ^{0.8} + 5 \sin x_i^3)$	(Deb, 2001)	Nonconvex
ZDT1	30	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x) [1 - \sqrt{\frac{x_1}{g(x)}}]$ $g(x) = 1 + 9 (\sum_{i=2}^n x_i) / (n-1)$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, 3, \dots, n$	Convex
ZDT2	30	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x) [1 - (x_1 / g(x))^2]$ $g(x) = 1 + 9 (\sum_{i=2}^n x_i) / (n-1)$	$x_i \in [0,1]$ $x_i = 0$ $i = 2, 3, \dots, n$	Nonconvex
ZDT3	30	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x) [1 - \sqrt{\frac{x_1}{g(x)}} - \frac{x_1}{g(x)} \sin(10\pi x_1)]$ $g(x) = 1 + 9 (\sum_{i=2}^n x_i) / (n-1)$	$x_i = 0$ $i = 2, 3, \dots, n$	Convex Disconnected
ZDT6	10	[0,1]	$f_1(x) = 1 - \exp(-4x_1) \sin^6(6\pi x_1)$ $f_2(x) = g(x) [1 - (\frac{f_1(x)}{g(x)})^2]$ $g(x) = 1 + 9 ((\sum_{i=2}^n x_i) / (n-1))^{0.25}$	$x_i \in [0,1]$ $x_i = 0$ $i = 2, 3, \dots, n$	Convex, Non-uniformly Spaced

$$C(P_{S1}, P_{S2}) = \frac{| \{y \in P_{S2} | \exists x \in P_{S1} : x > y\} |}{|P_{S2}|} \quad (1)$$

C represents the ratio of non-dominated solutions in  $P_{S2}$  that are dominated by non-dominated solutions in  $P_{S1}$  as well as the amount of solutions in  $P_{S2}$ . Therefore, when assessing a set PS, it is vital that the value of  $C(x, P_S)$  is minimised given that x represents another Pareto set.

The second measure refers to the hyper-volume metric (HV-metric or S-metric) which is widely utilised in evolutionary multi-objective optimisation in assessing the performance of search algorithms. It calculates the dominated portion's volume of the objective space in terms of a reference point. The greater the values of this performance indicator, the more desirable the solutions are.

The hyper-volume indicator is a measure of the convergence to the true Pareto front as well as the diversity of the achieved solutions. It can be given by the following equation. The hyper-volume indicator:

$$I_H^*(A) = \int_{(0,0,\dots,0)}^{(1,1,\dots,1)} \alpha_A(z) dz \quad (2)$$

Where:

$$\alpha_A(z) = \begin{cases} 1 & \text{if } A \geq \{z\} \\ 0 & \text{else} \end{cases}$$

Table 2: Evaluation measures

Measures	Class
Set coverage	Capacity
Number of non-dominated solutions	Capacity
Hyper-volume	Convergence and diversity

The last measure refers to the amount of non-dominated solutions used to express the efficacy of the optimisation algorithm in gathering solutions. One can calculate the size of  $P_S$  as follows:

$$NDS(N) = |P_S| \quad (3)$$

Higher values of NDS are preferred because they indicate the presence of adequate amounts of solutions. The evaluation measures that are generated are depicted in Table 2.

## RESULTS AND DISCUSSION

Different mathematical functions possess multiple points of local minima which complicate the optimisation and are subject to local minima. The problem becomes more complex in multi-objective optimisation cases. A number of these functions are utilised as benchmarking functions. They are given in Table 2. These functions were selected from previous studies in the field of optimisation.

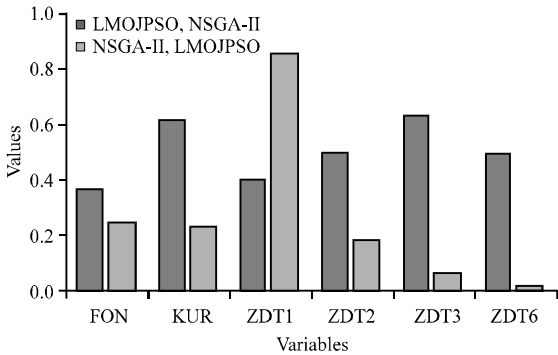


Fig. 2: Set coverage for LMOJPSO and NSGA-II for the benchmarking mathematical function

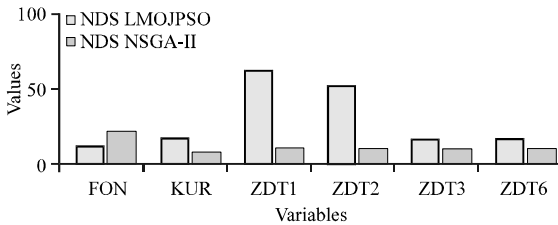


Fig. 3: NDS for LMOJPSO and NSGA-II for the benchmarking mathematical functions

Benchmark approach NSGA-II and optimisation approach LMOJPSO were tested on the given mathematical functions. The calculated multi-objective evaluation measures are the NDS, set coverage and hyper-volume. They are presented in Fig. 2. Based on Fig. 1, one can conclude that our developed approach performs better than NSGA-II based on set coverage for majority of mathematical functions. This signifies that LMOJPSO can offer more dominant solutions compared to NSGA-II.

The amount of non-dominated solutions is vital in showing the degree of freedom provided by the optimisation algorithm to the user. In Fig. 2, it can be observed that LMOJPSO produces higher amounts of non-dominated solutions compared to NSGA-II. This is true for all mathematical functions except FON. However, for FON, LMOJPSO has higher set coverage than NSGA-II. This maintains the superiority of LMOJPSO compared to FON.

The spread of the solutions within the solution space is another aspect that went through analysis. Hyper-volume was used to measure the aspect. The hyper volume can be seen in Fig. 3 and 4. It was observed that for 3 out of 6 functions, LMOJPSO has provided higher equal hyper-volume compared to NSGA-II. This also

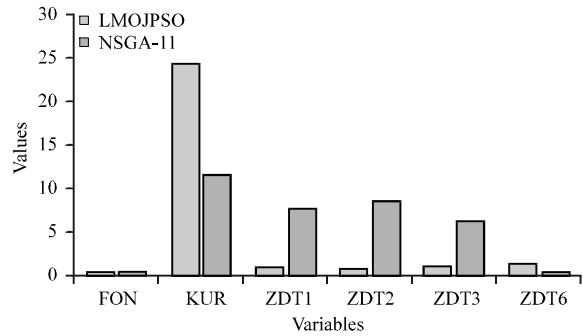


Fig. 4: Hyper-volume LMOJPSO and NSGA-II for the benchmarking mathematical functions

considers that there is higher domination for LMOJPSO solutions for all the given mathematical functions expect ZDT1.

## CONCLUSION

This study tested the LMOJPSO optimisation approach and the NSGA-II benchmark approach based on the given mathematical functions. The calculated multi-objective evaluation measures include NDS, set coverage and hyper-volume. The conclusion is that our developed approach performs better than NSGA-II for most mathematical functions. This shows that LMOJPSO can offer more dominant solutions compared to NSGA-II.

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