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On the Behavior of Solutions of a Fourth-Order Differential System at Infinity

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Abstract: The asymptotic behavior of the fundamental system of solutions of two fourth-order singular differential equations for large values of the spectral parameter is investigated in this article. The asymptotic formulas for the fundamental system of solutions are determined uniformly with respect to x when ly = λy, λϵΓ, λ→∞ in the case of slow rotation of the eigenvectors of the real symmetric matrix Q(x) with twice continuously differentiable elements. Replacing the variables in the system of equations of the fourth order allows us to pass to a system of equations of the first order with a new unknown vector function. An orthogonal matrix is introduced which can be reduced to diagonal form by means of transformations. For the system of equations in the space of vector-functions, asymptotic formulas are obtained and proved. Due to the uniformity of the asymptotic formulas, the asymptotics of the spectrum of the corresponding differential operator is calculated in this study. Using the obtained formulas, the defect indices of the corresponding differential operators are calculated.

Key words: Fundamental system of solutions, asymptotic behavior, uniformly with respect to x, symmetric matrix, system of differential equations, L-diagonal system, elements

INTRODUCTION

It happens that it is necessary to calculate a value determined in some way and this calculation leads to a very large number of actions and their execution becomes practically impossible. A real treasure during this time can be some other method of obtaining additional information about this quantity which allows us, at least approximately, to find its value. According to Laplace, such a method is "The more accurate the more it is necessary" that is we get a more accurate approximation to the desired value when more actions we perform for its direct calculation. In this case, we mean asymptotic estimates or asymptotic formulas.

Many problems, both theoretical and applied mathematics, deal with the behavior of solutions of differential equations near a singular point. Such problems are asymptotic in nature because by means of the transformation of an independent variable the singular point can always be transferred to infinity, after which the question arises: how do the solutions of a differential equation of the form $F(t, y, y', ..., y^{(m)}) = 0$ behave when we have $t \to \infty$ for unknown function $y = y(t)$?

These problems often arise in stability objectives, on linear and nonlinear oscillations, in quantum mechanics. Applications of a different nature consist in the fact that one can study the asymptotic behavior of solutions of the fundamental system of two singular differential equations in the space of vector-valued functions. In differential equations, it is possible to replace dependent and independent variables. After the completed replacements, the task often looks different.

Another characteristic feature of the asymptotic problems of differential equations is that one can guess which asymptotic formula or asymptotic series we need to obtain but to prove that this is indeed an asymptotic formula is rather difficult.

In the proof of a certain asymptotic formula for solving a certain system of equations, we must try to conclude this solution between two functions which asymptotic behavior is known. These inequalities are obtained by means of simple theorems of the type.

Let y(t) be solution for differential first-order equation $y' = (t, y)(a \le t \le b)$ but φ(t) function satisfying conditions: $\varphi'(t) < F(t, \varphi(t))$, $(a \le t \le b)$, $\varphi(a) \le y(a)$ then $\varphi(t) < y(t)(a < t \le b)$. In other words, to obtain the inequality of a function y(t), it is not necessary to solve the differential equation precisely, it is easier to find a function φ(t), satisfying condition $\varphi' < \bar{F}(t, \varphi)$.

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The aim of our research is to prove the theorem on the solution of differential equations in the space of vector functions whose behavior is described asymptotically. To describe the behavior of a function f(x) where $x\neg\infty$ in terms of a known function g(x), we use definitions introduced by Landau. Let $x\epsilon R$, ag (x) if $x\neg\infty$ converge to zero, to infinity or behave as you like.

Definition 1: If $f(x)/g(x)\neg 1$ where $x\neg\infty$, then it is written and said f asymptotically converge to g or g is an asymptotic approximation of the function f.

Definition 2: If $f(x)/g(x)\neg 0$ where $x\neg\infty$, then it is said that degree of order of f is less that g degree of order.

Definition 3: If the relation $|f(x)/g(x)|$ is limited, then it is written $f = O(g)$ and said that the function f has order not exceeding g order. That means that $\exists c\in R$ is such that if $x\geq x_0$ then $|f(x)|\leq c|g(x)|$ ($x\geq x_0$) inequality is satisfied.

MATERIALS AND METHODS

The research task of the asymptotical behaviour of solutions of ordinary differential equations depending on behaviour of coefficients is one of the central tasks in the theory of the ODE. A considerable number of works is devoted to the solution of this task (Fedoryuk, 1983). However, generally scalar differential equations are investigated in these works. We investigate the asymptotical behaviour of solutions of differential equations in space of vector functions. The following set of equations is considered:

$$ly = y^{(4)}+Q(x)y = \lambda y;\quad 0\leq x<+\infty \tag{1}$$

λ-complex parameter, $\lambda\epsilon\Gamma$, $\Gamma = \{\lambda:\lambda = \sigma+1\tau,\ \tau = \sigma\gamma,\ 0<\gamma<1\}$. $y = (y_1(x)/y_2(x))$-vector, $0\leq x<+\infty$. Q (x) is a real-valued symmetric matrix with twice continuously differentiable elements. $q_{i,\ j}(x)$, i, j = 1, 2, whose Eigenvalues are $\mu_i(x)\neg-\infty$ if $x\neg+\infty$. We introduce the following notation:

$$\varphi(x) = \frac{1}{2}\arctan\frac{q_{22}(x)-q_{11}(x)}{2q_{12}(x)}$$

$\varphi'(x)$ rotation speed of eigenvectors of matrix Q (x). We will be interested in asymptotical formulas for the fundamental system of solutions of the Eq. 1 if $\lambda\epsilon\Gamma$, $\lambda\neg\infty$, evenly on x: $0\leq x\leq+\infty$ in case of slow rotations of Eigenvectors of matrix Q(x). Asymptotical formulas, that are uniform on x are important both from the point of view of the asymptotical theory of differential equations and from the point of view of the spectral theory of differential operators. The matter is their knowledge gives the opportunity to investigate spectral properties of corresponding differential operators (Kostyuchenko and Sargsyan, 1979). Let us note that earlier one of the researchers of this research (Sultanayev, 1974) considered a question of the asymptotical behaviour of solutions of the second order system $-y''+Q(x)y = \lambda y$, $\lambda\epsilon\Gamma$, $\lambda\neg\infty$ evenly on x. In the same research, the asymptotic of a range of the relevant differential operator is calculated which is possible to do due to the uniformity of asymptotical formulas. Asymptotical formulas are found in research (Sultanayev and Myakinova, 2009; Sultanaev and Berdenova, 2014; Berdenova, 2015) for the Eq. 1 if $x\neg\infty$ which is enough for calculation of minimum indices of operator defect generated in $L^2(0, \infty)$ by differential expression of ly. For the research of the asymptotic of a range of the operator that will be studied separately, asymptotical formulas on λ are necessary that are uniform on x which is the main finding of the research.

RESULTS AND DISCUSSION

Theorem 1: Suppose the following conditions are fulfilled: for rather large x_0 and if $x\geq x_0$:

$$|\varphi'(x)|\leq c,\ c>0$$

$$0<A\leq\left|\frac{\mu_1(x)}{\mu_j(x)}\right|\leq B$$

$$\int_{x_0}^{\infty}\left|(\lambda-\mu_1(x))\right|^{-\frac{1}{4}}dx<\infty,\ \int_{x_0}^{\infty}\left|\frac{\varphi'(x)}{(\lambda-\mu_1(x))^{\frac{1}{4}}}\right|dx = o(1),\ \lambda\in\Gamma,\ \lambda\to\infty,\ i = 1, 2$$

$$\left|\mu_1'(x)\right|\leq o\left(\left|\mu_1(x)\right|^{\alpha}\right),\ x\to+\infty,\ i = 1, 2,\ 0<\alpha<\frac{5}{4}\mu_1'(x)$$

and $\mu_1''(x)$ maintain the sign if $x\geq x_0$. Then system 1 has eight linearly independent solutions of y_i (x, λ) such that $\lambda\neg\infty$, $\lambda\epsilon\Gamma$ is even on x, if $0\leq x<+\infty$.

$$y_{1,\ 2} = \phi_1(x,\lambda)\exp\left\{\pm\int_0^x(\lambda-\mu_1(t))^{\frac{1}{4}}dt\right\}(1+o(1))$$

$$y_{3,\ 4} = \phi_1(x,\lambda)\exp\left\{\pm i\int_0^x(\lambda-\mu_1(t))^{\frac{1}{4}}dt\right\}(1+o(1))$$

$$y_{5,6} = \phi_2(x,\lambda)\exp\left\{\pm\int_0^x(\lambda-\mu_2(t))^{\frac{1}{4}}dt\right\}(1+o(1))$$

$$y_{7,8} = \phi_2(x,\lambda)\exp\left\{\pm i\int_0^x(\lambda-\mu_2(t))^{\frac{1}{4}}dt\right\}(1+o(1))$$

$$\phi_1(x,\lambda) = \frac{1}{\sqrt[8]{(\lambda-\mu_1(x))^3}}\begin{pmatrix}\cos & \phi(x)\\ -\sin & \phi(x)\end{pmatrix}$$

$$\phi_2(x,\lambda) = \frac{1}{\sqrt[8]{(\lambda-\mu_2(x))^3}}\begin{pmatrix}\sin & \phi(x)\\ \cos & \phi(x)\end{pmatrix}$$

Proof: By the means of the change of variables:

$$z = \begin{pmatrix} y\\ y'\\ y''\\ y'''\end{pmatrix}$$

from system 1, we will pass to system of differential equations of first order $z' = Az$ where, $z = (z_1(x,\lambda), z_2(x,\lambda), z_3(x,\lambda), z_4(x,\lambda))$:

$$A = \begin{pmatrix} 0 & I & 0 & 0\\ 0 & 0 & I & 0\\ 0 & 0 & 0 & I\\ -Q+\lambda I & 0 & 0 & 0\end{pmatrix}$$

and I is the 2×2 identily matrix. Let us introduce an orthogonal matrix $U_1 = \begin{pmatrix}\cos\phi(x) & \sin\phi(x)\\ -\sin\phi(x) & \cos\phi(x)\end{pmatrix}$ such that $U_1^{-1}QU_1 = \Lambda = \mathrm{diag}\{\mu_1, \mu_2\}$:

$$\mu_1 = \frac{q_{11}+q_{22}+\sqrt{(q_{11}-q_{22})^2+4q_{12}^2}}{2}\quad \mu_2 = \frac{q_{11}+q_{22}-\sqrt{(q_{11}-q_{22})^2+4q_{12}^2}}{2}$$

Further, we will make a replacement:

$$z = \mathrm{diag}\{U_1, U_1, U_1, U_1\}\omega = U\omega$$

$$z' = U'\omega+U\omega'$$

$$U'\omega+U\omega' = AU\omega$$

$$\omega' = (U^{-1}AU)\omega-U^{-1}U'\omega$$

$$U^{-1}AU = \begin{pmatrix} 0 & I & 0 & 0\\ 0 & 0 & I & 0\\ 0 & 0 & 0 & I\\ -\Lambda+\lambda I & 0 & 0 & 0\end{pmatrix} \qquad (2)$$

$$U^{-1}U' = \mathrm{diag}\{p, p, p, p\},\ p = \phi'(x)\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

Further, as condition 1) of the theorem is fulfilled, the pivots in system 2 will be $U^{-1}AU$ matrix elements. Let us bring it to the diagonality. There is a matrix bringing $U^{-1}AU$ to the diagonality. Let us denote it by C (x, λ):

$$C^{-1}(U^{-1}AU)C = M = \mathrm{diag}\{\tilde{\mu}_1, \tilde{\mu}_2, \tilde{\mu}_3, \tilde{\mu}_4, \tilde{\mu}_5, \tilde{\mu}_6, \tilde{\mu}_7, \tilde{\mu}_8\}$$

$$\tilde{\mu}_1 = (\lambda-\mu_1)^{\frac{1}{4}},\quad \tilde{\mu}_2 = (\lambda-\mu_2)^{\frac{1}{4}}$$

$$\tilde{\mu}_3 = (\lambda-\mu_1)^{\frac{1}{4}},\quad \tilde{\mu}_4 = (\lambda-\mu_2)^{\frac{1}{4}}$$

$$\tilde{\mu}_5 = i(\lambda-\mu_1)^{\frac{1}{4}},\quad \tilde{\mu}_6 = i(\lambda-\mu_2)^{\frac{1}{4}}$$

$$\tilde{\mu}_7 = -i(\lambda-\mu_1)^{\frac{1}{4}},\quad \tilde{\mu}_8 = -i(\lambda-\mu_2)^{\frac{1}{4}}$$

Elements of C matrix are defined from a set of equations:

$$c_{1i}\overline{\mu}_i = c_{2i},\ c_{2i}\overline{\mu}_i = c_3,\ c_{3i}\overline{\mu}_i = c_{4i},\ c_{4i}\overline{\mu}_i = (-\Lambda+\lambda I)C_{1i}$$

where, $c_{ij}, i = \overline{1,4}, j = \overline{1,4}$ are two-dimensional matrixes, elements C:

$$\overline{\mu}_1 = \begin{pmatrix}\tilde{\mu}_1(x) & 0\\ 0 & \tilde{\mu}_2(x)\end{pmatrix},\ \overline{\mu}_2 = \begin{pmatrix}\tilde{\mu}_3(x) & 0\\ 0 & \tilde{\mu}_4(x)\end{pmatrix}$$

$$\overline{\mu}_3 = \begin{pmatrix}\tilde{\mu}_5(x) & 0\\ 0 & \tilde{\mu}_6(x)\end{pmatrix},\ \overline{\mu}_4 = \begin{pmatrix}\tilde{\mu}_7(x) & 0\\ 0 & \tilde{\mu}_8(x)\end{pmatrix}$$

From this system, matrix C is located ambiguously, with an accuracy to post-multiplication by block-diagonal matrix $\delta(x) = \mathrm{diag}\{\delta_1(x), \delta_2(x), \delta_3(x), \delta_4(x)\}$. Then elements C have the appearance:

$$c_{1i} = \delta_1(x),\ c_{2i} = \delta_1(x)\,\overline{\mu}_i(x),$$

$$c_{3i} = \delta_1(x)\,\overline{\mu}_i^2(x),\ c_i^4 = \delta_1(x)\,\overline{\mu}_i^3(x)$$

We find matrix C^{-1} from the condition $C^{-1}C = E$. Let us denote by $T = C^{-1}C^t$ and we will find its elements. Let us choose matrix δ (x), so that, the condition $(C^{-1}C^t)_{ii} = 0$ by $i = \overline{1,8}$ can be fulfilled. Then matrix δ (x) blocks have the appearance:

$$\delta_1(x) = (\overline{\mu}_1(x))^{-\frac{3}{2}},\ \delta_2(x) = (-\overline{\mu}_2(x))^{-\frac{3}{2}},$$

$$\delta_3(x) = (-i\overline{\mu}_3(x))^{-\frac{3}{2}},\ \delta_4(x) = (i\overline{\mu}_4(x))^{-\frac{3}{2}}$$

$$C=\begin{pmatrix} \overline{\mu_1}(x)^{\frac{3}{2}} & -\overline{\mu_2}(x)^{\frac{3}{2}} & -i\overline{\mu_3}(x)^{\frac{3}{2}} & i\overline{\mu_4}(x)^{\frac{3}{2}} \\ \overline{\mu_1}(x)^{\frac{1}{2}} & -(-\overline{\mu_2}(x))^{\frac{1}{2}} & i(-i\overline{\mu_3}(x))^{\frac{1}{2}} & -i(i\overline{\mu_4}(x))^{\frac{1}{2}} \\ \overline{\mu_1}(x)^{\frac{1}{2}} & (-\overline{\mu_2}(x))^{\frac{1}{2}} & -i(-i\overline{\mu_3}(x))^{\frac{1}{2}} & -(i\overline{\mu_4}(x))^{\frac{1}{2}} \\ \overline{\mu_1}(x)^{\frac{3}{2}} & -(-\overline{\mu_2}(x))^{\frac{3}{2}} & -i(-i\overline{\mu_3}(x))^{\frac{3}{2}} & i(i\overline{\mu_4}(x))^{\frac{3}{2}} \end{pmatrix}$$

$$C^{-1}=\frac{1}{4}\begin{pmatrix} \overline{\mu_1}(x)^{\frac{3}{2}} & \overline{\mu_1}(x)^{\frac{1}{2}} & \overline{\mu_1}(x)^{-\frac{1}{2}}-i\overline{\mu_3}(x)^{\frac{3}{2}} & \overline{\mu_1}(x)^{\frac{3}{2}} \\ (-\overline{\mu_2}(x))^{\frac{3}{2}} & -(-\overline{\mu_2}(x))^{\frac{1}{2}} & (-\overline{\mu_2}(x))^{-\frac{1}{2}}i(-i\overline{\mu_3}(x))^{\frac{1}{2}} & -(-\overline{\mu_2}(x))^{\frac{3}{2}} \\ (-i\overline{\mu_3}(x))^{\frac{3}{2}} & -i(-i\overline{\mu_3}(x))^{\frac{1}{2}} & -(-\overline{\mu_3}(x))^{\frac{1}{2}} & i(-i\overline{\mu_3}(x))^{\frac{3}{2}} \\ (i\overline{\mu_4}(x))^{\frac{3}{2}} & -i(i\overline{\mu_4}(x))^{\frac{1}{2}} & -(i\overline{\mu_4}(x))^{\frac{1}{2}} & -i(i\overline{\mu_4}(x))^{\frac{3}{2}} \end{pmatrix}$$

$$T=\frac{1}{8}\begin{pmatrix} 0 & 0 & \frac{-\mu'_1}{\lambda-\mu_1} & 0 & \frac{-\mu'_1(1+i)}{\lambda-\mu_1} & 0 & \frac{-\mu'_1(1-i)}{\lambda-\mu_1} & 0 \\ 0 & 0 & 0 & \frac{-\mu'_2}{\lambda-\mu_1} & 0 & \frac{-\mu'_2(1+i)}{\lambda-\mu_2} & 0 & \frac{-\mu'_2(1-i)}{\lambda-\mu_2} \\ \frac{-\mu'_1}{\lambda-\mu_1} & 0 & 0 & 0 & \frac{-\mu'_1(1-i)}{\lambda-\mu_1} & 0 & \frac{-\mu'_1(1+i)}{\lambda-\mu_1} & 0 \\ 0 & \frac{-\mu'_2}{\lambda-\mu_1} & 0 & 0 & 0 & \frac{-\mu'_2(1-i)}{\lambda-\mu_2} & 0 & \frac{-\mu'_2(1+i)}{\lambda-\mu_2} \\ \frac{-\mu'_1(1-i)}{\lambda-\mu_1} & 0 & \frac{-\mu'_1(1+i)}{\lambda-\mu_1} & 0 & 0 & 0 & \frac{-\mu'_1}{\lambda-\mu_1} & 0 \\ 0 & \frac{-\mu'_2(1-i)}{\lambda-\mu_2} & 0 & \frac{-\mu'_2(1+i)}{\lambda-\mu_2} & 0 & 0 & 0 & \frac{-\mu'_2}{\lambda-\mu_1} \\ \frac{-\mu'_1(1+i)}{\lambda-\mu_1} & 0 & \frac{-\mu'_1(1-i)}{\lambda-\mu_1} & 0 & \frac{-\mu'_1}{\lambda-\mu_1} & 0 & 0 & 0 \\ 0 & \frac{-\mu'_2(1+i)}{\lambda-\mu_2} & 0 & \frac{-\mu'_2(1-i)}{\lambda-\mu_2} & 0 & \frac{-\mu'_2}{\lambda-\mu_1} & 0 & 0 \end{pmatrix}$$

Suppose w = C(I + G)u where matrix G with elements g_{ij} satisfies the formula:

$$GM-MG=-T-C^{-1}PC$$

$$g_{ij}=0,\quad g_{ij}=\frac{(-C^{-1}C'-C^{-1}PC)_{ij}}{\tilde{\mu}_j-\tilde{\mu}_i},\quad i\neq j$$

$$\omega'=C'(I+G)u+CG'u+C(I+G)u'$$
$$C'(I+G)u+CG'u+C(I+G)u'=$$
$$U^{-1}AUC(I+G)u-PC(I+G)u$$

Let us pre-multiply this equation by C^{-1}:

$$T(I+G)u+G'u+(I+G)u'=M(I+G)u-C^{-1}PC(I+G)u$$
$$Tu+TGu+G'u+(I+G)u'=(M+MG)u-C^{-1}PC(I+G)u$$
$$Tu+TGu+G'u+(I+G)u'=(M+MG)u+Tu-C^{-1}PCu+$$
$$C^{-1}PCu-C^{-1}PCGu$$

By pre-multiplying this equation by $(I + G)^{-1}$, we will have the system of equations:

$$u'=(M+\theta(x,\lambda))u \qquad (3)$$

If we suppose that in Eq. 3 with constant i, $(i = \overline{1, 8})$:

$$u = s\times\exp\left\{\int_0^x \tilde{\mu}_i(t,\lambda)dt\right\}$$

where, $s = (s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8)$ is the unknown vector function, we will have a system of equations of first order:

$$\frac{d}{dx}s_i(x,\lambda) = v_i(x,\lambda)s_i(x,\lambda)+\sum_{m=1}^{\infty}\theta_{im}(x,\lambda)s_m(x,\lambda),\ i = \overline{1, 8} \tag{4}$$

Where:

$$v_i(x,\lambda) = \tilde{\mu}_j(x,\lambda)-\tilde{\mu}_i(x,\lambda)$$

$$\theta = \left\|\theta_{im}(x,\lambda)\right\|_{i,\ m=1}^{8}$$

The same as in Eq. 3. Let us demonstrate that:

$$\int_0^x \left\|\theta(x,\lambda)\right\|dx = o(1),\quad \lambda\in\Gamma,\ \lambda\to\infty \tag{5}$$

Here and thereafter, we will consider that the norm of matrix is the sum of absolute values of elements. Let us consider elements g_{ij} of matrix G. All $g_{ij}(x, \lambda)$ are constrained above by linear combinations of the kind:

$$\frac{\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}, \frac{\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{(\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]}{\sum_{i=1}^{2}K_i(\lambda-\mu_i(x))^{\frac{1}{4}}},\ K_i = \text{const.}$$

If $\lambda\in\Gamma$, $\lambda\to\infty$, then $\lambda = \sigma+i\tau$, $\sigma>0$, $\tau = \sigma^{\gamma}$, $0<\gamma<1$. It means that:

$$(\lambda-\mu_i(x)) = (\sigma+i\tau-\mu_i(x)) = (\sigma-\mu_i(x))\left(1+i\frac{\tau}{\sigma-\mu_i(x)}\right) = (\sigma-\mu_i(x))\left(1+i\frac{\sigma^{\gamma}}{\sigma-\mu_i(x)}\right) \tag{6}$$

As long as $\mu_i(x)<0$:

$$\frac{\sigma^{\gamma}}{\sigma-\mu_i(x)}\le\sigma^{\gamma-1}\to 0$$

If $\delta\to 0$ evenly on x, $0\le x<+\infty$. Consequently: from Eq. 6 we get that: $\lambda-\mu_i(x)\sim\sigma-\mu_i(x)\lambda\in\Gamma,\ \lambda\to\infty$ evenly on x, $0\le x<+\infty$. Next:

$$\frac{\left|\mu'_i(x)\right|}{\left|(\lambda-\mu_i(x))\right|^{\frac{5}{4}}}\to 0,\ \lambda\in\Gamma,\ \lambda\to\infty$$

evenly on x, $x\in[0, x_0]$ because:

$$\frac{\left|\mu_i'(x)\right|}{\left|(\lambda-\mu_i(x))\right|^{\frac{5}{4}}}\le\frac{1}{\sigma^{\frac{5}{4}}}\to 0\quad \sigma\to+\infty$$

If:

$$x\in\left[x_0,+\infty\right),\ \frac{\left|\mu_i'(x)\right|}{\left|(\lambda-\mu_i(x))\right|^{\frac{5}{4}}}\le K\frac{1}{(\sigma-\mu_i(x))^{\frac{5}{4}-\alpha}}\to 0$$

If $\sigma\to+\infty$ evenly on x. As for the summand:

$$\frac{\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]}{\sum_{i=1}^{2}K_i(\lambda-\mu(x))^{\frac{1}{4}}},\quad K_i = \text{const.}$$

Then the numerator is evenly constrained according to conditions 1 and 2. The consequent goes up infinitely if $\lambda\in\Gamma$, $\lambda\to\infty$ evenly on x because:

$$\left|\lambda-\mu_i(x)\right|^{\frac{1}{4}}\sim\left|\sigma-\mu_i(x)\right|^{\frac{1}{4}}>\sigma^{\frac{1}{4}}\to\infty$$

Thus, $\|G(x, \lambda)\|$ if $\lambda\in\Gamma$, $\lambda\to\infty$ evenly on x, $0 = x\le x<+\infty$ can be made, for instance, 1/2. Consequently, if $\lambda\in\Gamma$, $\lambda\to\infty$, matrix (I+G) has a bounded inverse matrix $(I+G)^{-1}$.

So, $\|I+G\|\le\text{const}$, $\|(I+G)^{-1}\| = \text{const}$, $\lambda\in\Gamma$, $\lambda\to\infty$ evenly on x. Therefore, for the correctness of Eq. 5 it is enough to show that these values are valid for matrixes TG, G', C^{-1} PCG. Let us now proceed to the evaluation of matrix TG elements. They are constrained above by linear combinations of functions of the kind:

$$\left|\begin{matrix}\frac{\mu_i'(x)}{\left(\lambda-\mu_i(x)\right)} & \frac{\mu_i'(x)}{\lambda-\mu_i(x)^{\frac{5}{4}}}\\ \frac{\mu_i'(x)}{\left(\lambda-\mu_i(x)\right)} & \frac{\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]}{(\lambda-\mu_i(x))^{\frac{1}{4}}}\end{matrix}\right|,\ i, j = 1, 2 \tag{7}$$

Let us prove that:

$$\int_0^{\infty}\|TG\|dx = o(1),\ \lambda\in\Gamma,\ \lambda\to\infty$$

$$\int_0^{\infty}\left|\frac{(\mu_i'(x))^2}{\lambda-\mu_i(x))^{\frac{9}{4}}}\right|dx = \int_0^{x_0}\left|\frac{(\mu_i'(x))^2}{\lambda-\mu_i(x))^{\frac{9}{4}}}\right|dx+\int_{x_0}^{\infty}\left|\frac{(\mu_i'(x))^2}{\lambda-\mu_i(x))^{\frac{9}{4}}}\right|dx \quad (8)$$

That the first summand in the right part of Eq. 8 is o(1) $\lambda\to\infty$, $\lambda\in\Gamma$ results from the continuity of the subintegral function and availability of λ in the consequent. As for the second summand, then:

$$\int_{x_0}^{\infty}\left|\frac{(\mu_i'(x))^2}{\lambda-\mu_i(x))^{\frac{9}{4}}}\right|dx \le K\int_{x_0}^{\infty}\frac{(\mu_i'^2(x))}{\sigma-\mu_i(x))^{\frac{9}{4}}}dx \le K_1\int_{x_0}^{\infty}\left|\frac{(\mu_i'(x))^2}{\lambda-\mu_i(x))^{\frac{9}{4}}}\right|dx =$$

$$K_2(\sigma-\mu_i(x))^{\alpha-\frac{5}{4}}\Big|_{x_0}^{\infty}\to 0 \text{ if } \delta\to+\infty \text{ because } \alpha<\frac{5}{4}$$

As to the integral in the second function in Eq. 7, again we break the integral into $\int_{x_0}^{\infty}$ and $\int_0^{x_0}\to 0$ if $\lambda\in\Gamma$, $\lambda\to\infty$ because the subintegral function is continuous and contains λ in the consequent. According to conditions 1 and 2:

$$\int_{x_0}^{\infty}\left|\frac{\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}\right]\right|dx \le$$

$$K\int_{x_0}^{\infty}\frac{|\mu_i'(x)|}{(\sigma-\mu_i(x))^{\frac{5}{4}}}dx = K_1(\sigma-\mu_i(x))^{\frac{1}{4}}\Big|_{x_0}^{\infty} = o(1) \text{ if } \delta\to\infty$$

Now, Let us prove that:

$$\int_0^{\infty}\|G'(x,\lambda)\|dx = o(1)$$

For that, we are going to use values that we got before from the elements of matrix G. In case when:

$$g_{ij}(x,\lambda) = \frac{K_{ij}\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}.$$

$$(g_{ij})' = \left(\frac{K_{ij}\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}\right)' = K_{ij}\frac{\mu_i''(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}+K_{ij}\frac{5\mu_i'(x)\mu_i'(x)}{4(\lambda-\mu_i(x))^{\frac{9}{4}}}.$$

Consequently:

$$\int_0^{\infty}|g_{ij}'|dx \le K_{ij}\int_0^{\infty}\left|\frac{\mu_i''(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}+\frac{\mu_i'^2(x)}{(\lambda-\mu_i(x))^{\frac{9}{4}}}\right|dx$$

Let us break this integral into $\int_0^{x_0}+\int_{x_0}^{\infty}$ then:

$$\int_0^{x_0}\left|\frac{\mu_i''(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}+\frac{\mu_i'^2(x)}{(\lambda-\mu_i(x))^{\frac{9}{4}}}\right|dx = o(1),\ \lambda\in\Gamma,\ \lambda\to\infty$$

As far as the subintegral functions are continuous and $\lambda-\mu_i(x)\neq 0$. Next because of condition 4:

$$\int_0^{x_0}\left|\frac{\mu_i'^2(x)}{(\lambda-\mu_i(x))^{\frac{9}{4}}}\right|dx \le K\int_{x_0}^{\infty}\frac{|\mu_i'(x)|}{(\sigma-\mu_i(x))^{\frac{9}{4}-\alpha}}dx = K\frac{(\sigma-\mu_i(x))^{a-\frac{5}{4}}}{a-\frac{5}{4}}\Bigg|_{x_0}^{\infty} =$$

$$o(1),\sigma\to\infty \text{ as } \alpha<\frac{5}{4},$$

$$\int_{x_0}^{\infty}\left|\frac{\mu_i''(x)}{(\lambda-\mu_i(x))^{\frac{9}{4}}}\right|dx \le K\int_{x_0}^{\infty}\frac{|\mu_i'(x)|}{|(\sigma-\mu_i(x))|^{\frac{5}{4}}}dx \le \frac{1}{\sigma^{\frac{5}{4}}}\to 0,\sigma\to+\infty$$

For elements:

$$(g_{ij})' = \left(\frac{K_{ij}\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]}{\sum_{i=1}^{2}K_i(\lambda-\mu_i(x))^{\frac{1}{4}}}\right)' =$$

$$K_{ij}\frac{\varphi''(x)}{\sum_{i=1}^{2}K_i(\lambda-\mu_i(x))^{\frac{1}{4}}}\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]+$$

$$\frac{K_{ij}}{8}\frac{\varphi'(x)-\left(\left(-\mu_i'(x)\right)(\lambda-\mu_j(x))-\mu_j'(x)(\lambda-\mu_i(x))\right)}{\sum_{i=1}^{2}K_i(\lambda-\mu_i(x))^{\frac{1}{4}}(\lambda-\mu_j(x))^2}$$

$$\left[\left(3\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{5}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{7}{8}}\right]-$$

$$K_{ij}\frac{\varphi'(x)K_i\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right](-\mu_i'(x))}{\sum_{i=1}^{2}K_i(\lambda-\mu_i(x))^{\frac{5}{4}}} \quad (9)$$

Then:

$$\int_0^{\infty} K_{ij} \frac{\varphi''(x)}{\sum_{i=1}^{2} K_i(\lambda-\mu_i(x))^{\frac{1}{4}}}\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]dx =$$

$$\int_0^{x_0} K_{ij} \frac{\varphi''(x)}{\sum_{i=1}^{2} K_i(\lambda-\mu_i(x))^{\frac{1}{4}}}\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]dx+$$

$$\int_{x_0}^{\infty} K_{ij} \frac{\varphi''(x)}{\sum_{i=1}^{2} K_i(\lambda-\mu_i(x))^{\frac{1}{4}}}\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]dx \quad (10)$$

The first summand in the right part of Eq. 10 is o(1) if $\lambda\in\Gamma$, $\lambda\to\infty$ results from the continuity of the subintegral function and availability of λ in the consequent. For the second summand in Eq. 10, the following values are correct:

$$\int_{x_0}^{\infty} K_{ij}\left|\frac{\varphi''(x)}{\sum_{i=1}^{2} K_i(\lambda-\mu_i(x))^{\frac{1}{4}}}\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\right|dx \le$$

$$\frac{K_{ij}}{K_1}\int_{x_0}^{\infty}\left|\frac{\varphi''(x)\left(B^{\frac{3}{8}}+B^{\frac{1}{8}}\right)}{(\sigma-\mu_i(x))^{\frac{1}{4}}}\right|dx \le \text{const}\int_{x_0}^{\infty}\left|\frac{\varphi''(x)}{(\lambda-\mu_i(x))^{\frac{1}{4}}}\right|dx = o(1)$$

If $\lambda\in\Gamma$, $\lambda\to\infty$ according to condition 3. Next, Let us estimate the second summand in Eq. 9 that is:

$$\int_0^{\infty} K_{ij}\left|\frac{\varphi'(x)-\left(\left(-\mu_i'(x)\right)(\lambda-\mu_j(x))-\mu_j'(x)(\lambda-\mu_i(x))\right)}{\sum_{i=1}^{2} K_i\left(\lambda-\mu_i(x)\right)^{\frac{1}{4}}(\lambda-\mu_j(x))^2}\right|$$

$$\left|\left[\left(\frac{3}{8}\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{5}{8}}+\frac{1}{8}\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{7}{8}}\right]\right|dx =$$

$$\int_0^{x_0} K_{ij}\left|\frac{\varphi'(x)-\left(\left(-\mu_i'(x)\right)(\lambda-\mu_j(x))-\mu_j'(x)(\lambda-\mu_i(x))\right)}{\sum_{i=1}^{2} K_i\left(\lambda-\mu_i(x)\right)^{\frac{1}{4}}(\lambda-\mu_j(x))^2}\right|$$

$$\left|\left[\left(\frac{3}{8}\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{5}{8}}+\frac{1}{8}\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{7}{8}}\right]\right|dx+$$

$$\int_x^{\infty} K_{ij}\left|\frac{\varphi'(x)-\left(\left(-\mu_i'(x)\right)(\lambda-\mu_j(x))-\mu_j'(x)(\lambda-\mu_i(x))\right)}{\sum_{i=1}^{2} K_i\left(\lambda-\mu_i(x)\right)^{\frac{1}{4}}(\lambda-\mu_j(x))^2}\right| \quad (11)$$

$$\left|\left[\left(\frac{3}{8}\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{5}{8}}+\frac{1}{8}\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{7}{8}}\right]\right|dx$$

The first integral in the right part of Eq. 11 is o(1) if $\lambda\in\Gamma$, $\lambda\to\infty$ results from the continuity of the subintegral function and $\lambda-\mu_i(x)\neq 0$. For the second summand in Eq. 11, the following values are correct:

$$\int_{x_0}^{\infty} K_{ij}\left|\frac{\varphi'(x)-\left(\left(-\mu_i'(x)\right)(\lambda-\mu_j(x))-\mu_j'(x)(\lambda-\mu_i(x))\right)}{\sum_{i=1}^{2} K_i\left(\lambda-\mu_i(x)\right)^{\frac{1}{4}}(\lambda-\mu_j(x))^2}\right|$$

$$\left|\left[\left(\frac{3}{8}\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{5}{8}}+\frac{1}{8}\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{-\frac{7}{8}}\right]\right|dx \le$$

$$\le K_{ij}\int_{x_0}^{\infty}\frac{\left|\varphi'(x)\right|\left|\left(\left(-\mu_i'(x)\right)(\lambda-\mu_j(x))-\mu_j'(x)(\lambda-\mu_i(x))\right)\right|\frac{3(B^{-\frac{5}{8}}+B^{-\frac{7}{8}})}{8}}{\sum_{i=1}^{2} K_i\left(\lambda-\mu_i(x)\right)^{\frac{1}{4}}(\lambda-\mu_j(x))^2}dx \le$$

$$\le A_1\int_{x_0}^{\infty}\frac{\left|\mu_i'(x)(\lambda-\mu_j(x))\right|+\left|\mu_j'(x)(\lambda-\mu_i(x))\right|}{(\sigma-\mu_i(x))^{\frac{1}{4}}\left|\sigma-\mu_j(x)\right|^2}dx \le$$

$$\le A_2\int_{x_0}^{\infty}\left|\frac{\mu_j'(x)}{(\lambda-\mu_j(x))^{\frac{5}{4}}}\right|dx = o(1),\quad A_1, A_2 = \text{const}, \lambda\to\infty, \lambda\in\Gamma$$

For the third integral in Eq. 9, the following values are correct:

$$\int_0^{\infty} K_{ij}\frac{\varphi'(x)K_1\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\mu_i'(x)}{4\sum_{i=1}^{2} K_i(\lambda-\mu_i(x))^{\frac{5}{4}}} =$$

$$\int_0^{x_0} K_{ij}\frac{\varphi'(x)K_i\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\mu_i'(x)}{4\sum_{i=1}^{2} K_i(\lambda-\mu_i(x))^{\frac{5}{4}}}dx +$$

$$\int_{x_0}^{\infty} K_{ij}\frac{\varphi'(x)K_i\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\mu_i'(x)}{4\sum_{i=1}^{2}K_i(\lambda-\mu_i(x))^{\frac{5}{4}}}dx \quad (12)$$

That the first summand in the right part Eq. 12 is o(1) if $\lambda\in\Gamma$, $\lambda\to\infty$ results from the continuity of the subintegral function and $\lambda-\mu_i(x)\neq 0$. For the second summand in Eq. 12, the following values are correct:

$$\int_{x_0}^{\infty}\frac{K_{ij}}{K_j}\frac{\left|\varphi'(x)\mu_i'(x)(B^{\frac{3}{8}}+B^{\frac{1}{8}})\right|}{\sum_{i=1}^{2}\left|\sigma-\mu_i(x)\right|^{\frac{5}{4}}}dx \le A_3\int_{x_0}^{\infty}\left|\frac{\mu_i'(x)}{\sigma-\mu_i(x))^{\frac{5}{4}}}\right|dx =$$

$o(1), A_3 = \text{const}, \lambda\to\infty, \lambda\in\Gamma$

Thus, $\int_0^{\infty}\|G'(x,\lambda)\|dx = o(1)$, if $\lambda\to\infty$, $\lambda\in\Gamma$. Let us prove that:

$$\int_0^{\infty}\left\|C^{-1}PCG\right\|dx = o(1)$$

Since, the elements of matrix C^{-1} PC are constrained above by linear combinations of the kind:

$$\left|\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}-\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\right|$$

Then, using the formulas we have found for the elements of matrix G, Let us write out the elements of matrix C^{-1} PCG:

$$\left(\left|\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}-\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\right|\right)K_{ij}\frac{\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}} \text{ and}$$

$$\left(\left|\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}-\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\right|\right)^2\frac{K_{ij}}{\sum_{i=1}^{2}(\lambda-\mu_i(x))^{\frac{1}{4}}}$$

Let us break the integral from the first formula into the sum:

$$\int_0^{\infty}\left|\left(\left|\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}-\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\right|\right)K_{ij}\frac{\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}\right|dx =$$

$$\int_0^{x_0}\left|\left(\left|\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}-\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\right|\right)K_{ij}\frac{\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}\right|dx+$$

$$\int_{x_0}^{\infty}\left|\left(\left|\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}-\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\right|\right)K_{ij}\frac{\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}\right|dx \quad (13)$$

As for the first summand in Eq. 13, it is o(1) if $\lambda\in\Gamma$, $\lambda\to\infty$ and results from the continuity of the subintegral function and $\lambda-\mu_i(x)\neq 0$. For the second summand in Eq. 13, the numerator is constrained according to conditions 1 and 2. The subsequent goes up infinitely if $\lambda\in\Gamma$, $\lambda\to\infty$ evenly on x as long as $|\lambda-\mu_i(x)|\sim|\sigma-\mu_i(x)|$:

$$\int_{x_0}^{\infty}\left|\left(\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\right)K_{ij}\frac{\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}\right|dx \le$$

$$K_{ij}K(B^{\frac{3}{8}}+B^{\frac{1}{8}})\int_0^{\infty}\left|\frac{\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}\right|dx \le K_{ij}\overline{K}\int_0^{\infty}\left|\frac{\mu_i'(x)}{(\lambda-\mu_i(x))^{\frac{5}{4}}}\right|dx = o(1),$$

if $\sigma\to+\infty$

For the second element of matrix C^{-1} PCG, the following values are correct:

$$\int_{x_0}^{\infty}\left|\left(\varphi'(x)\left[\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{3}{8}}+\left(\frac{\lambda-\mu_i(x)}{\lambda-\mu_j(x)}\right)^{\frac{1}{8}}\right]\right)\frac{K_{ij}}{\sum_{i=1}^{2}(\lambda-\mu_i(x))^{\frac{1}{4}}}\right|dx \le$$

$$K^2(B^{\frac{3}{8}}+B^{\frac{1}{8}})K_{ij}\int_{x_0}^{\infty}\frac{dx}{(\sigma-\mu_i(x))^{\frac{1}{4}}} = o(1)$$

If $\sigma\to\infty$ according to conditions 1 and 3. For the further proof, we need the lemma proved by Kostyuchenko and Belogrud (1979).

Lemma: Let the following conditions be fulfilled. Functions $\nu_k(x, \lambda)$ locally are summarised at any value $\lambda\in\Gamma$; At some i, $1\le i\le n$, $\nu(x, \lambda)\equiv 0$ and if $i\neq j$ of Re function $\{\nu_k(x, \lambda)\}$ do not change the sign if $x>x_0$, for sufficiently larger x_0 and $\lambda\in\Gamma$; Functions $\theta_{km}(x, \lambda)$ are summarised on $[0, \infty)$ and:

$$\int_a^b\|\theta(x, \lambda)\|dx = o(1), \lambda\to\infty, \lambda\in\Gamma$$

Then system 4 has the solution that is satisfying if $\lambda\to\infty$, $\lambda\in\Gamma$, $s_k(x,\lambda) = 1+o(1)$ $s_m(x,\lambda) = {}_0(1)$, $m\neq k$ is even relating to x, $x\in[0,\infty)$. Let us demonstrate that $\mathrm{Re}(\tilde{\mu}_i(x)-\tilde{\mu}_j(x))$ does not change the sign at sufficiently large x_0, $i,j=\overline{1,8}$, $i\neq j$. Let us demonstrate, for example, for case i = 1, j = 2:

$$\mathrm{Re}(\tilde{\mu}_1(x)-\tilde{\mu}_2(x)) = \mathrm{Re}((\lambda-\mu_1(x))^{\frac{1}{4}}+(\lambda-\mu_2(x))^{\frac{1}{4}}) =$$

$$\mathrm{Re}(2(\sigma+i\tau-\mu_i)^{\frac{1}{4}}) = 2\mathrm{Re}\left((\lambda-\tilde{\mu}_1(x))^{\frac{1}{4}}+(\lambda-\tilde{\mu}_1(x))^{\frac{1}{4}}\right)$$

As long as $\lambda = \sigma+i\tau$, $\tau = \sigma^{\gamma}$, $0<\gamma<1$, $\tau>0$, then if $x\to\infty$, we have:

$$2\mathrm{Re}\left\{(\sigma+i\tau-\mu_1(x))^{\frac{1}{4}}\right\} = 2\mathrm{Re}\left\{\left(-\mu_1(x)\left(\frac{\sigma+i\tau}{\mu_1(x)}+1\right)\right)^{\frac{1}{4}}\right\} =$$

$$2\mathrm{Re}\left\{\left(-\mu_1(x)\left(\frac{\sigma\left(1+\frac{i\sigma^{\gamma}}{\sigma}\right)}{\mu_1(x)}+1\right)\right)^{\frac{1}{4}}\right\} =$$

$$2\mathrm{Re}\left\{\left(-\mu_1(x)\left(\frac{\sigma(1+o(1))}{\mu_1(x)}+1\right)\right)^{\frac{1}{4}}\right\} = 2\mathrm{Re}\left\{\left(-\mu_1(x)(1+o(1))\right)^{\frac{1}{4}}\right\}$$

When $|\mu_i(x)|\to\infty$, $x\to\infty$ there are two possible cases: $|\mu_1(x)|\to-\infty$ then $-\mu_1(x)>0$, consequently, $(-\mu_1(x))^{1/4}$ real number and $\mathrm{Re}\{(\mu_1(x))|^{1/4}\}$ does not change the sign at larger x. $\mu_1(x)\to+\infty$ then $= \mu_1(x)<0$, consequently:

$$(-\mu_1(x))^{\frac{1}{4}} = |\mu_1(x)|^{\frac{1}{4}}\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)$$

Then:

$$2\mathrm{Re}\left\{|\mu_1(x)|^{\frac{1}{4}}\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right\} = \sqrt{2}|\mu_1(x)|^{\frac{1}{4}}$$

Also does not change the sign at large x. Next as we showed that:

$$\int_0^{\infty}\|\theta(x,\lambda)\|dx = o(1)$$

Then system 4 is L-diagonal and the Lemma is applicable to it. Now by the means of formulas z = Uw, w = C(E+G)u, we can return from vector u to the required vector z. Then taking into account the formulas for the elements of matrixes U and C and that $(G)_{ij}(x,\lambda)\to0$ if $\lambda\to\infty$, $\lambda\in\Gamma$ evenly on x, we will receive the necessary asymptotical formulas.

CONCLUSION

The asymptotic formulas obtained for the fundamental system of solutions of the Eq. 1 if $\lambda\in\Gamma$, $\lambda\to\infty$, evenly on x: $0\le x<+\infty$ in case of slow rotation of the eigenvectors of the matrix Q(x) are necessary for investigating the asymptotic behavior of the spectrum of a differential operator in the space vector-valued functions. Asymptotic formulas that are uniform in x are important both from the point of view of the asymptotic theory of differential equations and from the point of view of differential operators.

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REFERENCES

Berdenova, G.Z., 2015. On the deficiency indices of differential operators of vector space. Proceedings of the 11th International Scientific Conference on Students, Masters and Young Scientists, April 10-11, 2015, Lomonosov Moscow State University, Moscow, Russia, pp: 28-29.

Fedoryuk, M.V., 1983. Asymptotical Methods for the Linear Ordinary Differential Equations. Russian Academy of Science, İoscow, Russia, Pages: 352.

Kostyuchenko, A.G. and I.S. Sargsyan, 1979. The Distribution of Eigenvalues: Self Adjoint Ordinary Differential Operators. Nauka, Moscow, Russia, Pages: 402.

Sultanaev, Y.T. and G. Berdenova, 2014. The deficiency indices of singular differential operators in the space of vector-valued functions. Proceedings of the 2nd International Conference on Analysis and Applied Mathematics (ICAAM'14), September 11-13, 2014, International Advisory Board (IAB), Shymkent, Kazakhstan, pp: 54-55.

Sultanayev, Y.T. and O.V. Myakinova, 2009. On indexes of the defect of singular differential operator of fourth order in space of vector functions. Matem. Notes, 86: 950-953.

Sultanayev, Y.T., 1974. On the asymptotics of a range of the differential operator in the space of vector functions. Dif. Equ., 10: 1673-1683.