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Incorporating Computer Algebra System in Differential Equations Syllabus

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Abstract: Innovative teaching and learning is an alternative way for educators to deliver the lessons to students. Incorporating computer algebra system is a branch under the innovative teaching and learning. This study incorporates computer algebra system in teaching differential equations subject, a common engineering mathematics subject in Universiti Kebangsaan Malaysia (UKM). This study aims to introduce laboratory classes for engineering mathematics students in UKM. These laboratory sessions align with the course outcome of the subject. The 14 weeks syllabus of the subject is compared with that of 60 universities across the world. The common topics between UKM's differential equations syllabus with other countries are identified. First-order differential equations, higher-order differential equations and Laplace transform are the common topics with UKM's differential equations syllabus. Thus, the 3 topics will be chosen for laboratory sessions. In addition, recent researchers who had conducted laboratory sessions in differential equations subject is discussed in this study. Positive feedback and enormous benefits were obtained by students through this innovative way of teaching differential equations. Thus, by incorporating computer algebra system in differential equations syllabus in UKM it is hoped that the engineering mathematics students will get a better understanding of the conceptual knowledge.

Key words: Computer algebra system, differential equations, curriculum, common topics, syllabus, UKM

INTRODUCTION

Differential equations is one of the common engineering mathematics subjects in UKM. This 4 credit-hours subject is taken by students from electrical, mechanical, civil and chemical engineering in UKM. There are two pre-requisite subjects to be taken by students prior to this subject. They are vector calculus and Linear algebra subjects. Differential equations is taken in the third semester of the study in an engineering course.

The course study trend of engineering mathematics in UKM is lecturing and tutorial. These traditional methods of teaching need to be improved by means of incorporating computer algebra system in teaching engineering mathematics. This means that students study via. lecture, tutorial and lab. Incorporating computer algebra system proved to increase student's learning interest (Chen, 2013). Students tend to feel excited and computer algebra system prevents students from losing sight of the subjects (Suanmali, 2008). Students potray higher level of confidence in studying engineering mathematics when computer algebra system was implemented in their syllabus (Godarzi *et al*., 2009).

MATERIALS AND METHODS

The detail of the weekly syllabus of the differential equations for engineering students at UKM is given in Table 1. In the first week, students are introduced to differential equations. From the second week to the fourth week, students are exposed to first-order differential equations and modeling with systems of the first-order differential equations. From week 5 until week 7, students are taught higher-order differential equations and modeling with systems of higher-order differential equations. The students will learn Laplace transform in week 8 and 9. This is followed by a series of solutions of

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linear differential equations in week 10. The topics for week 11 are orthogonal functions and Fourier series. In week 12 and 13 students will be introduced to boundary value-problem in rectangular coordinates. Lastly, they will study integral transform in week 14.

The UKM's differential equations syllabus is compared with 60 other universities. The 20 universities from the United States, 20 universities from the United Kingdom, 10 universities from Ocenia, 5 universities from Asia and 5 universities from Malaysia are chosen for this study.

The 20 United States universities chosen for this study are Massachusetts Institute of Technology, Stanford University, University of California, California Institute of Technology, Princeton University, Georgia Institute of Technology, Carnegie Mellon University, University of Texas at Austin, University of Michigan, Cornell University, University of Illinois at Urbana Champaign, Northwestern University, University of Wisconsin Madison, Columbia University, University of Washington, University of Minnesota, Rice university, Purdue University, Ohio State University and Pennsylvania State University.

Table 1: Weekly syllabus of differential equations in UKM

Weeks	Syllabus
1	Introduction to differential equations
2	First-order Differential Equations and modeling with systems of first-order DEs
3	First-order Differential Equations and modeling with systems of first-order DEs
4	First-order Differential Equations and modeling with systems of first-order DEs
5	Higher-order Differential Equations and modeling with systems of higher-order DEs
6	Higher-order Differential Equations and modeling with systems of higher-order DEs
7	Higher-order Differential Equations and modeling with systems of higher-order DEs
8	The Laplace transform
9	The Laplace transform
10	Series solutions of linear differential equations
11	Orthogonal functions and Fourier series
12	Boundary value-problem in rectangular coordinates
13	Boundary value-problem in rectangular coordinates
14	Integral transforms

The 20 United Kingdom Universities are University of Cambridge, University of Oxford, Imperial College London, University of Manchester, University College London, University of Edinburgh, University of Nottingham, University of Bristol, University of Southhampton, University of Leeds, University of Sheffield, University of Liverpool, The University of Warwick, University of Bath, University of Strathclyde, Cardiff University, New Castle University, Queen Marry University of London, University of Glassgow and University of Surrey.

Ten universities from Ocenia are chosen for this comparative study. They are University of Melbourne, University of Queensland Australia, University of Sdyney, Monash University, University of New South Wales, University of Auckland, University of Otago, University of Canterbury, Victoria University of Wellington and Massey University.

Five Asian Universities are the National University of Singapore, Nanyang Technological University, Hong Kong University of Science and Technology, The University of Hong Kong and Kyoto University.

University Malaya, Universiti Teknologi Malaysia, Universiti Sains Malaysia, Universiti Putra Malaysia and Universiti Teknologi Mara are the 5 Malaysian Universities selected for this study.

Firstly, each university website was explored to get into the engineering faculty. Then, the engineering mathematics subjects are explored. Next, the differential equations syllabus of UKM is mapped with other universitie's differential equation syllabus. A table was prepared for the convenience whereby if the other universitie's syllabi have the same weekly topic as UKM's 'x' will be given for that week's syllabus. Table 2 and 3 illustrate the mapping of UKM's syllabus with other universitie's syllabi.

Table 2: The matching of UKM's and other universities differential equations syllabus

		Values																												
Weeks	Topic/University	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	Introduction to differential equations				x		x		x		x	x	x	x	x	x	x	x		x					x		x	x		x	
2	First-order differential equations and modeling with systems of first-order DEs	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
3	First-order differential Equations and modeling with systems of first-order DEs	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
4	First-order differential	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x

Table 2: Continue

		Values																													
Weeks	Topic/University	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
	Equations and modeling with systems of first-order DEs																														
5	Higher-order differential equations and modeling with systems of higher-order DEs	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
6	Higher-order differential equations and modeling with systems of higher-order DEs	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
7	Higher-order differential equations and modeling with systems of higher-order DEs	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
8	The Laplace transform	x	x		x	x	x		x	x	x	x		x		x	x		x	x	x	x	x		x	x	x	x		x	x
9	The Laplace transform	x	x		x	x	x		x	x	x	x		x		x	x		x	x	x	x	x		x	x	x	x		x	x
10	Series solutions of linear differential equations					x	x			x	x			x	x				x	x	x		x			x		x			
11	Orthogonal functions and fourier series					x			x		x	x		x									x			x	x	x		x	
12	Boundary value problems in rectangular coordinates					x			x		x	x		x													x			x	
13	Boundary value problems in rectangular coordinates																														
14	Integral transforms																								x	x	x			x	x

Table 3: The matching of UKM's and other universities differential equations syllabus

		Values																													
Weeks	Topic/University	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
1	Introduction to differential equations	x			x		x				x	x	x	x	x	x	x			x		x				x	x	x	x	x	x
2	First-order differential equations and modeling with systems of first-order DEs	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x			x	x		x	x	x	x	x
3	First-order differential equations and modeling with systems of first-order DEs	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x			x	x		x	x	x	x	x
4	First-order differential equations and modeling with systems of first-order DEs	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x			x	x		x	x	x	x	x
5	Higher-order differential equations and modeling with systems of higher-order DEs	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x		x	x	x	x					x	x	x	x	x
6	Higher-order differential equations and modeling with systems of higher-order DEs	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x		x	x	x	x					x	x	x	x	x
7	Higher-order differential equations and modeling with systems of higher-order DEs	x	x	x	x	x	x	x	x	x		x	x	x	x	x	x		x	x	x	x					x	x	x	x	x
8	The Laplace transform	x	x	x	x		x	x		x		x		x	x	x	x			x		x	x	x	x	x		x	x	x	
9	The Laplace transform	x	x	x	x		x	x		x		x		x	x	x	x			x		x	x	x	x	x		x	x	x	

Table 3: Continue

Weeks	Topic/University	Values (31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60)
10	Series solutions of linear differential equations	x x x x x
11	Orthogonal functions and fourier series	x x x x x x x x x x
12	Boundary value problems in rectangular coordinates	x x x x x
13	Boundary value problems in rectangular coordinates	x x x x x
14	Integral transforms	x x x

RESULTS AND DISCUSSION

Once Table 2 and 3 are complete, the total 'x' for 60 universities for week 1 row is calculated. For example, introduction to differential equation's is being taught in 34 out of 60 universities. Therefore, the percentage value is 57% as calculated in the following manner:

$$\frac{34}{60}\times 100 = 57\%$$

With a similar method, the other 13 weeks percentages of weekly topic for differential equations are calculated and tabulated in Table 4. From Table 4, a bar chart is plotted to represent the distribution percentages of weekly topic of differential equation. Figure 1 shows the distribution of the bar chart. The heighest bar is the first-order differential equation. This is followed by higher-order differential equation, Laplace transform, orthogonal functions, series solutions and boundary value-problems. The least percentages are scored by integral transforms. This means that the integrals transform is not a common topic taught in differential equation in the sixty universities. Recently many researchers had incorporated teaching differential equations via. the computer algebra system. A study was incorporated using MATLAB at the University of Queensland where students underwent a self-study session with worksheets consisting of syntaxes and problems (Tonkes *et al*., 2005). A weekly lab session was introduced in 2002 and thereafter a learning model was developed using MATLAB. The learning model was targeted for first year students which was revised over a 3 years period. Learning modules were designed for several subjects including ordinary differential equations subject. Students were introduced to simple commands of MATLAB initially. The next GUIs from MATLAB were used to visualize further mathematical concepts. Students wrote and ran their own coding. Students gave positive comments on incorporating MATLAB in their learning module as the software helped to visualize abstract concepts.

Table 4: Percentage of weekly topic for differential equation

Weeks	Total	Percentage
1	34	57
2	55	92
3	55	92
4	55	92
5	53	88
6	53	88
7	53	88
8	44	73
9	44	73
10	18	30
11	21	35
12	11	18
13	11	18
14	8	13

The two components of education technology were introduced using Maple for ordinary differential equations subject (Kovacheva, 2007). The 1st technology is used in the study process which is Maple itself. The next one is the technology of study process which includes planning, organizing, carrying out and evaluating the entire process. The seven laboratory exercises include the 1st and 2nd orders of differential equations and Laplace transform. Maple aims at exposing student's knowledge for them to be able to work hands on towards a ready-made system. Other than enhancing the depth of a subject, Maple managed to develop student's motivation.

MATLAB was introduces to 2nd year students at Drexel University in laboratory exercises (Abichandani *et al*., 2010). It was introduced in ENGR 232 dynamics engineering system in modeling, simulation and analysis from the first and second orders of ordinary differential equations. A 2 h weekly laboratory session targeted at a number of modeling problems. MATLAB improved student's understanding compared to the traditional teaching method. Scientific software should be embedded throughout the engineering curriculum for students of every course year to provide alternative ways of problrm solving.

A study which selected 10 engineering technology students was carried out to solve ordinary differential equations particularly the 1st and 2nd order differential equations (Maat and Zakaria, 2011). This task was a

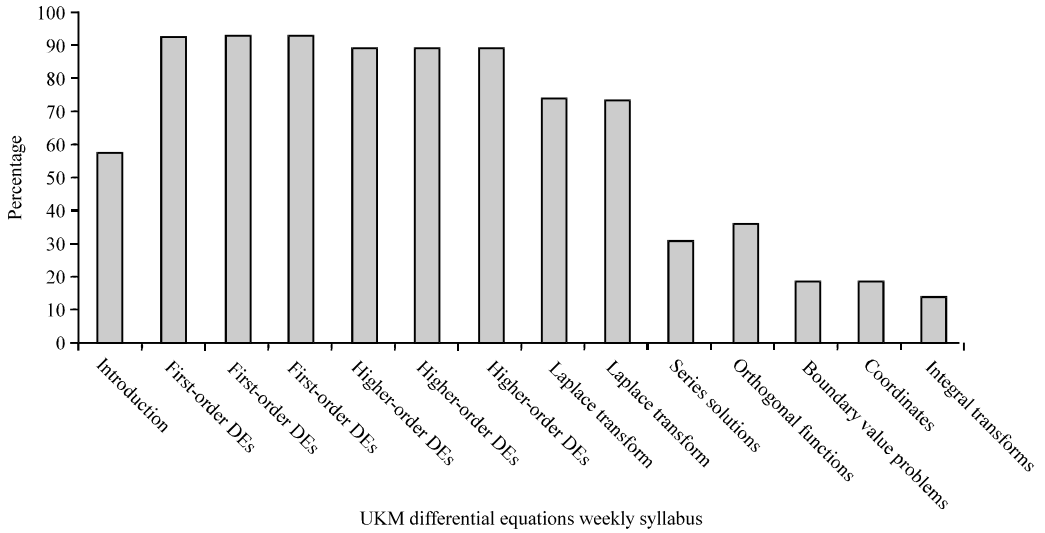

Fig. 1: Distribution of weekly topics of differential equation

comparison between the traditional teaching versus technological tool, Maple. The results from the participants were very positive. Maple enhanced student's understanding and developed creative thinking. This boost their interest in mathematics. Tedious and complex calculations were managed well with the aid of Maple. Interactive activities via. Maple are able to link with real-life engineering applications. Students were very confident studying odinary differential equations with Maple. Maxima, a computer software was introduced to students to enhance the mathematical thinking powers in learning differential equations (Zeynivannezhad, 2014). A preliminary test was conducted prior to the teaching experiments to prove that students were weak in plotting graph. Later, the software was used as an aid to solve 1st and 2nd order differential equations and Laplace transforms. Students were exposed to draw the graphs using Maxima. The step-by-step method of drawing graph helped student to visualize the graph as well as enhanced student's conceptual understanding of the subjects.

MATLAB and CoNum Software were incorporated in the differential equations course at the University of Minho (Carneiro *et al*., 2010). Students study either in the university campus or the lecturers would go to the industrial centre or video conference used as the tool to conduct lectures. CoNum was chosen due to its simplicity and user-friendly manner while MATLAB is due to handling matrix operation. First order differential equation was explored using CoNum and the output was presented via. MATLAB. Students were exposed to solving real problems using effective tools (Table 5).

Table 5: Suggested lab session for differential equation subjects in UKM

Weeks	Lab session
1	First-order Differential Equations and modeling with systems of first-order DEs
2	First-order Differential Equations and modeling with systems of first-order DEs
3	First-order Differential Equations and modeling with systems of first-order DEs
4	Higher-order Differential Equations and modeling with systems of higher-order DEs
5	Higher-order Differential Equations and modeling with systems of higher-order DEs
6	Higher-order Differential Equations and modeling with systems of higher-order DEs
7	The Laplace transform
8	The Laplace transform

Since, computer solutions can be incorrect or inaccurate, they are definitely necessary to validate (Shacham *et al*., 2008). The 1st order differential equation, expecially, the Runge-Kutta algorithm for an interacting tanks simulation problem was analyzed and compared using two different mathematical packages, namely Polymat 6.1 and MATLAB. The algorithm, appropriateness of the default parameter, stiffness of the problem and solution with stiff algorithms and non-stiff algorithms were compared.

Teaching via. computer algebra system in UKM will be conducted by teaching differential equation in labs. A total of eight lab sessions are suggested for this subject. The duration of each lab session are 2 h. The heighest 8% of differential equation topics from Table 4 will be chosen as the lab topics. They are first-order differential equations and modeling with systems of first-order differential equations, higher-order differential equations and modeling with systems of higher-order differential

equations and Laplace transforms. Table 5 illustrates the suggested lab sessions for the differential equations subject in the Faculty of Engineering and Built Environment, UKM.

From Table 5, lab for week 1-3 is first-order differential equations. Researchers (Kovacheva, 2007; Abichandani *et al*., 2010; Maat and Zakaria, 2011; Zeynivannezhad, 2014; Carneiro *et al*., 2010; Shacham *et al*., 2008) are the researchers who have incorporated computer algebra system in the first-order differential equation. Maple, MATLAB, Maxima and Polymat 6.1 are among the software used to teach in the lab.

Lab for week 4-6 is on higher-order differential equations and modeling with systems of higher-order differential equations. Several researchers Kovacheva (2007), Abichandani *et al*. (2010), Maat and Zakaria (2011), Zeynivannezhad (2014) conducted the lab sessions for these topics and found them to be beneficial for students. Among the software used to conduct the lab are Maple, MATLAB and Maxima. The last topic for the suggested lab session is Laplace transform. Maple and Maxima were utilized in teaching Laplace transform via the computer algebra system (Kovacheva, 2007; Zeynivannezhad, 2014).

CONCLUSION

Incorporating computer algebra system in engineering mathematics curriculum is an innovative way of teaching compared to the traditional style of teaching. MATLAB, Maple, Maxima and Polymat 6.1 are among the software introduced in teaching differential equation by some recent researchers. From UKM's 14 weeks of differential equation syllabus, the common topics among UKM and other universities were listed. First-order differential equation and modeling with systems of first-order differential equations, higher-order differential equations and modeling with systems of higher-order differential equations and Laplace transform are among the common topics selected for lab sessions. Students will be studying these topics in lab using selected software for 8 weeks. It is hoped that UKM's engineering mathematics students will have positive feedback on incorporating computer algebra system in their differential equations curriculum.

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