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Fully Stable Quasi-Prime Module

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Abstract: In this research, we shall introduce the concept of fully stable quasi-prime module and give some properties of this type of module.

Key words: Prime module, quasi-prime module, weakle quasi-prime module, fully stable quasi-prime module, properties, module

INTRODUCTION

Let R be a commutative ring uith unity and M be an R-module, we introduce that an R-module M is called fully stable quasi-prime module iff annRM = annR f(N), for every non-zero submodule N of M and f be homomorphism function f: N→M where:

$$\text{annRM} = \{r: r\in R \text{ and } rM = 0\}$$

The main purpose of this research is to investigate the properties of full stable quasi-prime module and we give several characterization of fully stable quasi-prime module. Recall that an R-module is called prime, if annRM = annRN for every non-zero submodule N of M (Al-Bahrany, 1996). The concept of quasi-prime module is introduced by Hassin (1999) where an R-module M is quasi-prime if ann_RN is prime ideal for every non-zero submodule N of M. Recall that M is called weakly quasi-prime module, if annRM = annR rM for each r∉annRM (Hassin, 2011). We show that every quasi-prime module is fully stable quasi-prime module and every fully stable quasi-prime module is weakly quasi-prime module but the converse is not true.

MATERIALS AND METHODS

Fully stable quasi-prime module: In this study, we introduce the concept of fully stable quasi-prime module and give several results about it.

Definition 2.1: Let, N be submodule of an R-module M, then, N is called stable quasi-prime module (briefly sqp) module, if there exist homomorphism function:

$$f: N \rightarrow M; \text{annR}f(N) = \text{annRM}$$

Definition 2.2: An R-module M is called fully stable quasi-prime module (briefiy fsqp) module, if for each N submodule of M is sqp module.

Examples and remarks: Every simple R-module is fsqp module. But the converse is not true, for example, let:

$$f: 2Z \rightarrow Z; f(2a) = 2a+1; a\in Z$$
$$\text{AnnR } f(2a) = 0 = \text{annRZ}$$

So, Z as Z-module is fsqp module but Z is not simple module. Z_8 as Z-module is not fsqp module, since, $ann_ZZ_8 = 8Z$. Let:

$$f:[2]Z \rightarrow Z8; f([2]) = [2]+1 \quad \text{so,}$$
$$\text{annR } f([2]) \neq 8Z$$

So, Z_n as Z-module is fsqp module iff n is prime. Let , $N = 1/p^n+Z$ be submodule of amodule $M = Z_{P^\infty}$. Let, f: N→M; $f(N) = 1/p^n$, so, $ann_Z f(N) = 0$ and $ann_ZM = 0$, so, N is sqp module of M, so, M is fsqp module. Let $N = (0)+Z_6$ be submodule of a module $M = Z+Z_6$ where, f:N→M. f((0)+Z6) = Z6, so, $ann_ZZ_6 = Z_6$ but $ann_ZM = 0$. So, N is not sqp module, so, M is not fsqp module.

Theorem 2.4: Every quasi-prime module is fsqp module.

Proof: Let, M be q.p module to prove M is fsqp module, i.e., to prove annRM = annR f(N) for each submodule N of M and homomorphism function:

$$f: N \rightarrow M, \text{ since, } f(N) \subseteq M, \text{ so, } ann_RM \subseteq ann_Rf(N)$$

To prove $ann_Rf(N)\subseteq ann_RM$: Let x∈annR f(N) and suppose x∉ANNrm, so, xM≠0, so, ther exist $m_1\in M$;

$x(m_1) \neq o$ implies $x \notin$ annR (m_1), since, $x \in$annRf(N) implies xf (N) = 0, since, f is homomorphism, so, f(xN) = f(0) = 0 implise XN for each $n \in N \subseteq M$, so, $n \in M$.

If $(m_1) \subseteq (n)$ then, $n = cm_1$, so, $xn = xcm_1$ implies $0 = x (cm_1)$, so, $x \in ann_R (cm_1)$ but M is qp module, so, $ann_R(cm_1) = ann_R(m_1)$ by Abdul Razak, so, $x \in annR(m_1)$ which is contradiction.

If $(n) \subseteq (m_1)$ then, $m_1 = cn$ then, $xm_1 = x$ cn where, xn = 0 implies $xm_1 = 0$ which is contradiction with hypothesis, so, $x \in ann_R M$.

The converse of theorem is not true, for example, $M = Z_p\infty$ is fsqp module by example 3 but is not qp (Hassin, 1999). But if we addition the condition, we can satisfy the converse.

Theorem 2.5: If M is cyclic fsqp module then M is qp module.

Proof: Let, f: N→M be homomorphism, annRf(N) = annRM, we must prove that M is qp, i.e., $ann_R M$ is prime ideal.

Let $a,b \in R$, $ab \in ann_R M$, suppose $a \notin ann_R M$ and $b \notin ann_R M$, so, abM = 0, so, abM = 0 but $aM \neq 0$ and $bM \neq 0$. There exist $x \in M$, $ax \neq 0$ and $bx \neq 0$, since, M is cyclic, let N = (bx), so, abx = 0 implies $a \in ann_R N$, so, aN = 0, so, f (αN) = F(0) = since, f is homomorphism, so, af(N) implies $a \in ann_R f(N)$ but M is f.s.q.p module, so, $a \in ann_R M$ which is a controduction.

Theorem 2.6: Let, M be cyclic R-module then, the following statement are equivalent:

- M is fsqp module
- M is qp module
- annR(m) = annR(rm), $r \notin$annR(m)

Proof:

- 1→2 (by th. 2.5)
- 2→3 by Hassin (1999)
- 3→1, since, M is cyclic, so, M = (ry), $r \in R$, $y \in M$

To prove annRf(N) = annRM for each f is homomorphism. f: N→M, since, $f(N) \subseteq M$, so, $ann_R M \subseteq ann_R f$ (N). To prove $ann_R f(N) \subseteq ann_R M$. $x \in$annRf(N), so, xf(N) = 0, let $y \in f(N)$, so, $y \in M$. Implies xy = 0 for each $y \in M$, so, $x \in ann_R(y) = ann_R(ry)$, $r \in R$ by hypothesis. So, x(ry) = 0 implies $x \in ann_R M$. So, annRf(N)⊆AnnRM implies. So, M is fsqp module.

RESULTS AND DISCUSSION

Relation between fsqp module and wqp module: In this study, we give the relation between fsqp module and weakly quasi-prime module where an R-module M is called weakly quasi-prime module (briefly wqp), if $ann_R M = ann_R rM$ for $r \notin ann_R M$ (Hassin, 2011).

Theorem 3.1: Every f.s.q.p module is w.q.p module.

Proof: Let, N be submodule of an R-module M. Let, f: N→M be homomorphism function st $ann_R f$ (N) = $ann_R M$. To prove $ann_R rM = ann_R M$ for each $r \notin ann_R M$. Since,

$$rM\ M, \text{ so, let } N = rM, \text{ so, annRM annRN} \quad (1)$$

To prove $ann_R N \subseteq ann_R M$. Let $x \in ann_R N$ implies xN = 0, so, xrM = 0 but f is homomorphism, so, f (xrM) = f (0) = 0, so, x f (rM) = 0 implies $x \in ann_R f$ (rM) but M is f.s.q.p module, so, $x \in ann_R M$.

Note 3.2: The convers of theorem 3-1 is not true, for example. Let, $M = Z+Z_P$; p is prime number is wqp module (Hassin, 2011) but is not f.q.p module see [example and remark 2-3. But, we can give the equivalent by the theorem.

Theorem 3.3: Let, M be cyclic R-module then the following statement are equivalent:

- M is qp module
- M is fsqp module
- M is wqp module

Proof: 1→2 by (2-4 theorem) and 2→3 by (3-1 theorem) and 3→2 by [3, 2-8 prosition] implies M is q.p module and by (2-4 theorem), so, M. Is fsqp module and 3→1 by Hassin (2011).

Recall that an R-module M is called multiplication if for each submodule N of M; N = IM for some ideal I of R (Atani, 2004; Kasch, 1982).

Theorem 3.4: Let, I be ideal of aring R, then, every multiplication wqp module is fs qp module.

Proof: Let, N be submodule of an R-module M, since, M is multiplication, then, N = IM, so, ther exist a homomorphism function.

f: IM M, to prove annRf (N) = annRM

i.e., annR f (IM) = annRM

Since,

$$f\ (IM)\ M, \text{ so, annRM annRf (IM)} \quad (1)$$

To prove $ann_R f(IM) \subseteq ann_R M$. Let, $x \in ann_R f(IM)$, so, xf (IM) = 0, since, f is homomorphism then:

f(xIM) = f(0) implies xIM = o, so,
x annRIM but M is multiplication, so
by Atani (2004) annRIM = annR(Rm) (2)
where annRRm annRrM, so,
x∈ annRrM but Mis w.q.p, so,
x∈ annRM implies annRf(IM) ⊆ annRM

From Eq. 1 and 2, implies M is fsqp module.

CONCLUSION

From this research, we conclude that every quasi-prime module is (fsqp) module. If M is a cyclie (fsqp) module, then, M is a (qp) module and every (fsqp) module is (wqp) module but the converse is not true, if, we put the condition cyclic. Finally, we conclude that every multiplication (wqp) module is (fsqp) module.

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