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Stochastic Neutron Density Employing Different Uncertainties

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Abstract: In this study, a numerical study of the stochastic dynamics of a nuclear reactor is carried out using the Euler-Maruyama method to resolve the stochastic point kinetic equations. The method was tested using different uncertainties, initial conditions and different forms of reactivity with one and six groups of delayed neutron precursors using up to 5000 Brownian trajectories for each numerical results. The average expected values, standard deviations of neutron density and delayed neutron precursors were calculated in each computational experiment. It was compared with other methods reported in literature and on average with the deterministic model of point kinetics, demonstrating that the Euler-Maruyama method for this proposal, apart from being easy to implement is extremely precise in obtaining expected values.

Key words: Brownian motion, Euler-Maruyama, neutron density, nuclear reactors, stochastic point kinetic equations, literature

INTRODUCTION

A nuclear reactor is a device in which controlled nuclear reactions are carried out to free energy. The dynamics of this device is described in deterministic form by point kinetic equations which are a set of strongly linked non-linear differential equations which describe an average value of the neutron density and delayed neutron precursor concentrations. However, in levels of low power, high randomness occurs in the dynamics of the neutrons in the presence of a weak source (Stacey, 2007), therefore, the process must be described with the stochastic point kinetic equations.

There are various researches in literature in which the stochastic point kinetic equations are solved: the Piecewise Constant Approximation (PCA) and Monte Carlo (Hayes and Allen, 2005), Euler-Maruyama and Taylor 1.5 (Ray, 2012; Ray and Patra, 2013) the Simplified Stochastic Point Kinetics (SSPK) (Ayyoubzadeh and Vosoughi, 2014) the Analytical Exponential Model (AEM) (Nahla and Edress, 2016a) the Efficient Stochastic Model (ESM) (Nahla and Edress, 2016b) and the Double Diagonalization-Decomposition Method (DDDM) (Da Silva *et al*., 2016).

In this research, due to the difficulty in generating sufficiently random numbers, consideration was given to different seeds, this is a set of values to begin the generation of random numbers by means of the randn function of MATLAB and with this, simulation can be carried out of different Brownian motions whose differential is known as white noise which express as uncertainties which serve to resolve the stochastic point kinetic equation using the Euler-Maruyama method. Considering the average of numerical experiments obtained for each seed, the results are compared with the methods reported in literature and on average with the deterministic model calculated with Runge-Kutta O (h^4).

MATERIALS AND METHODS

The stochastic point kinetic equations: It is possible to obtain the stochastic point kinetic equations based on the analysis of the events of birth and capture of neutrons and to assume that the changes in neutron population follow a normal distribution (Hayes and Allen, 2005). This analysis results from the following non-linear system of m+1 Ito stochastic differential equations for the neutron density and delayed neutron precursor concentrations:

$$\frac{d\vec{P}(t)}{dt} = A\vec{P}(t)+\vec{Q}(t)+B^{\frac{1}{2}}\frac{d\vec{W}(t)}{dt} \quad (1)$$

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Where:

$$\vec{P}(t)=\begin{bmatrix} n(t) \\ C_1(t) \\ C_2(t) \\ \vdots \\ C_m(t) \end{bmatrix} \vec{Q}(t)=\begin{bmatrix} q(t) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \overrightarrow{W}(t)=\begin{bmatrix} W_1(t) \\ W_2(t) \\ W_3(t) \\ \vdots \\ W_{m+1}(t) \end{bmatrix} \quad (2)$$

for i = 1, ..., m, where, m is the number of groups of concentration of delayed neutrons precursors n(t) and $C_i(t)$ are the neutron density and the ith density of delayed neutrons precursors, respectively, q(t) is the external source of neutrons and $W_1(t)$, $W_2(t)$, ..., $W_{m+1}(t)$ are Wiener processes or Brownian motion and $dW_1(t)$, $dW_2(t)$, ..., $dW_{m+1}(t)$ are uncertainties or white noise. On the other hand, the matrices A and B are given by:

$$A=\begin{bmatrix} \frac{\rho(t)-\beta}{\Lambda} & \lambda_1 & \lambda_2 & \cdots & \lambda_m \\ \frac{\beta_1}{\Lambda} & -\lambda_1 & 0 & \cdots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & -\lambda_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_m}{\Lambda} & 0 & 0 & \cdots & -\lambda_m \end{bmatrix} \quad (3)$$

$$B=\begin{bmatrix} \zeta & a_1 & a_2 & \cdots & a_m \\ a_1 & b_{1,1} & b_{1,2} & \cdots & b_{1,m} \\ a_2 & b_{2,1} & b_{2,2} & \cdots & b_{2,m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_m & b_{m,1} & b_{m,2} & \cdots & b_{m,m} \end{bmatrix} \quad (4a)$$

Doing the required multiplication, using Eq. 4a and 2 in the term $B^{1/2}d\vec{W}(t)$ given in Eq. 1, provides that to each variable contained in $\vec{P}(t)$ would give us (m+1) uncertainties, this is one for neutrons density and m for each concentration of precursors as can be seen in Eq. 1-2.

B represents the covariance matrix as it is in Eq. 4a, it is a symmetric, positive matrix where the main diagonal represents the variance and the other elements represent the covariance. With the disadvantage of calculating $B^{1/2}$ that would give a computational effort. We propose that the calculation of each of the variables contained in $\vec{P}(t)$ is associated with only one uncertainty and that it depends on the same variable that will be calculated, this means that we assume that the variables contained in $\vec{P}(t)$ have zero covariance, this it is possible to write the Eq. 4a in the following way:

$$B^{**}=\begin{bmatrix} \zeta & 0 & 0 & \cdots & 0 \\ 0 & b_{1,1} & 0 & \cdots & 0 \\ 0 & 0 & b_{2,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & b_{m,m} \end{bmatrix} \quad (4b)$$

where, the elements of matrix B or B** are defined as:

$$\zeta=\left[\frac{-1-\rho(t)+2\beta+(1-\beta)^2\nu}{\Lambda}\right]n(t)+\sum_{i=1}^{m}\lambda_i C_i(t)+q(t) \quad (5)$$

$$a_i=\frac{\beta_i}{\Lambda}\left[(1-\beta)\nu-1\right]n(t)-\lambda_i C_i(t) \quad (6)$$

$$b_{i,j}=\frac{\beta_i\beta_j\nu}{\Lambda}n(t)+\delta_{i,j}\lambda_i C_i(t) \quad (7)$$

Where:

- p(t) = The reactivity
- ν = The average neutron number per fission
- λ_i = The decay constant of the ith precursor which emits delayed neutron
- β_i = The fraction of delayed neutrons of the ith group of precursors
- $\beta=\sum_{i=1}^{m}\beta_i$ = The total fraction of delayed neutrons
- Λ = The prompt neutron generation time
- $\delta_{i,j}$ = The Kronecker delta, defined in accordance with the following Eq. 9

$$\delta_{i,j}=\begin{cases} 1 \text{ if } i=j \\ 0 \text{ if } i\neq j \end{cases} \quad (8)$$

Equation 1 is known as the stochastic point kinetic equations and generalizes the deterministic equations, since when B = 0, it returns to the deterministic model (Kinard and Allen, 2004). The importance of (B**) is that it is not required to compute the square root of a matrix which is a great advantage when compared with B. The matrix B** was found in the literature (Ayyoubzadeh and Vosoughi, 2014) considering a poisson distribution, using the stirling's approximation, stable distributions with the disadvantage of additionally requiring two Wiener processes to solve Eq. 1 and finally using the independence of normally distributed random variables to simplify the problem. In this research, it was necessary to suppose only the covariance is zero.

The Euler-Maruyama method: The Euler-Maruyama approximation is a continuous multidimensional stochastic process $\vec{Y}_t=\{\vec{Y}(t),t_0<t<T\}$ which satisfies the following iterative scheme:

$$\vec{Y}_{n+1} = \vec{Y}_n + a\left(t_n, \vec{Y}_n\right)h + b\left(t_n, \vec{Y}_n\right)\Delta\vec{W} \quad (9)$$

For n = 0, 1, 2, ..., N-1 where $\vec{Y}_0 = \vec{Y}(t_0) = \left[Y_0^1, Y_0^2, Y_{0,...,}^3 Y_0^k\right]^T$ is defined as the initial conditions, the time step defined as $h = t_{n+1}-t_n$ and $\Delta\vec{W} = \vec{W}_{n+1}-\vec{W}_n$ is the increase of the Brownian motion or uncertainties defined as $\Delta\vec{W} = \sqrt{h}\vec{\eta}$ where, $\vec{\eta}$ is a vector of random variables normally distributed with a mean zero and unit variance. Applying the Euler-Maruyama method given for Eq. 10 to the expression given for Eq. 1 gives the result:

$$\vec{P}_{n+1} = \vec{P}_n + \left(A_n\vec{P}_n + \vec{Q}_n\right)h + B\frac{1}{m}\Delta\vec{W} \quad (10)$$

With initial condition $\vec{P}_0 = \vec{Y}_0$ .Equation 11 is the solution of the point kinetic stochastic equations using the Euler-Maruyama method in Eq. 1.

RESULTS AND DISCUSSION

The following are the results of the proposed method, using different uncertainties $\Delta\vec{W}$ for multiple computational experiments. Calculation of the expected values and standard deviations for different uncertainties is carried out by means of different seeds. The results are subsequently compared with stochastic methods reported in literature such as PCA and Monte Carlo (Hayes and Allen, 2005), Euler-Maruyama and Taylor 1.5 (Saha, 2012, 2013), AEM (Nahla and Edress, 2016a) ESM (Nahla and Edress, 2016b) DDDM (Da Silva *et al*., 2016) and on average with Runge-Kutta $O(h^4)$ deterministic. In all the computational experiments shown, up to 5000 Brownian trajectories were used for each seed.

To test the proposed method, the first experiment consists of a step-reactivity insertion ρ = -1/3 when one precursor (m = 1) is considered in a nuclear reactor with the following parameters: decay constant λ_1 = 0.1 sec^{-1}, fraction of delayed neutrons β_1 = 0.05, average neutron number per fission ν = 2.5, generation time Λ = 2/3 sec, external source q = 200 sec^{-1} and an initial condition of $\vec{P}_0 = [400, 300]^T$. The simulation was made over N = 40 steps with an interval [0, 2] sec. Table 1 shows the mean of the expected values and standard deviations of the neutron density n(t) and the total concentration of precursors $C_\tau(t)$ in t = 2 sec using different seeds with one uncertaint $\Delta\vec{W} = \Delta W_1$ where, $C_\tau(t) = \sum_{i=}^{m} C_i(t)$ .These results indicate that, for 10 different seeds, each value of the neutron density and concentration of precursors is very similar to the average value, respectively when we use the proposed method (EM*), this is using the Euler-Maruyama method with a given uncertainty for Eq .4.

Table 1: Expected values and standard deviations using different seeds for one precursor with one uncertainty

Seeds	E (n (2 ses))	σ (n (2 sec))	E (C_τ (2 sec))	σ (C_τ (2 sec))
100	400.1973	31.18396	299.7546	7.929925
200	399.7253	31.98934	299.7701	7.966396
300	399.2918	31.00181	299.9346	8.056304
400	400.4318	31.56806	299.8427	7.794507
500	399.8844	31.56903	300.0402	7.840968
600	400.3342	31.42084	299.9319	8.028536
700	399.4346	31.62014	299.8181	7.955117
800	400.2483	30.98744	299.8795	7.883205
900	399.0195	31.09449	299.8779	8.017407
1000	399.8987	31.49116	300.1506	8.062298
Mean (EM*)	399.8466	31.39263	299.9000	7.953466

Table 2: Expected values and standard deviations using different seeds for one precursor with two uncertainties

Seeds	E (n (2 sec))	σ (n (2 sec))	E (C_τ (2 sec))	σ (C_τ (2 sec))
100	400.1021	31.57557	299.7669	10.24146
200	399.6746	32.40962	299.6904	10.34983
300	399.2808	31.43599	299.8284	10.32776
400	400.3723	32.00041	299.8863	10.14301
500	399.9058	31.96058	300.0430	10.13379
600	400.3183	31.83929	299.9589	10.32397
700	399.3583	32.01649	299.7299	10.25591
800	400.1835	31.37446	299.9067	10.13157
900	398.9828	31.50246	299.7565	10.30098
1000	399.9648	32.03126	300.1290	10.47774
Mean (EM**)	399.8143	31.81461	299.8696	10.26860

Table 3: Comparison of methods in a problem of one precursor

Methods	E (n (2 sec))	σ (n (2 sec))	E (C_τ (2 sec))	σ (C_τ (2 sec))
Monte Carlo	400.0300	27.31100	300.0000	7.806300
PCA	395.3200	29.41100	300.6700	8.356400
EM (Saha)	412.2300	34.39100	315.9600	8.265600
Taylor 1.5	412.1000	34.51900	315.9300	8.315800
AEM	396.2800	31.21200	300.4200	7.957600
ESM	396.6200	0.919900	300.3900	0.001600
DDDM	402.1300	28.93000	305.8400	7.924900
EM**	399.8143	31.81461	299.8696	10.26860
EM*	399.8466	31.39263	299.9000	7.953466
RK $O(h^4)$	400.0000	-	300.0000	-

Considering that there are two uncertainties ΔW_1 and ΔW_2 associated with each one of the variables we want to know in this case n(t) and $C_\tau(t)$. Table 2 shows the mean of expected values and standard deviations of the neutron density n (t) and the total concentration of precursors $C_\tau(t)$ in t = 2 sec. The results shown in Table 1 and 2 indicate that there is a slight change in the expected values but a significant change in the standard deviation for the concentration of precursors which seems reasonable, since, we have considered one uncertainty more than in the first case, represented, by (EM*) and (EM**), in this second case the Euler-Maruyama method with two uncertainties given by Eq. 5 is used.

Table 3 shows the comparison of the previous results with other works reported in literature and their validation with the deterministic model. First, we can see that the current proposal has a mean value for the neutron densities and for the very approximate concentration of precursors when we compare it with the deterministic

Table 4: Expected values and standard deviations for ρ = 0.003 (or 300 pcm) using different seeds for one uncertainty

Seeds	E (n(0.1 sec))	σ (n(0.1 sec))	E (C_τ(0.1 sec))	σ (C_τ(0.1 sec))
100	179.3362	217.3994	448867.8	1989.857
200	179.0589	219.5875	448847.9	1987.940
300	177.1956	215.0377	448875.7	2042.874
400	179.1409	217.5602	448877.8	1996.082
500	171.9135	215.7338	448918.2	1960.309
600	178.4538	220.5902	448882.3	2000.672
700	177.1948	216.1799	448864.9	2003.152
800	179.5957	215.6196	448915.2	2002.388
900	178.7917	216.6938	448848.5	2021.231
1000	181.7231	217.0913	448932.4	2039.020
Mean (EM*)	178.2404	217.1493	448883.9	2004.353

value when the Runge-Kutta method is used. In addition, the values obtained in the current proposal, especially when one uncertainty is used are very much in agreement when compared with the results obtained using the Monte Carlo and PCA methods (Hayes and Allen, 2005) and with the AEM method (Nahla and Edress, 2016a).

The second and third experiment consist of the insertion of two reactivity constants for a prompt subcritical insertion ρ = 0.003 (or equivalently 300 pcm, parts per hundred thousand) and for a prompt critical insertion ρ = 0.007 (or equivalently 700 pcm), respectively in a problem of six groups of precursors (m = 6) for a nuclear reactor with the following parameters: decay constants λ_i = [127, 317, 1150, 3110, 14000, 38700]×10^{-4} sec^{-1} fractions of delayed neutrons β_i = [266, 1491, 1316, 2849, 896, 182]×10^{-6}, total fraction of delayed neutrons, β = 0.007, fission neutrons ν = 2.5, generation time $\Lambda = 2\times10^{-5}$ sec and an external source q = 0 sec^{-1}. Both computational experiments were carried out over N = 40 steps at intervals of [0, 0.1] and [0, 0.001] sec with the initial condition $\vec{P}_0 = [1, \beta_1/\lambda_1\Lambda, \beta_2/\lambda_2\Lambda, \cdots, \beta_6/\lambda_6\Lambda]^T$.

Table 4 shows the mean values expected and standard deviations of the neutron density n(t) and the total concentration of precursors C_τ(t) for a prompt subcritical insertion ρ = 0.003 (or equivalently 300 pcm) in t = 0.1 sec, using different seeds with one uncertainty $\Delta\vec{W} = \Delta W_1$. Again the results using 10 different seeds, each value with the neuron densities and concentration of precursors are very similar to the average value.

Table 5 shows the mean of the expected values and standard deviations of the neutron density n(t) and the total concentration of precursors C_τ(t) for the same case as Table 4 with the difference that seven uncertainties($\Delta\vec{W} = \Delta W_i, i = 1, \cdots, 7$)are used. These results are approximately the same for the neutron density and concentration of precursors when compared with their average value when using 10 different seeds.

Table 6 compares the previous results with other works reported in literature and the deterministic point kinetic model. It can be observed that the results obtained

Table 5: Expected values and standard deviations for ρ = 0.003 (or 300 pcm) using different seeds with seven uncertainties

Seeds	E (n (0.1 sec)	σ (n (0.1 sec))	E (C_τ(0.1 sec))	σ (C_τ(0.1 sec))
100	179.0575	217.8088	448867.8	2174.250
200	179.3576	219.8941	448845.9	2179.808
300	177.5753	216.1228	448876.6	2241.775
400	179.6542	218.4496	448875.3	2186.352
500	171.9785	216.7798	448922.3	2144.035
600	178.5670	221.5763	448880.4	2188.824
700	176.9575	216.8416	448863.3	2197.016
800	179.4934	216.8765	448916.7	2186.658
900	178.9073	217.7606	448853.3	2217.282
1000	181.8915	218.4109	448940.5	2235.841
Mean (EM**)	178.3439	218.0521	448884.1	2195.184

Table 6: Comparison of methods for ρ = 0.003 (or 300 pcm) in a problem of six precursors

Methods	E (n (0.1))	σ (n (0.1))	E (C_τ (0.1))	σ (C_τ (0.1))
Monte Carlo	183.0400	168.7900	447800.0	1495.700
PCA	186.1600	164.1600	449100.0	1917.200
EM (Saha)	208.6000	255.9500	449800.0	1233.380
Taylor 1.5	199.4000	168.5400	449700.0	1218.800
AEM	186.3000	164.1400	449000.0	1911.900
ESM	179.9300	10.55500	448900.0	94.75000
DDDM	187.0500	167.8300	448800.0	1475.600
EM**	178.3439	218.0521	448884.1	2195.184
EM*	178.2404	217.1493	448883.9	2004.353
RK O(h^4)	179.9500	-	448878.0	-

Table 7: Expected values and standard deviations for ρ = 0.003 (or 700 pcm) using different seeds with one uncertainty

Seeds	E (n (0.001 sec))	σ (n (0.001 sec))	E (C_τ (0.001 sec))	σ (C_τ (0.001 sec))
100	134.2827	92.56374	446360.5	18.79142
200	134.0279	93.27359	446359.9	18.77698
300	134.1766	92.73754	446360.3	18.62391
400	135.6616	94.29252	446360.4	18.81022
500	136.6873	92.72723	446360.6	18.62025
600	134.4107	94.01819	446360.1	18.90852
700	134.3027	92.79483	446360.2	18.52236
800	135.9779	93.44768	446360.5	18.66368
900	133.0583	92.33267	446359.8	18.66976
1000	137.6626	93.66755	446360.9	18.91518
Mean (EM*)	135.0248	93.18555	446360.3	18.73023

using the proposed method with one and seven uncertainties (EM* and EM**) when compared with an expected value in the deterministic case and is very approximate to the Runge-Kutta methods and the recently published ESM method (Nahla and Edress, 2016b). In the standard deviation, it is similar to the PCA method (Hayes and Allen, 2005). However, there is a difference in the results when compared with the Euler-Maruyama method (Ray, 2012).

For a prompt critical insertion ρ = 0.007 (or equivalently 700 pcm), Table 7 shows the mean of the expected values and standard deviations of the neutron density n (t) and the total concentration of precursors C_τ(t) in t = 0.001 sec, using different seeds with one uncertainty $\Delta\vec{W} = \Delta W_1$. The results are very similar and there is practically no difference in each calculation carried out with regard to the expected values and standard deviations.

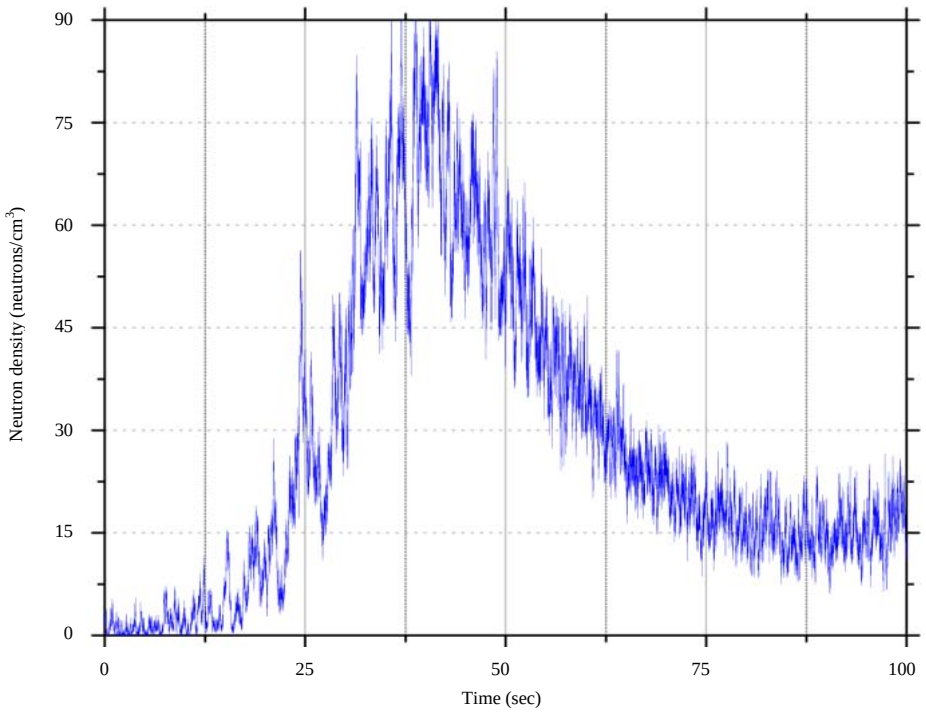

Fig. 1: Density of neutrons for a sinusoidal reactivity through one period

Table 8: Expected values and standard deviations for ρ = 0.003 (or 700 pcm) using different seeds with seven uncertainties

Seeds	E (n (0.001 sec))	σ (n (0.001 sec))	E (C_τ (0.001 sec))	σ (C_τ (0.001 sec))
100	134.3062	92.62652	446360.4	29.09122
200	134.0041	93.54258	446359.8	29.27451
300	134.2316	93.10647	446360.2	28.98068
400	135.7164	94.47060	446360.6	29.35535
500	136.6362	92.77626	446360.7	28.89884
600	134.3586	94.10905	446360.0	29.54918
700	134.2543	93.02594	446360.1	28.91319
800	135.9165	93.53007	446360.6	29.02346
900	133.0774	92.46126	446359.6	28.93699
1000	137.6863	93.81167	446361.2	29.54030
Mean (EM^{**})	135.0188	93.34604	446360.3	29.15637

Table 9: Comparison of methods for ρ = 0.003 (or 700 pcm) in a problem of six precursors

Method	E (n (0.001 sec))	σ (n (0.001 sec))	E (C_τ (0.001 sec))	σ (C_τ (0.001 sec))
Monte Carlo	135.6700	93.37600	446400.0	16.22600
PCA	134.5500	91.24200	446400.0	19.44400
EM (Saha)	139.5680	92.04200	446300.0	6.071000
Taylor 1.5	139.5700	92.04700	446300.0	18.33700
AEM	134.5400	91.23400	446400.0	19.23500
ESM	134.9600	6.852000	446400.0	2.529000
DDDM	135.8600	93.21000	446300.0	17.84500
EM^{**}	135.0188	93.34604	446360.3	29.15637
EM^{*}	135.0248	93.18555	446360.3	18.73023
RK O(h^4)	135.0000	-	446360.0	-

Table 8 shows the mean of expected results and standard deviations of the neutron density n(t) and the total concentration of precursors $C_\tau(t)$ for the same case as Table 7 except that seven uncertainties $\Delta\vec{W} = \Delta W_i, i = 1,\cdots,7$ are used associated with each of the variables that we want to know in this case (t) and $C_\tau(t)$ (Fig. 1-4).

The results obtained for a prompt critical insertion ρ = 0.007 (or equivalently 700 pcm) are presented in Table 9 and compare the results of other methods reported in literature, together with their validation with the deterministic model. From these results we can see that the proposed model when an uncertainty (EM^{*}) is used or seven uncertainties, (EM^{**}) in the case of the expected values are very precise when compared with the methods in literature and their exactness can be proven with the Runge-Kutta method. For the standard deviation value for the nuclear density we can say that all methods are very similar also for the standard deviation in the density of precursors the methods presented in the current proposal (EM^{*}) and PCA (Hayes and Allen, 2005) practically coincide.

The results given in Table 4, 5, 7, 8, indicate improvements in the approximations by using only one uncertainty. On the other hand, according to Table 6 and 9, it can be seen that the present proposal is in line with the values reported by other methods (Hayes and Allen, 2005; Nahla and Edress, 2016a, b; Da Silva *et al*., 2016) while in another publication (Ray, 2012) in which the same method is used there are significant differences.

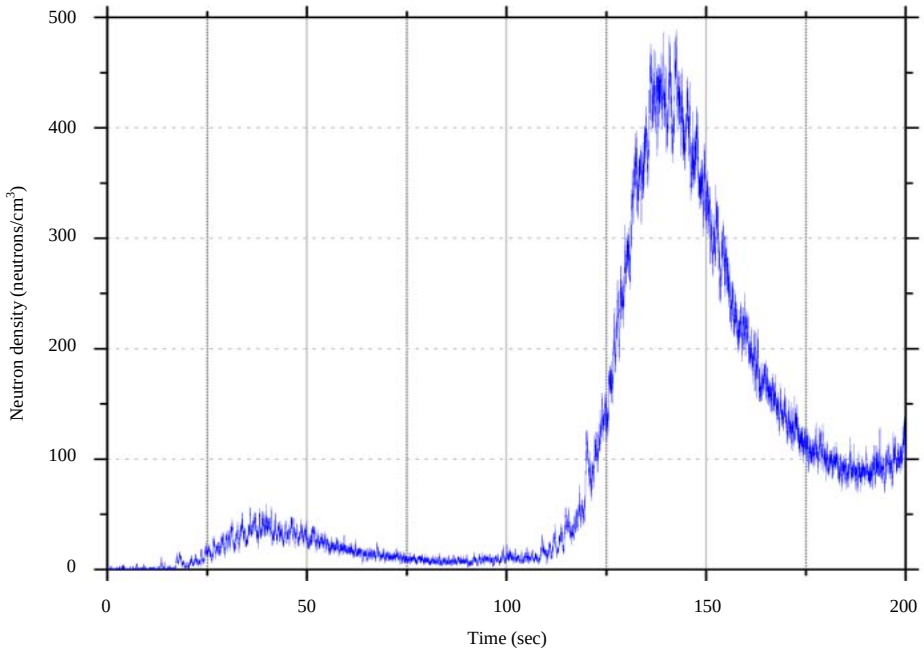

Fig. 2: Density of neutrons for a sinusoidal reactivity through two periods

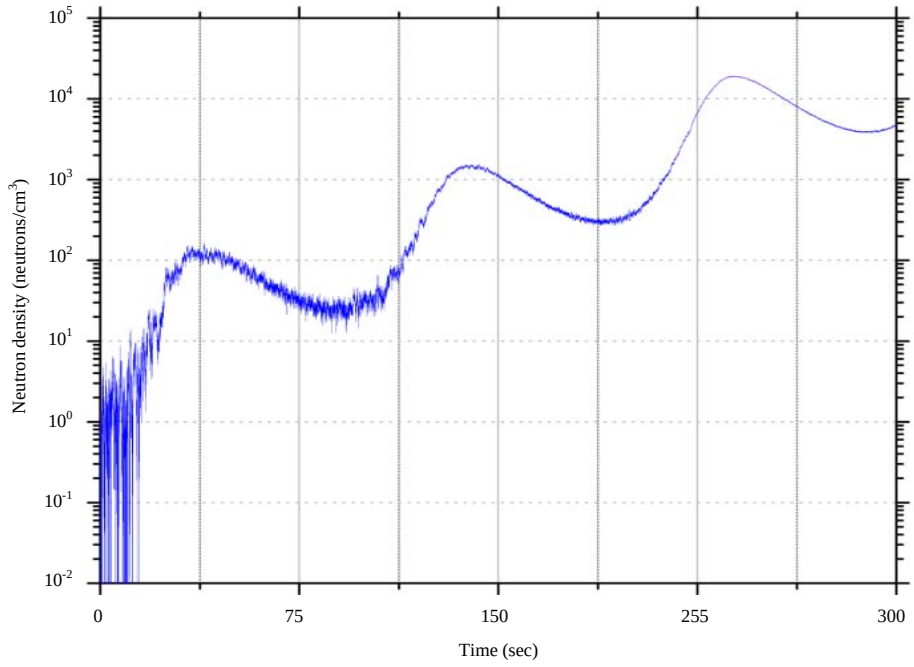

Fig. 3: Density of neutrons for a sinusoidal reactivity through three periods

Finally, the last computational experiments shown (Ray, 2013) consist of a sinusoidal reactivity $\rho(t)$ $\rho_0 Sin(\pi t/T)$ in a problem of one precursor (m = 1). The parameters used for this simulation consist of an initial reactivity $\rho_0 = 0.005333$ (or equivalently 533.3 pcm), decay constant $\lambda_1 = 0.077$ sec^{-1}, fraction of neutron precursor $\beta_1 = 0.0079$, time of neutron generation $\Lambda = 10^{-3}$sec, external neutron source q = 0 sec^{-1}, an average period T = 50 sec and initial condition $\vec{P}_0 = ,[1,\beta_1 / \lambda_1 \Lambda]^T$. The results obtained are shown in Fig. 1-4 show the expected value

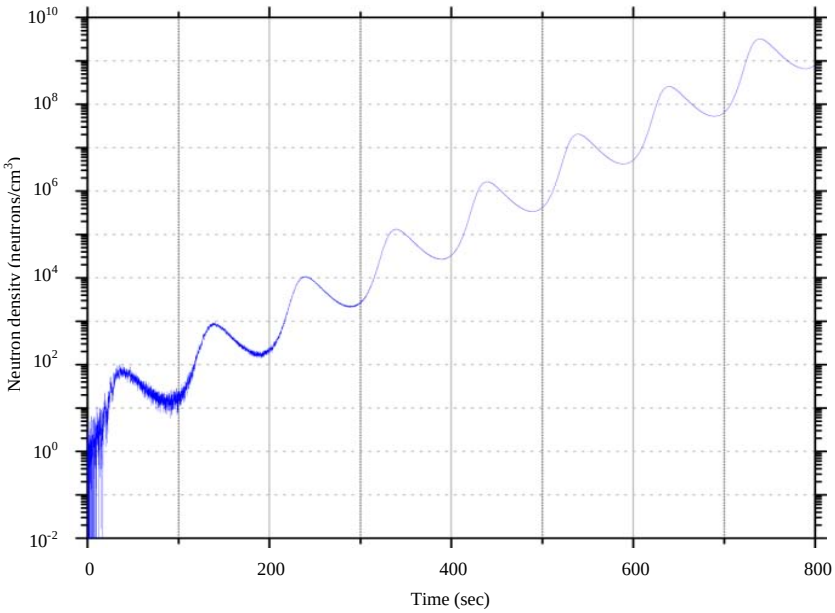

Fig. 4: Density of neutrons for a sinusoidal reactivity through eight periods

of the neutron density for a time interval of [0.100, 0.200, 0.003 and 0.800 sec], respectively. Comparing the results obtained with the current proposal, under the same parameters we found that there are time intervals considered as one, two, three and eight spikes, respectively. This analysis is supported by recent studies (Nahla and Edress, 2016).

CONCLUSION

In this study, the stochastic point kinetic equations were numerically solved using the Euler-Maruyama method in the cases in which each random variable n(t) $y_i(t)$ depends on one or several uncertainties $\Delta\vec{w}$. In this way, approximations were obtained of the expected values for the neutron density and delayed neutron precursor concentrations using different seeds with various types of reactivity and initial conditions. The results were compared with other methods reported in literature for which it was possible to demonstrate that this proposal achieved better approximations, confirming that the method, apart from being easy to implement is efficient in the study of stochastic kinetics in a nuclear reactor.

RECOMMENDATIONS

The researchers consider and recommend making greater efforts in research to propose a new matrix B and/or for the calculation of the square root of the matrix B presented in Eq. 7-5), since, this would enable to establish if it is necessary to use all the uncertainties($\Delta\vec{w}_i$, i = 1, ..., 7) for each of the unknowns and with this to be able to determine the standard deviation of neutron densities and delayed neutron concentrations. At the same time, more numerical simulations should be carried out with different seeds that provide numbers which would be as random as possible to reconfirm that the results presented in the literature are correct.

The researchers also consider that there is a potential for work to be done by including different considerations in the study and granting different possibilities that can be presented, making the problem a little more general, considering, for example, the inclusion of 8 groups of delayed neutrons and determining how this would affect the average value and standard deviations of neutron densities and concentration. The effect of feedback was not considered in the present research and it would be very interesting to know what the stochastic behavior of the nuclear density would be and if it is also possible to determine the reactivity which is perhaps the most important parameter in a nuclear reactor. Finally, another possibility would be to take the work to stochastic spatial kinetics in 2D and 3D problems, possible in this way we could be able to better describe the real behavior of a nuclear reactor.

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REFERENCES

Ayyoubzadeh, S.M. and N. Vosoughi, 2014. An alternative stochastic formulation for the point reactor. Ann. Nucl. Energy, 63: 691-695.

Da Silva, M.W., R. Vasques, B.E. Bodmann and M.T. Vilhena, 2016. A nonstiff solution for the stochastic neutron point kinetics equations. Ann. Nucl. Energy, 97: 47-52.

Hayes, J.G. and E.J. Allen, 2005. Stochastic point-kinetics equations in nuclear reactor dynamics. Ann. Nucl. Energy, 32: 572-587.

Kinard, M. and E.J. Allen, 2004. Efficient numerical solution of the point kinetics equations in nuclear reactor dynamics. Ann. Nucl. Energy, 31: 1039-1051.

Nahla, A.A. and A.M. Edress, 2016a. Analytical exponential model for stochastic point kinetics equations via eigenvalues and eigenvectors. Nucl. Sci. Tech., 27: 19-27.

Nahla, A.A. and A.M. Edress, 2016b. Efficient stochastic model for the point kinetics equations. Stochastic Anal. Appl., 34: 598-609.

Ray, S.S. and A. Patra, 2013. Numerical solution for stochastic point-kinetics equations with sinusoidal reactivity in dynamical system of nuclear reactor. Intl J. Nucl. Energy Sci. Technol., 7: 231-242.

Ray, S.S., 2012. Numerical simulation of stochastic point kinetic equation in the dynamical system of nuclear reactor. Ann. Nucl. Energy, 49: 154-159.

Stacey, W.M., 2007. Nuclear Reactor Physics. 2nd Edn., Wiley-Vch, Weinheim, Germany, USA., ISBN: 9783527406791, Pages: 736.