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Pythagorean Proposition and Geometric Demonstration: The Sum of Squares of Catheti is Equal to the Square of the Hypotenuse

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Abstract: This study arose from the need to provide known elements that allow to more easily reason about the importance and contribution of geometry in the solution of the Pythagorean proposal, assigned to squares of catheti and the square of the hypotenuse in a triangle rectangle. For this, we take a circle inscribed in a quadrilateral whose middle points form a square inscribed in the circle and diagonals of this second square form four isosceles triangles, one of these is taken to demonstrate by construction a particular case (isosceles right triangle) the Pythagorean proposition or theorem.

Key words: Pythagorean theorem, circumference, triangles, square, congruence, contribution of geometry

INTRODUCTION

The correct concepts emitted by scholarly researchers by means of articles about the origins: practical-empirical or lacking theories that obeyed the playful or simply ornamental needs of the moment and the etymological one, coming from Greek observation and wisdom, geometry and its subsequent evolution are accompanied by irrefutable truths and therefore, axiomatic over the centuries.

These events of each period of time that had passed, presented a result from which the search for data that were transformed into new knowledge or new links in the scientific chain being constructed was reinitiated. This modern constructivist concept is observed from beyond centuries BC.

Recurrently and in the civilizations that have been taken as cones or avant-garde raizal scientific research: Egyptian, Sumerian, Greek, Chinese, Cretan, Aztec, Inca, Zenu, among others and in all the activities of man, this constant progression that concerns his intellectual capacity in permanent development has been noticed. Each advance in condition of new proposal, meant a great step of the applied science in all the disciplines known millennia ago, calculated until today and from now on.

Just as some primitive inhabitants of planet Earth were restless observers of nature phenomena, later on in prudential times, becoming researchers of their reason, they were transformed into admired scientists: chemists, physicists, geographers, hydrologists, engineers, sculptors, geometers; sorcerers who cured diseases and illnesses who stimulated by their findings, put the first, stones of the solid base of our collection in the multiplicity of areas of human knowledge through the centuries.

Geometry has not been alien to these evolutionary processes of initial stages and assigned chronological spaces, it has been present throughout the development of human civilization, it is not a result of an individual, Gonzalez and Cantor. Its growth has been on par with areas that arbitrarily can be taken as the most advanced of the ancient and modern sciences. If it were not, so, it would be withdrawn and condemned to be a simple gregarious or even worse, ashen as opposed to the other branches of scientific knowledge.

Pythagorean theorem has been present in many events of our life and has been of vital importance for the development of humanity, since, without this inventions such as radio, cell phone, television, internet, flight, pistons, cyclic movement, topography and development of infrastructure and stellar measurements would not have been possible as (Sparks, 2013) affirms it.

Having powerful applications in almost all branches of science and being the basis of trigonometry, it is considered one of the fundamental theorems of

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mathematics. The theorem states that "the sum of the squares of catheti of a right triangle is equal to the square of the hypotenuse". It receives this name after Pythagoras of Samos in honor of the Greek mathematician (Moledina, 2013).

According to Ratner (2009) the Pythagorean theorem was discovered probably by mathematicians from Babylon, 1000 years before Pythagoras birth, there are more than 371 proofs of this theorem which are grouped in a book created in 1927.

The purpose of this document is to present a new proof of the Pythagorean theorem, taking into account a particular case, it is applied to a right isosceles triangle. Since, many original and long tests have been left aside by the subsequent discovery of shorter tests (Basu, 2015), this fact has been taken into account when making this demonstration.

MATERIALS AND METHODS

Mathematical foundation

Definition 1.1: The Pythagorean theorem states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two catheti, Shu and Silva.

Most people who know the Pythagorean theorem learned it in high school or middle school. This theorem states that "the hypotenuse squared equals the sum of the squares of catheti in a right triangle". In Fig. 1, c represents the hypotenuse (it is the longest side of the right triangle which as seen in the figure is the side opposite the right angle), sides a and b are called cathetus, like this:

$$c^2 = a^2 + b^2 \tag{1}$$

Euclid of Alexandria was the first to mention and prove this theorem. The sum is the sum of the areas of the squares whose lengths are a-c where, the area of the square whose length is a plus the area of the square of length b is equal to the area of the square of length c as shown in Fig. 2.

According to Stillwell (2010) the Pythagorean theorem was the first indication between a hidden and deeper relationship between arithmetic and geometry, arithmetic is based on counting while geometry requires continuous places such as lines and curves.

The Pythagorean theorem in the particular case of the right isosceles triangle appears in the dialogue Meno (82d-83e) of Plato, Gonzalez.

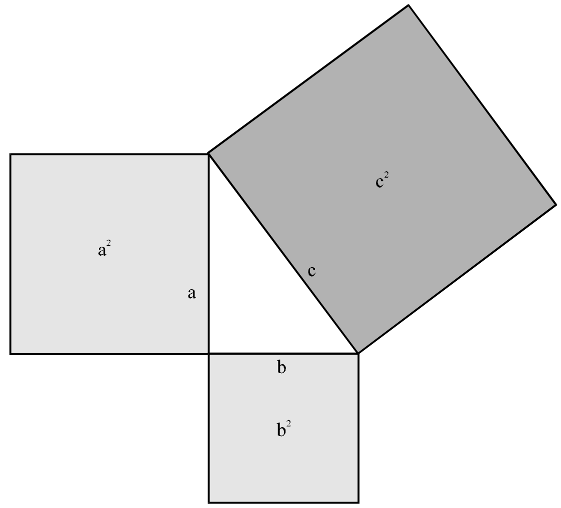

Fig. 1: Pythagorean theorem

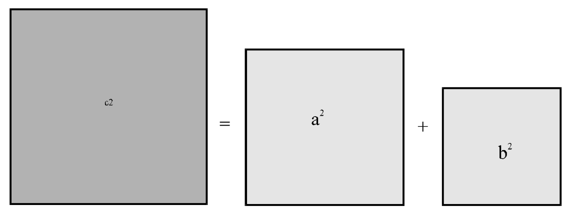

Fig. 2: Sum of areas

Tests: On the Pythagorean theorem, classical demonstrations have been carried out by civilizations such as Chinese, Indian and Arabic. There are tests where tessellations are used such as the test done by Euclides or that of Leonard Pisano, among others (Ostermann and Wanner, 2012).

Loomis (1940) shows a compilation of 370 demonstrations of the Pythagorean theorem, on the other hand as far as the teaching of Pythagorean theorem, Vargas and Gamboa show the design of a methodological strategy using geometry software (GeoGebra), applied to a group of students.

RESULTS AND DISCUSSION

Pythagorean theorem

Theorem 1.1: In a right isosceles triangle, the sum of the squares of catheti is equal to the square hypotenuse (particular case).

Demonstration: The square ABCD is built. Since, the square is a regular polygon, it can be inscribed or circumscribed in a Pogorélov circle. The center circle is inscribed in 0 in the regular quadrilateral ABCD. The midpoints of the sides of the square ABCD are determined, we have:

$$\begin{aligned} \overline{AE} &= \overline{AD}2 \\ \overline{AF} &= \overline{AB}2 \\ \overline{BG} &= \overline{BC}2 \\ \overline{CH} &= \overline{CD}2 \end{aligned} \quad (2)$$

The square EFGH is built inscribed in the center circumference O. Diagonals to the square EFGH: OE, OF, OG, OH are drawn which are apothems of the square ABCD. In addition, apothems are radii of the center circumference O. The center of the regular polygon is the common center of the inscribed and circumscribed circle Bos.

Triangles • EFO, • FGO, • GHO, • HEO all right in O. Property of the diagonals of the square. Right triangles are congruent because they have their three congruent sides, like this:

$$\overline{EF} \cong \overline{FG} \cong \overline{GH} \cong \overline{EH} \quad (3)$$

For being sides of the square EFGH. Further:

$$\overline{EO} \cong \overline{FO} \cong \overline{GO} \cong \overline{EO} \quad (4)$$

For being radii of the same circumference or for being apothems of the square ABCD. In O four right angles are formed by being the midpoint of the diagonals of the square. As shown in Fig. 3. These triangles considered:

- • AEF (Right in A by construction)
- • BFG (Right in B by construction)
- • CGH (Right in C by construction)
- • DEH (Right in D for construction)

These triangles are isosceles, congruent rectangles. Each triangle rectangle external to the square EFGH, is congruent with the triangle •EFO. If we consider the triangle •AEF as shown in Fig. 4, we have:

$$\overline{AE} = \text{cathetusa} = \overline{AD}2 \quad (5)$$

$$\overline{AF} = \text{cathetusb} = \overline{AB}2 \quad (6)$$

$$\overline{AE} = \text{hypotenuse} \quad (7)$$

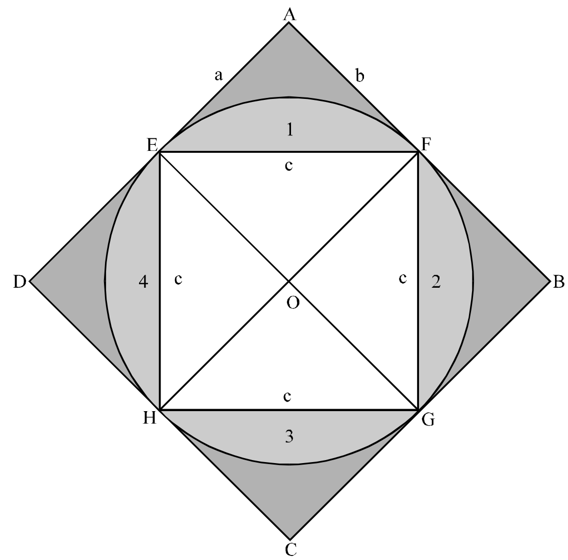

Fig. 3: Circle with center in O inscribed in the four-sided ABCD

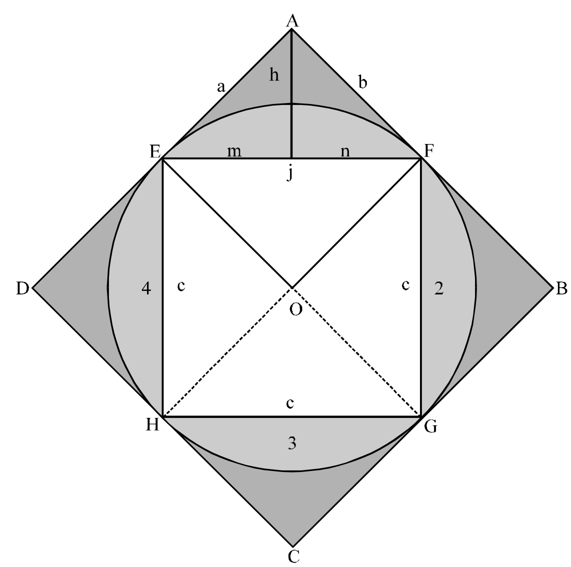

Fig. 4: Triangles formed by diagonals and the quadrilateral EFGH

m and n projection of a and b on c:

$$h = \text{height} = \overline{AJ} \quad (8)$$

$\overline{Ej}$ • $\overline{jF}$ por for being projections of congruent catheti j is the midpoint of $\overline{EF}$:

- If J is the midpoint of $\overline{EF}$, we have $m = c^2$ and $n = c^2$
- As in any right triangle $h = \sqrt{m \times n}$, then, we have:

$$h = \sqrt{c2 \times c2}$$

$$h = \sqrt{c^2 \times 2^2}$$

$$h = c2$$

The area of the triangle • AFE equals 1/4 of the square area EFGH and as the triangles external to the square EFGH are congruent to each other, then the area of the four triangles equals:

$$4c^2 4 = c^2 \quad (9)$$

The area of the square ABCD is equal to the areas of the four outer triangles, plus the area of the square EFGH. Then the area of the square ABCD = c^2+c^2. The area of the square ABCD can also be written as:

$$\left(\overline{AD}\right)^2 \quad (10)$$

$$\text{where } \left(\overline{AD}\right)^2 = 2c^2 \quad (11)$$

Solving for, we have:

$$\left(\overline{AD}\right)^2 2 = c^2 \quad (12)$$

From Eq. 14, we have:

$$a = \overline{AD}2 \quad (13)$$

Then:

$$2a^2 = \left(\overline{AD}\right)^2 2 \quad (14)$$

Equalizing expressions 12 and 14, we have:

$$2a^2 = c^2 \quad (15)$$

But as a = b for being sides of the quadrilateral AF0E or for being catheti of the isosceles triangle • AEF. However, $2a^2 = a^2+a^2$, then, $a^2+a^2 = a^2+b^2$ you also, have $a^2+a^2 = c^2$ then you have $a^2+b^2 = c^2$, Carmona.

CONCLUSION

Geometry through the times continues in force and is part of the transformation of humanity. Despite the many demonstrations throughout history for the Pythagorean theorem, it is still possible to get new demonstrations. A demonstration of the Pythagorean theorem was made using geometric construction and a particular case when the triangle is isosceles.

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