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A Comparative Study on Reliability Attributes of Software Reliability Model Dependent on Minimax and Lomax Lifetime Distribution

Hee-Cheul Kim
Department of Industrial and Management Engineering, Namseoul University,
Cheonan, South Korea, kim1458@nsu.ac.kr

Abstract: In this study was applied the lifetime distribution which the Lomax distribution with heavy-tail probability and the Minimax distribution which the special form of the beta distribution follow. Using this lifetime distribution, reliability model based on the non-homogeneous Poisson process was analyzed. As a result, the mean squared error, mean value function and intensity function in order to recognize the failure type for software reliability were estimated. The results of this study show that the mean squared error of the Lomax Model is smaller than that of the Minimax distribution model and the mean value function trend is smaller in terms of the true value. Therefore, the Lomax distribution Model can be regarded as an efficient model than the Minimax distribution Model. In the form of the reliability function, it gradually appears as a non-incremental pattern as the mission time elapses and the Lomax distribution Model is higher than the Minimax distribution model. Through this study, software operators will be able to use the mean square error, the mean value and the change in the intensity function to identify the type of failure in software reliability that reflects the characteristics of various lifetime distributions.

Key words: Mean squared error, Minimax-distribution, mission time, Lomax-distribution, NHPP, box plot

INTRODUCTION

Software working systems have long been an essential condition of our lives. The reliability of these operating systems can be a fundamental aspect in a very basic operating environment, since, improved quality of software service is available to software workers. Thus, the failure of a computer system due to a software defect reason can be caused tremendous damage to software. However, the development of a software operating system can be observed as a complicated and difficult process due to time and cost. Therefore, software operations and designers have tried to improve the stability of software systems. Therefore, the software reliability model is used to analyze the software failure phenomenon by estimating the reliability of the software, the remaining number of failures, the failure intensity and the software development cost. Many software reliability analysis models have been proposed to solve this situation. Among the proposed models, the model using the Non-Homogeneous Poisson Process (NHPP) (Gokhale and Trivedi, 1999) is a reliable model in terms of defect search analysis, it is known that the model assumes that if a fault occurs, it is removed immediately and no other faults occur during the debugging process. In terms of fault observation, the S-shape model was presented a learning process technique that software operators can use in software failure inspection tools (Chiu *et al*., 2008). Using Burr-XII and Type-2 Gumbel life time distributions, also reliability characteristics were studied (Kim, 2019).

In this study was applied the lifetime distribution which the Lomax-distribution with heavy-tail probability and the Minimax-distribution which the special form of the beta distribution follow. The reliability characteristics of the software reliability model based on the non-homogeneous Poisson process with a finite number of failures were was discussed using this lifetime distribution.

MATERLALS AND METHODS

Parameter estimation using finite-failure NHPP based on Minimax-distribution: The Minimax-distribution is a special form of the beta distribution and the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) are known as follows (Oguntunde and Adejumo, 2015) Eq. 1:

$$f(t) = a\,b\,t^{a-1}\left(1\text{-}t^{a}\right)^{b-1} \quad F(t) = 1\text{-}\left(1\text{-}t^{a}\right)^{b} \tag{1}$$

Note. $t\in(0,\ 1]$, $a>0$ and $b>0$ are the shape parameter. In this Minimax-distribution (Oguntunde and Adejumo, 2015) were analyzed the case of a = 1.

Therefore, this study also seeks to analyze the software reliability model by choosing the shape parameter a = 1. The mean value function and the intensity function of the Minimax-distribution finite-failure NHPP Model are as in Eq. 2 and 3 (Song *et al*., 2017; Yamada and Osaki, 1985):

$$\lambda(t|\theta, a, b) = \theta f(t) = \theta a b t^{a-1}(1-t^a)^{b-1} \tag{2}$$

$$m(t|\theta, a, b) = \theta F(t) = \theta\left[1-(1-t^a)^b\right] \tag{3}$$

In finite failure NHPP Model, θ was specified the expected value of faults that would be discovered observing time (0, t]. In Eq. 3, time t and x_n are replaced with the last failure time point, the likelihood function is known as follows (Goel and Okumoto, 1979; Kim, 2019):

$$\begin{aligned} L_{NHPP}(\Theta|\underline{x}) &= \prod_{i=1}^{n}\lambda(x_i)\exp\left[-m(x_n)\right] \\ &= \left(\prod_{i=1}^{n}\theta a b\, x_i^{a-1}(1-x_i^a)^{b-1}\right) \\ &\times\exp\left[-\theta\left(1-(1-x_n^a)^b\right)\right] \end{aligned} \tag{4}$$

Note $\underline{x} = (x_1 \le x_2 \le x_3, \cdots, \le x_n)$, $i = 1, 2, \ldots, n$,Θ = {θ, a, b} indicates parameter space. When shape parameter a = 1 is fixed, using Eq. 4, the log-likelihood function is derived as follows (Al Turk, 2019):

$$\begin{aligned} \ln L_{NHPP}(\Theta|\underline{x}) &= -m(x_n)+\sum_{i=1}^{n}\ln\lambda(x_i) \\ &= -\theta\left(1-(1-x_n^a)^b\right)+n\ln\theta+n\ln a+n\ln b \\ &+(a-1)\sum_{i=1}^{n}\ln x_i+(b-1)\sum_{i=1}^{n}\ln(1-x_i^b) \end{aligned} \tag{5}$$

Considering the shape parameter a = 1, the estimator $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ must be assessed the following structure for the maximum likelihood estimation about all parameter by means of Eq. 5:

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\theta}-\left(1-(1-x_n^a)^b\right) = 0 \tag{6}$$

$$\begin{aligned} \frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} &= \frac{n}{b}+\sum_{i=1}^{n}\ln(1-x_i^a)+ \\ &\theta(1-x_n^a)^b\ln(1-x_n^a) = 0 \end{aligned} \tag{7}$$

Parameter estimation using finite-failure NHPP based on Lomax-distribution: The Lomax-distribution, a special form of the Pareto distribution is known to be a distribution often used to model social phenomena in the business and economic sectors due to its characteristics with a trend of heavy-tail probability distribution. The probability density function and the distribution function of the Lomax distribution used in this social phenomenon field are as follows (http://www.math.wm.edu):

$$f(t) = \frac{\lambda k}{(1+\lambda t)^{k+1}},\ F(t) = 1-(1+\lambda t)^{-k} \tag{8}$$

Note. t∈(0,∞], λ>0 scale is parameter and k>0 is the shape parameter. This study also seeks to analyze the software reliability model by choosing the shape parameter k = 1. The mean value function and the intensity function of the Lomax-distribution finite-failure NHPP Model are as follows (Song *et al*., 2017):

$$\lambda(t|\theta, a, b) = \theta f(t) = \theta\frac{\lambda k}{(1+\lambda t)^{k+1}} \tag{9}$$

$$m(t|\theta, a, b) = \theta F(t) = \theta\left[1-(1+\lambda t)^{-k}\right] \tag{10}$$

Note. $\underline{x} = (x_1 \le x_2 \le x_3, \cdots, \le x_n)$, i = 1, 2, K, n, Θ = {θ, λ, k} indicates parameter space. When shape parameter k = 1 is fixed, using Eq. 9 and 10, the log-likelihood function is derived as follows (Al Turk, 2019):

$$\begin{aligned} \ln L_{NHPP}(\Theta|\underline{x}) &= -m(x_n)+\sum_{i=1}^{n}\ln\lambda(x_i) \\ &= -\theta\left[1-(1+\lambda x_n)^{-1}\right]+n\ln\theta+n\ln\lambda \\ &-2\sum_{i=1}^{n}\ln(1+\lambda x_i) \end{aligned} \tag{11}$$

The estimator $\hat{\theta}_{MLE}$ and $\hat{\lambda}_{MLE}$ must be assessed the following structure for the maximum likelihood estimation about all parameter by means of Eq. 11:

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\theta}-\left[1-(1+\lambda x_n)^{-1}\right] = 0 \tag{12}$$

$$\begin{aligned} \frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} &= \frac{n}{\lambda}-2\sum_{i=1}^{n}\frac{x_i}{(1+\lambda x_i)} \\ &-\theta x_n\frac{1}{(1+\lambda x_n)^2} = 0 \end{aligned} \tag{13}$$

RESULTS AND DISCUSSION

Analysis of the software failure time using Minimax and Lomax-distribution: In this study, using the failure time structure, the property for software reliability model considering Minimax and Lomax-distribution that controls the several life style distribution were studied. Table 1 is information of the software failure time (Hayakawa and

Telfar, 2000). Furthermore, the trend test should be headed in order to assure reliability of data. In this study, the trend analysis used was the box-plot test (Kim, 2017). Therefore, in Fig. 1, since, there is no data information that is out of the range between the upper limit (= 0.1564+1.5×(0.1564-0.04849) = 0.318265) and the lower limit (= 0.04849-1.5×(0.1564-0.04849) = -0.11338), it can be seen that no abnormal value or extreme value occurs (Kim and Kim, 2016).

Table 1: Failure time data (Hayakawa and Telfar, 2000)

Failure number	Failure time (hours)	Failure time×10^{-2}
1	0.4790	0.00479
2	0.7450	0.00745
3	1.0220	0.01022
4	1.5760	0.01576
5	2.6100	0.02610
6	3.5590	0.03559
7	4.2520	0.04252
8	4.8490	0.04849
9	4.9660	0.04966
10	5.1360	0.05136
11	5.2530	0.05253
12	6.5270	0.06527
13	6.9960	0.06996
14	8.1700	0.08170
15	8.8630	0.08863
16	10.7710	0.10771
17	10.9060	0.08863
18	11.1830	0.11183
19	11.7790	0.11779
20	12.5360	0.12536
21	12.9730	0.12973
22	15.2030	0.15203
23	15.6400	0.15640
24	15.9800	0.15980
25	16.3850	0.16385
26	16.9600	0.16960
27	17.2370	0.17237
28	17.6000	0.17600
29	18.1220	0.18122
30	18.7350	0.18735

Table 2: MLE, MSE and for the each model

Model	MLE	Model comparison (MSE
Minimax (a = 1)	$\hat{b}_{MLE} = 3.13\times10^{-1}$	1.7821
	$\hat{\theta}_{MLE} = 477.188$	
Lomax (k = 1)	$\hat{\lambda}_{MLE} = 1.93\times10^{-1}$	1.4006
	$\hat{\theta}_{MLE} = 849.489$	

MLE: Maximum Likelihood Estimation; MSE: Mean Square Error; R^2: Coefficient of determination

The estimation approximation value of the parameters for the projected model was used the maximum likelihood method. In this study, so, as to facilitate the estimation of the parameters of the two models (Minimax and Lomax), the numerical conversion (Failure time (hours)×0.01) used. The calculations to estimate the parameter, solving mathematically because the initial values were given 0.0001 and 5.000 and tolerance value for the measurement of interval (10^{-5}) were specified were accomplished repetition of 100 times using R-language (https://www.r-project.org/) checking acceptable convergent.

Based on the parameter estimates listed in Table 2, the transition of the mean value function using Eq. 3 and 10 is shown in Fig. 2. Figure 2 shows that the Lomax (k = 1) distribution Model is nearly similar to the Minimax (a = 1) distribution Model in the comparison of the mean value function patterns but the Lomax (k = 1) distribution has a relatively smaller width in terms of real value than Minimax (a = 1) distribution Model. Also, based on the parameter estimates listed in Table 2, the transition of the

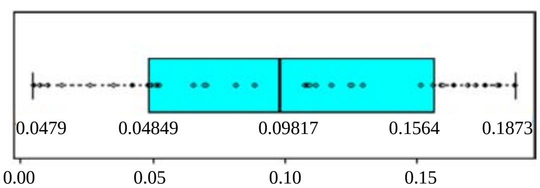

Fig. 1: Result of box plot; Software failure time

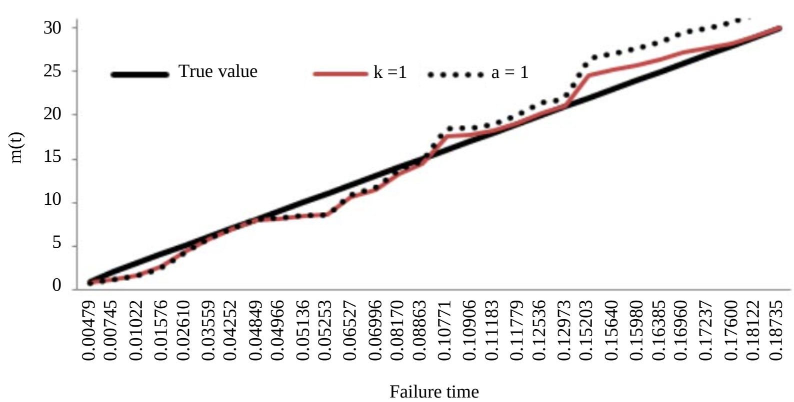

Fig. 2: Types of mean value function; Mean value of function vs. failure time

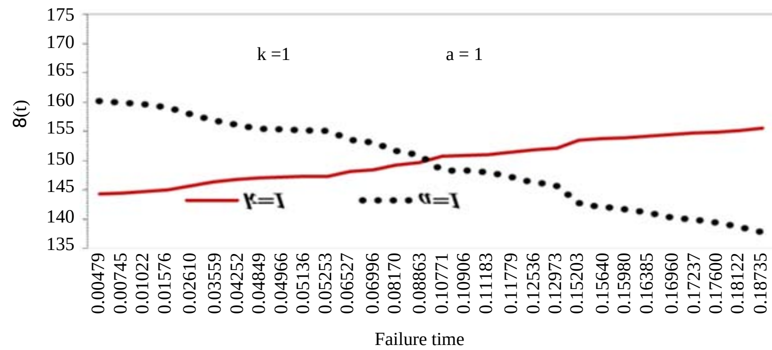

Fig. 3: Types of intensity function

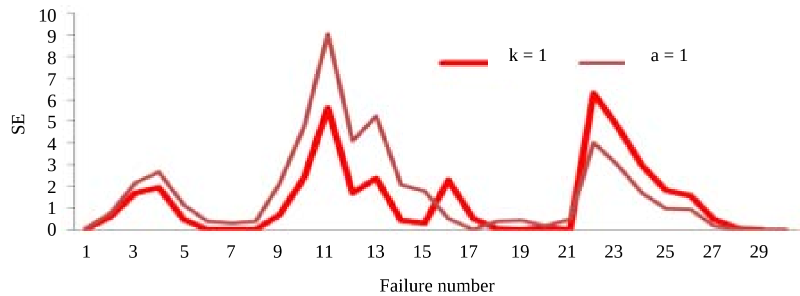

Fig. 4: Estimation of square error for each time

intensity function using Eq. 2 and 9 is shown in Fig. 3. In intensity function pattern, Fig. 3 shows that the Minimax (a = 1) distribution model shows an increase pattern and the Lomax (k = 1) distribution model shows a decrease pattern. Particularly, the first half of the failure time, the Minimax (a = 1) distribution model is lower than the Lomax (k = 1) and in the second half, the Lomax (k = 1) distribution model is lower:

The mathematical formula of Mean Square Error (MSE) (Chiu *et al*., 2008; Kim, 2015) indicates measure that is often used to tell the difference between the actual value (observed value) and the estimated value that is the residual value. It was obtained as next measure.

$$MSE = \frac{\sum_{i=1}^{n}\left[m(x_i)-\hat{m}(x_i)\right]^2}{n-k} \tag{14}$$

Note that $m(x_i)$ is the cumulated number of the faults detected in $(0, x_i)$ and $\hat{m}(x_i)$ estimating cumulated number of the faults detected in $(0, x_i)$, n specifies the number of realizing values and is the number of the parameter (Tae-Hyun, 2015). In Table 2, the mean squared error using (Eq. 14) shows the Lomax (k = 1) distribution model is smaller than the Minimax (a = 1) distribution model. Therefore, the Lomax (k = 1) distribution model can be considered as an efficient model than the Minimax (a = 1) distribution model.

In order to settle this situation, a summary picture of the comparison of estimated values of square error $\left(SE=\left[m(x_i)-\hat{m}(x_i)\right]^2, i=1, 2K\ 30\right)$ for each failure time points are abridged in Fig. 4. In this figure in squared error value, the Lomax (k = 1) shows a smaller than Minimax (a = 1) distribution model as the failure time increases. Also, the Lomax (k = 1) distribution model can be considered as an efficient model than the Minimax (a = 1) distribution model.

In the NHPP Model, a software failure occurs at the time of testing x_{30} = 0.18735 and reliability which is the probability that a software failure does not occur between 0.18735 and 18.735+t (where t is the mission time) can be stated using the ensuing construction (Kim and Kim, 2016; Kim, 2019):

$$\begin{aligned}\hat{R}(t|0.18735) &= \exp\left[-\int_{0.18735}^{0.18735+t}\lambda(\eta)\,d\eta\right] \\ &= \exp\left[-\left\{m(t+10.18735)-m(0.18735)\right\}\right]\end{aligned} \tag{15}$$

In the form of the reliability function of Fig. 5 using Eq. 15, it gradually appears as a non-increasing pattern as the mission time pass. Therefore, Lomax (k = 1) distribution Model is higher than Minimax (a = 1) distribution Mode in terms of reliability.

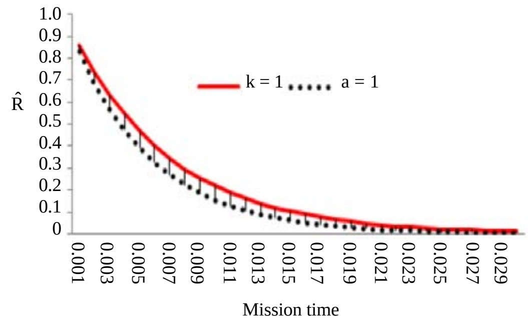

Fig. 5: Transition of reliability pattern function

CONCLUSION

In the process of software development, it is possible to evaluate the efficiency by comparing and analyzing the software safety by quantitatively modeling the characteristics of the failure or the occurrence of the failure during the execution of the test or actual software operation. In this study, using the lifetime distribution following the Minimax-distribution which is a special form of the beta distribution and the lomax-distribution property with the heavy-tail distribution, reliability characteristics of software reliability model based on non-homogeneous Poisson process with finite number of failures was discussed. In this study, the Lomax-distribution model is smaller than Minimax-distribution model in the mean squared error and mean value function. Therefore, the Lomax-distribution model can be regarded as an efficient model than the Minimax-distribution model. In the comparison of the intensity function patterns, the Minimax distribution model shows an increase pattern and the Lomax-distribution model shows a decrease pattern. In the form of the reliability function, the non-incremental pattern gradually appears as the mission time elapses and the Lomax-distribution model is higher than the Minimax-distribution model. Through this study, software operators will be able to use the mean square error, the mean value function and intensity function to identify the type of failure in software reliability that reflects the characteristics of various lifetime distribution.

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