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SPC Monitoring Method of FRAR Control Chart

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Abstract: In this study, a new control chart called, Full Range Autoregressive (FRAR) chart is introduced following the Jiang *et al*. A comparative study of the FRAR chart with existing ARMA and EWMA control charts are provided to study the performance of FRAR chart.

Key words: Control chart, ARMA chart, FRAR Model, EWMA, existing, India

INTRODUCTION

Statistical Process Control (SPC) is a powerful collection of problem solving tools useful in achieving process stability and improving capability through the reduction of variability and monitoring the performance of the process. For monitoring autocorrelated observations, various control charts have been developed to detect shifts in the mean of the process.

Alwan and Roberts (1988) have discussed, standard applications of statistical process control, a state of statistical control is identified with a random process assuming independent and identically distribution (iid) random variables.

Departures from a state of statistical control are discovered by plotting and viewing data on a variety of control charts such as Shewhart, Cumulative Sum (CUSUM), Exponential Weighted Moving Average (EWMA) and moving average charts.

In practice, however, it may be difficult either to recognize a state of statistical control or to identify departures from one because systematic non-random partterns-reflecting common causes may be present throughout the data. When systematic non-random patterns are present, casual inspection makes it hard to separate special causes and common causes.

A natural solution to this difficulty is to model systematic non-random partners by time series model that go beyond the simple benchmark of iid random variables. Hence, when the data suggest lack of statistical control, one should attempt to model systematic non random behaviour by time series model-autoregressive or other types models. In particular, using the Autoregressive Moving Average (ARMA) Models of Box and Jenkins (1970), identified and estimated by standard techniques, to supplement the iid model. This approach leads to two basic charts rather than one:

A chart of fitted values based on ARMA Models. This chart provides guidance in seeking better understanding of the process and in achieving real-time process control, called common cause chart. Chart of residual (or one-step prediction errors) from fitted ARMA Models. This chart can be used in traditional ways to detect any special causes, called special cause chart.

There have been many applications of time series concepts in process control, the thrust of these applications has been directed toward testing for randomness, not toward modelling of departures from randomness. Use of ARMA Models in other areas of statistical process control and designing control chart.

Determination of appropriate ARMA (p,q) Model, namely, appropriate p and q values is a very difficult task in time series data. Hence, a new family of time series model is used to avoid the order determination problem with minimum number of parameters.

MATERIALS AND METHODS

Shewhart control chart: Control charts of Shewhart (1926) are widely used to monitor a stable process by plotting a sequence of sample data in time order on the charts. Such samples are usually taken as independent samples from the process either with fixed sample size or with fixed sampling interval.

Shewhart control chart is based on the information about the process contained in the last point plotted on the control chart, it ignores any information given by the entire sequence of points obtained from the experiment. This feature makes the Shewhart chart relatively insensitive to small shifts in the process when the change is very small compared to standard deviation or less.

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There are many effective alternatives to the Shewhart control chart can be found in the literature, when small shifts of interest. The CUSUM and EWMA are most popular and best alternatives for the Shewhart control chart. The CUSUM chart directly incorporates all the information in cumulative sums of the deviations of the sample value from the target value, more details can be found by Montgomery (2002).

Exponential weighted moving average is a type of control chart used to monitor either variables, attribute type data using the monitor process. The EWMA emerges as a flexible time-series model that, for many but not all processes may be a satisfactory approximation.

Roberts (1959) first introduced the Exponential Weighted Moving Average (EWMA) control scheme. Using simulation to evaluate its properties, he showed that the EWMA is useful for detecting small shifts in the mean of a process.

One appealing interpretation of EWMA is that the process being studied can be decomposed into two components: iid random disturbances with mean 0. A random walk which is the sum of a fraction of all past iid random disturbances.

Autoregressive moving average chart: The statistical control chart is an effective tool for achieving process stability. For monitoring autocorrelated observations, various control charts have been developed to detect shifts in the mean of the process. Among those that have been widely discussed are the Special Cause Chart (SCC) Alwan and Roberts (1988). The basic idea of the SCC chart involves filtering techniques to whiten an autocorrelated process and then monitoring the residuals by traditional control charts are used (Zhang, 1998). This chart is more effective when detecting large shifts. On the other hand, the EWMAST chart applies the Exponential Weighted Moving Average (EWMA) statistic directly to the autocorrelated process without identifying the process parameters and is more efficient in some parameter regions. More details regarding EWMA charts and its proportion can be found by Chao-Wen and Reynolds (1999).

This ARMA chart provides a more flexible choice of parameters to relate the autocorrelation of the statistics to the chart performance and includes the SCC chart and EWMAST chart as a special case. It can be shown that an ARMA chart with appropriate parameter values will outperform both the SCC and EWMAST charts for autocorrelated processes.

Suppose that, we are monitoring a production process and with to detect the shift in the mean of the process if any. Using ARMA (1, 1) which is more general model then AR (1) is fitted to the data that is:

$$X_t = \theta_0 e_t - \theta_1 e_{t-1} + \varnothing_1 X_{t-1} = \theta_0 (e_t - \beta e_{t-1} + \varnothing_1 X_{t-1}$$

where, $\beta = \theta/\theta_0$ and θ_0 is chosen, so that, the sum of the coefficient is unity when X_t is expressed in terms of e_t 's that is:

$$\frac{\theta_0 - \theta_1 B}{1-\varnothing_1 B} = \frac{\theta_0 - \theta_1}{1-\varnothing_1 = 1}$$

where, B is the backshift operator and $Be_t = e_{t-1}$. Thus, $\theta_0 = 1+\theta_1-\varnothing_1$. To guarantee that the monitoring process is reversible and stationary, we have the constrains that $|\beta|<1$ and $|\varnothing_1|<1$ the ARMA chart signals when $|X_t|>L\sigma_x$. The ARMA chart reduces to the EWMA chart when $\theta_1 = 0$ with $\varnothing_1 = 1-\lambda$, thus, the ARMA chart can be considered as an extension of the EWMA chart. More generalized version of AR Model called FRAR Model is now considered to construct the control chart which is explained in the following section.

Full range autoregressive model: Full range autoregressive model is introduced by Venkatesan *et al.* (2017) is a new family of time series models which avoid the problem of order determination and explained in the following.

The various models discussed in the literature are all of finite order type. That is each involves only a finite order, at least as far as real life applications are concerned. That is, they are all generally based on the questionable assumption that the future value would be influenced only by a limited number of past values.

Moreover, the existing theory of autoregressive models assumes that the coefficients of the model are not connected in any way among each other. That is they are treated as independent constants. Therefore, it would be useful from practical point of view, to propose new models which can accommodates long range dependence and have the property that the coefficients of the past values in the model are functions of a limited number of parameters.

Further, most of the research in time series analysis are concerned with series having the property that the degree of dependence between observations separated by a long time span, given the intermediate observations is zero or highly negligible.

A family of models, called full range autoregressive models and denoted as FRAR Model for short are defined in such a way that they possesses the following basic features.

The models should be capable of representing long term persistence. This is justified by the fact that the future may not depend on the present and a few past values alone but may depend on the present and the whole past. The models should be flexible enough to explain both of short-term and the long-term correlation structure of a series. The parameters of the model which are likely to be large in number due to (1) should exhibit some degree of dependence among themselves which would avoid the difficult task of order determination of the model.

Therefore, the new models are expected to have infinite structure with a finite number of parameters and so, completely avoid the problem of the order determination. FRAR Model is defining a family of models by a discrete-time stochastic process $\{X_t\}$, t = 0, ±1, ±2, ±3, ..., called the Full Range Autoregressive (FRAR) Model by the difference equation:

$$X_t = \sum_{r=1}^{\infty} \frac{k\sin(r\theta)\cos(r\varnothing)}{\alpha^r} X_{t-r} + e_t \quad (1)$$

$$= \sum_{r=1}^{\infty} a_r X_{t-r} + e_t \quad (2)$$

Where:

$$a_r = \frac{k\sin(r\theta)\cos(r\varnothing)}{\alpha^r},\ \ r = 1, 2, 3, ...,$$

k, α, θ and $\varnothing$ are real parameters. The initial assumptions about the parameters are as follows. It is assumed that X_t will influence X_{t+n} for all positive n and the influence of X_t on X_{t+n} will decrease, at least for large n and become insignificant as n becomes very large because more important for the recent observations and less important for an older observations. Hence, a_n must tends to zero as n goes to infinity. This achieved by assuming that a>1. The feasibility of X_t having various magnitudes of influence on X_{t+n} when n is small is made possible by allowing k to take any real value. Because of the periodicity of the circular functions sine and cosine, the domain of θ and $\varnothing$ are restricted to $\theta \epsilon (0, \pi)$ and $\varnothing \epsilon (0, \pi/2)$, respectively.

The region of identifiability of the models is given by S = {a, k, θ, $\varnothing$[kϵR, α>1, $\theta \epsilon [0, \pi$ and $\varnothing \epsilon [0, \pi/2)$} and more details can be found by Venkatesan *et al*. (2017).

Full range autoregressive control chart: Introduce the new FRAR chart for monitoring the mean of a production process. which is expected to be stationary and random process. Suppose that, we are monitoring the process $\{X_t\}$ which is assumed to follow a time series pattern and e_i, i = 1, 2, 3, …, n are normal variation with mean zero and variance σ^2_e. We wish to detect shifts in the mean of the process, if any using FRAR chart.

The mean, variance and other statistical properties of the FRAR process can be found by Venkatesan *et al*. (2017). One can show that:

$$\text{Mean} = \mu_0$$

$$E[X_t, X_{t+s}] = \sigma_e^2 \frac{k^2}{(1+k)^2} \frac{k(1+k)^{s-1}}{\alpha^r} \left[\frac{\alpha^2}{\alpha^2-(1+k^2)}\right]$$

and

$$var(X_t) = \sigma^2_{X_t} = \sigma_e^2 \frac{k^2}{(1+k)^2}\left[\frac{\alpha^2}{\alpha^2-(1+k^2)}\right]$$

When autocorrelation exists between the observations. Then, the control limits for FRAR chart are obtained and is given by:

$$UCL = \mu_0 + L\sigma_{X_t}$$
$$CL = \mu_0$$
$$LCL = \mu_0 - L\sigma_{X_t}$$

RESULTS AND DISCUSSION

Numerical examples: A numerical example is considered for illustrating the applicability of FRAR control chart and to comparative performance of FRAR chart over other charts, considered by other researchers available in the literature. We use a same set of simulated observations from Jiang *et al*. (2000). The data, together with the corresponding FRAR values are shown in Table 1. The target value is taken to be 0 and standard deviation is 1, so, the process is in-control for the first 11 observations. The mean level was shifted upward by approximately one standard deviation for the last 8 observations.

The same parameter taken from Jiang *et al*. (2000), the EWMA with parameter λ = 0.15 and L = 2.913, giving actual control limits of ±0.829. For the ARMA chart, the parameter are chosen as $\varnothing$ = 0.85 which corresponding to λ = 0.15 of the EWMA chart and θ = -0.03. The control

Table 1: The comparison of EWMA, ARMA and FRAR charts

		1 σ_ϵ shift			0.75 σ_ϵ shift			
Observation No.	Observation	EWMA	ARMA	FRAR	Observation	EWMA	ARMA	FRAR
1	1	0.15	0.12	0.000	1	0.15	0.12	0.000
2	-0.5	0.053	0.072	0.117	-0.5	0.053	0.072	0.117
3	0	0.045	0.046	0.079	0	0.045	0.046	0.079
4	-0.8	-0.082	-0.057	0.053	-0.8	-0.082	-0.057	0.053
5	-0.8	-0.190	-0.168	-0.059	-0.8	-0.190	-0.168	-0.059
6	-1.2	-0.341	-0.311	-0.182	-1.2	-0.341	-0.311	-0.182
7	1.5	-0.065	-0.12	-0.334	1.5	-0.065	-0.120	-0.334
8	-0.6	-0.145	-0.129	-0.154	-0.6	-0.145	-0.129	-0.154
9	1	0.026	-0.008	-0.136	1	0.026	-0.008	-0.136
10	-0.9	-0.113	-0.085	0.010	-0.9	-0.113	-0.085	0.010
11	1.2	0.084	0.045	-0.046	0.95	0.047	0.015	-0.046
12	0.5	0.147	0.134	0.083	0.25	0.077	0.071	0.053
13	2.6	0.515	0.441	0.172	2.35	0.418	0.350	0.108
14	0.7	0.543	0.537	0.478	0.45	0.423	0.422	0.384
15	1.1	0.626	0.609	0.585	0.85	0.487	0.474	0.467
16	2	0.832	0.791	0.652	1.75	0.676	0.639	0.516
17	1.4	0.917	0.900	0.799	1.15	0.748	0.733	0.651
18	1.9	1.065	1.035	0.878	1.65	0.883	0.856	0.722
19	0.8	1.025	1.033	0.979	0.55	0.833	0.843	0.817

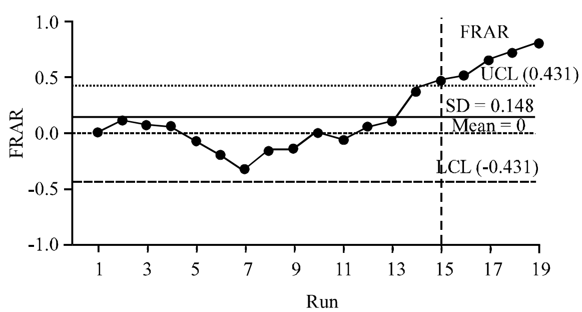

Fig. 1: FRAR control chart

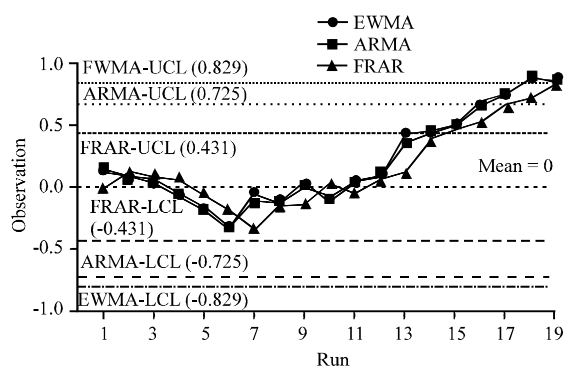

Fig. 2: The comparison of EWMA, ARMA and FRAR charts

limits ±0.725 are chosen, so that, the in-control ARL remains at 500. The FRAR parameters are chosen as k = 2, α = 1.7, θ = 0.1736 and ⌀ = 0.5736. Giving the actual control limits ±0.431.

To see how the EWMA and ARMA chart react the mean shifts, we know a shift of one standard deviation occur at the 11th run and the FRAR chart react the mean shift, of one standard deviation occur at the 12th run. As shown in Table 1, the EWMA chart and ARMA chart both signal at the 16th run and the FRAR is signal at the 14th run.

To illustrate the difference between the three charts, we consider a small mean shift of 0.75 standard deviations. As shown in Table 1, the EWMA chart signals the shift at the 18th observation and ARMA chart detects the shift at the 17th but the FRAR chart detects the shifts at 15th. From Table 1 and Fig. 1 and 2 it is obtained FRAR chart quickly detect the change in the mean of the production process compared to that of ARMA and EWMA charts.

From Table 1 and Fig. 2, it is observed that, FRAR chart perform better then ARMA and EWMA chart in the sense that quickly detecting the change in the process. Hence, FRAR chart may be considered on variable alternative to ARMA and other charts.

CONCLUSION

FRAR charts is proposed in this study by using FRAR Model which could completely avoid the order determination problem, to monitor the production process and shown that its performance is a relately better than ARMA and EWMA to detect the change in the mean of the process and quickly detect the change in the mean of the production process.

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