# Study of the Nuclear Structure of ${ }_{58}^{134} \mathrm{Ce}_{76}$ Isotope within the Framework of the Interacting Boson Model (IBM-1) 

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#### Abstract

The nuclear structure of ${ }^{134} \mathrm{Ce}$ isotope has been studied within the framework of the Interacting Boson Model (IBM-1). The dynamical symmetry has been determined depending on the ratios of the Experimental Energy of $E 4_{1}+/ E 2_{1}+$ which is in agreement with theoretical values. Also, the energy levels, $g, \gamma, \beta 1$ bands have been calculated and compared with the experimental data and which is showed agreement between them. In addition to that, the reduced probability of quadrupole electric transitions, $B(E 2)$ values for ${ }^{134} \mathrm{Ce}$ isotope have been calculated and compared with available experimental data. The calculated results are in good agreement with the experimental data. The potential Energy surface ( $\mathrm{E}(\mathrm{N}, \beta, \gamma)$ ) have been calculated and these properties show that the shape of ${ }^{134} \mathrm{Ce}$ isotope isotopes is $\gamma$-unstable and it has the $\mathrm{O}(6)$ limit.


Key words: Nuclear structure, dynamical symmetry, electric transitions, isotope, properties, $\gamma$-unstable

## INTRODUCTION

There have been numerous efforts in the history of nuclear physics to model the characteristic structure of nuclei. The quantity of data created requires more models of the nuclear structure to access it. It has been linked with our computational capability (Cohen, 1971; Abdulkadhim and Hussain, 2017). Arima and Iachello proposed a new model in 1974, called Interacting Boson Model (IBM) "of nuclear structure (Iachello and Arima, 1987; Arima and Iachello, 1975, 1976; Abrahams et al., 1981). It has been very successful and widely used to the structure of low-lying states in even even nuclei. IBM assumed that the even even nucleus is a collection of interacting $s$ and $d$ bosons with" angular momentum $(\mathrm{L})=0$ and 2 , respectively. This model is linked with an inherent group structure which allows the introduction of limiting symmetries called $\mathrm{U}(5), \mathrm{SU}(3)$ and $\mathrm{O}(6)$ (Iachello and Arima, 1987; Arima and Iachello, 1975, 1976; Abrahams et al., 1981). The degree of freedom for proton-and neutron-boson is not distinguished in the interacting boson model in its original version. It is supposed that, the excitations of the valence protons and neutrons caused the dominance of low-lying collective states in medium and heavy even-even nucleus away from closed shells (i.e., particles outside the major closed shells at $2,8,20,28,50,82$ and 126 ) only while the closed shell core is inert. It is also assumed that the particle configurations which are identical (coupled particles) with each other forming pairs of angular momentum 0 and 2. As well, these pairs "proton (neutron) have been treated as bosons. Furthermore, if the angular momentum $\mathrm{L}=0$
then proton (neutron) bosons are called s-bosons and are denoted by $\mathrm{s}_{\pi}\left(\mathrm{s}_{v}\right)$ while if the angular momentum $\mathrm{L}=2$ then proton (neutron) bosons are called d-bosons and are denoted by $\mathrm{d}_{\pi}\left(\mathrm{d}_{\mathrm{v}}\right)$. The symbol $\pi(v)$, recognizes protons and neutrons. The Hamiltonian corresponding to the IBM has a group structure $U(6)$ because that the $s$ and d bosons span a six-dimensional Hilbert space. The geometrical shapes, spherical vibrator, symmetric rotor and $\gamma$-unstable rotor, respectively, correspond to the three limiting symmetries of this Hamiltonian, $\mathrm{U}(5), \mathrm{SU}(3)$ and O(6) (Iachello and Arima, 1987; Sethi et al., 1991). At last, the number of valence proton (neutron) pairs which is symbolized by $\mathrm{N}_{\pi}\left(\mathrm{N}_{v}\right)$ is calculated from the nearest closed shell taking into consideration the particle-hole conjugation that means that the number of pairs of particles is considered as the number of bosons if less than half of the shell is filled while that the number of bosons is considered equal to the number of pairs of holes if more than half of the shell is filled. The IBM Sharrad et al. (2013) was successful in reproducing the nuclear collective levels in terms of $s$ and d bosons which are essentially the collective $s$ and d pairs of valence nucleons with angular momentum $L=0$ and $L=2$ (Otsuka et al., 1978; Hussain et al., 2018). Al-Hilfy et al. (2013) studied the even-even ${ }^{130-136} \mathrm{Ce}$ isotopes within the framework of the Interacting Boson Model (IBM), investigated the electric quadruple transitions and energy levels of these isotopes and compared the calculated results with the experimental data. Zhu et al. (2017) measured new lifetime of excited states in ${ }^{134} \mathrm{Ce}$, populated excited states of ${ }^{134} \mathrm{Ce}$ by the fusion evaporation reaction ${ }^{122} \mathrm{Sn}\left({ }^{16} \mathrm{O}, 4 \mathrm{n}\right){ }^{134} \mathrm{Ce}$, employed the

[^0]recoil distance Doppler shift method and discussed the systematic evolution in the collectivity of the Ce isotopes.

## Theoretical basis

Hamiltonian operator: The IBM-1 Hamiltonian can express it as (Iachello and Arima, 1987; Sharrad et al., 2012):

$$
\begin{align*}
& \mathrm{H}=\varepsilon_{\mathrm{s}}\left(\mathrm{~s}^{\dagger} . \tilde{\mathrm{s}}\right)+\varepsilon_{\mathrm{d}}\left(\mathrm{~d}^{\dagger} . \tilde{d}\right)+\sum_{\mathrm{L}=0,2,4} \frac{1}{2}(2 \mathrm{~L}+1)^{\frac{1}{2}} \\
& \mathrm{C}_{\mathrm{L}}\left[\left[\mathrm{~d}^{\dagger} \times \mathrm{d}^{\dagger}\right]^{(\mathrm{L})} \times[\tilde{\mathrm{d}} \times \tilde{\mathrm{d}}]^{(\mathrm{L})}\right]^{(0)}+\frac{1}{\sqrt{2}} \mathrm{v}_{2} \\
& {\left[\left[\mathrm{~d}^{\dagger} \times \mathrm{d}^{\dagger}\right]^{(2)} \times[\tilde{\mathrm{d}} \times \tilde{\mathrm{s}}]^{(2)}+\left[\mathrm{d}^{\dagger} \times \mathrm{s}^{\dagger}\right]^{(2)} \times[\tilde{\mathrm{d}} \times \tilde{\mathrm{d}}]^{(2)}\right]^{(0)}+} \\
& \frac{1}{2} \mathrm{u}_{0}\left[\left[\mathrm{~d}^{\dagger} \times \mathrm{d}^{\dagger}\right]^{(0)} \times[\tilde{\mathrm{s}} \times \tilde{\mathrm{s}}]^{(0)}+\left[\mathrm{s}^{\dagger} \mathrm{s}^{\dagger}\right]^{(0)} \times[\tilde{\mathrm{d}} \times \tilde{\mathrm{d}}]^{(0)}\right]^{(0)}+ \\
& \frac{1}{2} \mathrm{u}_{0}\left[\left[\mathrm{~s}^{\dagger} \times \mathrm{s}^{\dagger}\right]^{(0)} \times[\tilde{\mathrm{s}} \times \tilde{\mathrm{s}}]^{(0)}\right]^{(0)}+\mathrm{u}_{2}\left[\left[\mathrm{~d}^{\dagger} \times \mathrm{s}^{\dagger}\right]^{(2)} \times[\tilde{\mathrm{d}} \times \tilde{\mathrm{s}}]^{(2)}\right]^{(0)} \tag{1}
\end{align*}
$$

The raising operator is $\left(\mathrm{s}^{\dagger}, \mathrm{d}^{\dagger}\right)$ for s -bosons and lowering operator is $(\tilde{\mathrm{s}}, \tilde{\mathrm{d}})$ for d-bosons (Casten and Warner, 1988). The Hamiltonian includes two terms of one body interactions are ( $\varepsilon_{\mathrm{s}}$ and $\varepsilon_{\mathrm{d}}$ ) which represent the single-boson energies and seven terms of the two-body interactions are $\left[C_{L}(L=0,2,4), v_{L}(L=0,2), v_{L}(L=0\right.$, 2) which describe the interactions of two-boson but it has been shown that for a fixed boson Number ( N ), only one of the one-body term and five of the two body terms are independent. As can be observed by $\mathrm{N}=\mathrm{n}_{\mathrm{s}}+\mathrm{n}_{\mathrm{d}}$. However, it is mostly the Hamiltonian of the IBM-1 written as a multipole expansion, grouped into different boson-boson interactions Eq. 1 (Sharrad et al., 2012; Casten and Warner, 1988):

$$
\hat{\mathrm{H}}=\varepsilon \hat{\mathrm{n}}_{\mathrm{d}}+\mathrm{a}_{0} \hat{\mathrm{p}} \cdot \hat{\mathrm{p}}+\mathrm{a}_{1} \hat{\mathrm{~L}} \cdot \hat{\mathrm{~L}}+\mathrm{a}_{2} \hat{\mathrm{Q}} \cdot \hat{\mathrm{Q}}+\mathrm{a}_{3} \hat{\mathrm{~T}}_{3} \cdot \hat{\mathrm{~T}}_{3}+\mathrm{a}_{4} \hat{\mathrm{~T}}_{4} \cdot \hat{\mathrm{~T}}_{4}(2)
$$

Where:

$$
\begin{aligned}
& \hat{n}_{d}=\left(d^{\dagger} \cdot \tilde{d}\right) \text { is represents the total number of } \\
& d_{\text {boson }} \text { operator } \\
& \hat{p}=1 / 2[(\tilde{d} . \tilde{d})-(\tilde{s} . \tilde{s})] \text { represents the } \\
& \quad \text { operator of pairing }
\end{aligned}
$$

$$
\hat{\mathrm{L}}=\sqrt{10}\left[\mathrm{~d}^{+} \times \tilde{\mathrm{d}}\right]^{1} \text { presents the operator }
$$

of angular momentum

$$
\begin{align*}
\hat{\mathrm{Q}}= & {\left[\mathrm{d}^{\dagger} \times \tilde{\mathrm{s}}+\mathrm{s}^{\dagger} \times \tilde{\mathrm{d}}\right]^{(2)}+\chi\left[\mathrm{d}^{\dagger} \times \tilde{\mathrm{d}}\right]^{(2)} \text { represents the } }  \tag{6}\\
& \text { operator of quadrupole }
\end{align*}
$$

where, $\chi$ is the parameter of quadrupole structure and its values 0 and $\pm \sqrt{7 / 2}$ (Iachello and Arima, 1987; Casten and Warner, 1988):

$$
\begin{align*}
& \hat{\mathrm{T}}_{\mathrm{r}}=\left[\mathrm{d}^{+} \times \tilde{d}\right]^{(r)} \text { operator of the octoupole }  \tag{7}\\
& (\mathrm{r}=3) \text { and hexadecapole }(\mathrm{r}=4) \\
& \varepsilon=\varepsilon_{d}-\varepsilon_{\mathrm{s}} \text { energy of boson } \tag{8}
\end{align*}
$$

The strength of the pairing, angular momentum, quadrupole, octoupole and hexadecapole interaction between the bosons were assigned by the parameters $\mathrm{a}_{0}-\mathrm{a}_{4}$.

Electromagnetic transitions: IBM has been used to describe electromagnetic transition rates in addition to excitation energy spectra. One must detect the transition operators in terms of the boson operators in order to do so (Sethi et al., 1991). It is assumed that the transition operators will include one-body terms only in minimum order, clearly in IBM-1 that the more general form of this operator can be given by Sethi et al. (1991), Iachello and Arima (1987), Casten and Warner (1988), Yazar and Erdem (2008):

$$
\begin{align*}
& \mathrm{T}_{\mathrm{m}}^{1}=\alpha_{2} \delta_{12}\left[\mathrm{~d}^{\dagger} \times \tilde{\mathrm{s}}+\mathrm{s}^{\dagger} \times \tilde{\mathrm{d}}\right]_{\mathrm{m}}^{2}+ \\
& \beta_{1}\left[\mathrm{~d}^{\dagger} \times \tilde{d}\right]_{\mathrm{m}}^{1}+\gamma_{0} \delta_{10} \delta_{\mathrm{m} 0}\left[\mathrm{~s}^{\dagger} \times \tilde{\mathrm{s}}\right]_{0}^{0} \tag{9}
\end{align*}
$$

Also, in case ( $l=2$ transitions) can the first term be presented while in case ( $\mathrm{l}=0$ transitions) can the last term be presented. This is confirmed by Kronecker delta ( $\delta$ ) associated them. The transition operator specified form in the special cases of electric monopole, quadrupole and hexadecapole transitions is $\gamma_{0}, \alpha_{2}$ and $\beta_{1}(l=0,1,2,3,4)$, respectively which are parameters determining the various terms in the corresponding operators. The electric quadrupole transition is:

$$
\begin{align*}
& \mathrm{T}_{\mathrm{m}}^{\mathrm{E} 2}=\alpha_{2}\left[\mathrm{~d}^{\dagger} \times \tilde{\mathrm{s}}+\mathrm{s}^{\dagger} \times \tilde{\mathrm{d}}\right]_{\mathrm{m}}^{2}+\beta_{2}\left[\mathrm{~d}^{\dagger} \times \tilde{d}\right]_{\mathrm{m}}^{2}=  \tag{10}\\
& \alpha_{2}\left(\left[\mathrm{~d}^{\dagger} \times \tilde{\mathrm{s}}+\mathrm{s}^{\dagger} \times \tilde{d}\right]_{\mathrm{m}}^{2}+\chi\left[\mathrm{d}^{+} \times \tilde{d}\right]_{\mathrm{m}}^{2}\right)=\mathrm{e}_{\mathrm{B}} \tilde{\mathrm{Q}}
\end{align*}
$$

The $\alpha_{2}$ and $\beta_{2}$ are two parameters where:
$\beta_{2}=\chi \alpha_{2}$
$\alpha_{2}=\mathrm{e}_{\mathrm{B}}$
$\mathrm{e}_{\mathrm{B}}=$ Effective charge of boson

The quadrupole operator $\hat{Q}$ show in Eq. 6. Rates of electromagnetic transition can be calculated in the usual way by using the element of reduced matrix for the corresponding transition operator between the state of initial and the final. The reduced matrix element symbol is $\left\langle\mathrm{L}_{\mathrm{f}} \mid \mathrm{T}^{1} \| \mathrm{L}_{\mathrm{i}}\right\rangle$ (Krane and Shobaki, 1977). So, from definition, the $\mathrm{B}(\mathrm{El})$ values will be:

$$
\begin{equation*}
B\left((E l), L_{i} \rightarrow L_{f}=\frac{1}{2 L_{i}+1}\left|\left\langle L_{f}\left\|T^{(E)}\right\| L_{i}\right\rangle\right|^{2}\right. \tag{11}
\end{equation*}
$$

## MATERIALS AND METHODS

Potential energy surface basis: The final form of the nucleus corresponding to the Hamiltonian function is determined by the potential Energy surface ( $\mathrm{E}(\mathrm{N}, \beta, \gamma$ ) ) as shown in this Eq. 12 (Casten and Warner, 1988; Iachello and Arima, 1987; Hamilton, 1975):

$$
\begin{equation*}
E(N, \beta, \gamma)=\langle N, \beta, \gamma| H|N, \beta, \gamma\rangle /\langle N, \beta, \gamma \mid N, \beta, \gamma\rangle \tag{12}
\end{equation*}
$$

The expected IBM-1 value was used by the surface energy with the coherent state ( $|\mathrm{N}, \beta, \gamma\rangle$ ) in order create the IBM (Sharrad et al., 2013; Casten and Warner, 1988). The state is a product of boson creation operators ( $\mathrm{b}_{\mathrm{c}}^{\dagger}$ ) with:

$$
\begin{equation*}
|\mathrm{N}, \beta, \gamma\rangle=1 / \sqrt{\mathrm{N}!}\left(\mathrm{b}_{\mathrm{c}}^{\dagger}\right)^{\mathrm{N}}|0\rangle \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{b}_{\mathrm{c}}^{\dagger}=\left(1+\beta^{2}\right)^{-1 / 2}\left\{\mathrm{~s}^{\dagger}+\beta\left[\cos \gamma\left(\mathrm{d}_{0}^{\dagger}\right)+\sqrt{1 / 2} \sin \gamma\left(\mathrm{~d}_{2}^{\dagger}+\mathrm{d}_{-2}^{\dagger}\right)\right]\right\} \tag{14}
\end{equation*}
$$

The energy surface as a function of $\beta$ and $\gamma$ has been given by (Casten and Warner, 1988):

$$
\begin{align*}
& E(N, \beta, \gamma)=\frac{N \varepsilon_{d} \beta^{2}}{\left(1+\beta^{2}\right)}+\frac{N(N+1)}{\left(1+\beta^{2}\right)^{2}}  \tag{15}\\
& \left(\alpha_{1} \beta^{4}+\alpha_{2} \beta^{3} \cos 3 \gamma+\alpha_{3} \beta^{2}+\alpha_{4}\right)
\end{align*}
$$

$\alpha_{1}-\alpha_{4}$ were related to the coefficients $C_{L}, v_{2}, v_{0}, u_{2}$ and $u_{0}$ of Eq. 1. The total nucleus deformation is a measured by $\beta$ when $\beta=0$ the shape will be spherical and be distorted when $\beta \neq 0, \gamma$ represents the quantity of deviation from the focus symmetry and it's associated with the nucleus. When the value equal to 0 , the shape be prolate but when it is value equal to 60 the shape becomes oblate. The potential energy surface can be represented by Eq. 16 for three dynamic symmetries:

$$
E(N, \beta, \gamma) \propto \begin{cases}U(5): & \varepsilon_{d} N \frac{\beta^{2}}{1+\beta^{2}} \\ S U(3): & k N(N-1) \frac{\frac{3}{4} \beta^{4}-\sqrt{2} \beta^{3} \cos 3 \gamma+1}{\left(1+\beta^{2}\right)^{2}}  \tag{16}\\ O(6): & k^{\prime} N(N-1)\left(\frac{1-\beta^{2}}{1+\beta^{2}}\right)^{2}\end{cases}
$$

Where, $\mathrm{k} \propto \mathrm{a}_{2}$ and $\mathrm{k} \propto \mathrm{a}_{0}$ in Eq. 2.

## RESULTS AND DISCUSSION

IBM was used to calculate the properties of the ${ }_{58}^{134} \mathrm{Ce}_{76}$ issotope. The ratio $\mathrm{E}(4) / \mathrm{E}(2)$ for the nucleus ${ }_{58}^{134} \mathrm{Ce}_{76}$ which is equal to 2.5 , i.e, it is a deformed nucleus and belongs to the limit O6 (dynamic symmetry O6). One of the most important concepts in the nuclear structure is the concept of symmetry that must be determined accurately. The form of the nucleus has influential relationship in determining the nuclear quantities such as energy levels, the electromagnetic transitions probability and the electric quadrupole moment.

Energy levels: The energy levels of ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotopes have been classified according to three bands (gr-, $\gamma$ and $\beta$-bands). Adopted values for the parameters used in IBM-1 calculations are shown in Table and 2.
${ }_{58}^{134} \mathrm{Ce}_{76}$ nucleus: The ${ }_{58}^{134} \mathrm{Ce}_{76}$ nucleus has 58 protons and 76 neutrons and then the number of bosons is 7 . The levels $0^{+}, 2^{+}{ }_{3}$ and $4^{+}{ }_{3}$ with energies $1.533,1.9644$ and 1.812 MeV , respectively were confirmed with the states that are not well established experimentally (Sonzogni, 2004) and it can be seen in Fig. 1.

## $B(E 2)$ values and related quantities

Absolute B(E2) values: Much information can be obtained by studying the reduced transition probabilities B(E2). The computer code IBMT Scholten (1980) was applied to calculate the values of $B(E 2)$ and it must

Table 1: Adopted values of the parameters, measured in MeV units, excepted N and CHI

| A | N | EPS | $\mathrm{a}_{0}$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{3}$ | $\mathrm{a}_{4}$ | CHI |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 134 | 76 | 0.0000 | 0.1916 | 0.0246 | 0.0000 | 0.1868 | 0.0000 | 0.0000 |




Fig. 1: Comparison the IBM-1 calculations with the experimental data (Sonzogni, 2004) for ${ }^{134} \mathrm{Ce}$ isotope
specify values of effective charge ( $\mathrm{e}_{\mathrm{B}}$ ). To reproduce the experimental $B(E 2)$, the effective charge values $\left(\mathrm{e}_{\mathrm{B}}\right)\left(\alpha_{2}\right)$ were estimated and it is tabulated in Table 2.

Table 3 shows a comparison between B(E2) values in IBM-1 and in the experimental data (Sharrad et al., 2012, 2013; Casten and Warner, 1988; Yazar and Erdem, 2008; Arima and Iachello, 1976; Krane and Shobaki, 1977; Hamilton, 1975; Sonzogni, 2004) for ${ }_{58}^{134} \mathrm{Ce}_{76}$ nucleus. There is no existing experimental transition data for most transitions in Table 3. Thus, it was predicted by using IBM-1. Also, Table 3 shows that IBM values are in good agreement with $\mathrm{B}(\mathrm{E} 2)$ values experimentally.

B(E2) ratio: By using other important quantities, the $\mathrm{B}(\mathrm{E} 2)$ ratio shows that the ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotope is deformed nucleus with a dynamical symmetry $\mathrm{O}(6)$. The formulas for calculating the $\mathrm{B}(\mathrm{E} 2)$ ratio are (Iachello and Arima, 1987; Casten and Warner, 1988):

Table 3: B (E2) values for ${ }^{134} 58 \mathrm{Ce}_{76}$ nucleus (in e ${ }^{2} . \mathrm{b}^{2}$ )

| ${ }^{134} \mathrm{Ce}$ (Sonzogni, 2004) |  |  |
| :---: | :---: | :---: |
| $\mathrm{J}_{\mathrm{i}} \rightarrow \mathrm{J}_{\mathrm{f}}$ | IBM-1 | Exp. |
| $0^{+}{ }_{3} 2^{+}{ }_{2}$ | 0.2976 | - |
| $2_{1}^{+}{ }_{1} 0^{+}{ }_{1}$ | 0.2115 | 0.2118 |
| $2^{+}{ }_{3} \rightarrow 0^{+}{ }_{2}$ | 0.1236 | - |
| $2^{+}{ }_{2} \rightarrow 2^{+}{ }_{1}$ | 0.2826 | - |
| $2_{4}^{+} \mathrm{C}^{+}{ }_{3}$ | 0.1570 | - |
| $4_{1}^{+} \rightarrow 2^{+}{ }_{1}$ | 0.2826 | 0.15885 |
| $4_{2}^{+} \mathrm{C}^{+}{ }_{2}$ | 0.1559 | - |
| $4_{3}^{+}{ }^{+}{ }^{+}{ }_{3}$ | 0.1570 | - |
| $4^{+}{ }_{2} 4^{+}{ }_{1}$ | 0.1417 | - |
| $6_{1}^{+} \rightarrow 4^{+}$ | 0.2976 | 0.057 |
| $6_{2}^{+} \rightarrow 4^{+}$ | 0.1907 | - |
| $6_{2}^{+} \rightarrow 6^{+}{ }_{1}$ | 0.0890 | - |
| $8^{+}{ }_{1} \rightarrow 6^{+}{ }_{1}$ | 0.2797 | 0.0987 |
| $8^{+}{ }_{2} \rightarrow 6^{+}{ }_{2}$ | 0.1807 | - |
| $8^{+}{ }_{2} \rightarrow 8^{+}{ }_{1}$ | 0.0571 | - |
| $10^{+} \rightarrow 8^{+}{ }_{1}$ | 0.2377 |  |
| $10^{+}{ }_{2} 8^{+}{ }_{2}$ | 0.1419 |  |
| $12^{+}{ }_{1} \rightarrow 10^{+}{ }_{1}$ | 0.1758 |  |
| $12^{+} \rightarrow 10^{+}$ | 0.0806 |  |



Fig. 2: The potential energy surface in $\gamma-\beta$ plane for nucleus

Table 4: Comparison the experimental data (Sonzogni, 2004) with IBM-1 calculations for ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotope

| IBM-1 calculations for ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotope |  |  |  |
| :--- | ---: | :--- | ---: |
|  |  |  | O(6) Iachello and Arima (1987) <br> Isotope |
| N | IBM-1 | Casten and Warner (1988) |  |

$$
\left.\begin{array}{c}
\frac{\mathrm{B}\left(\mathrm{E} 2 ; 4_{1}^{+} \rightarrow 2_{1}^{+}\right)}{\mathrm{B}\left(\mathrm{E} 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)}=\frac{\mathrm{B}\left(\mathrm{E} 2 ; 2_{2}^{+} \rightarrow 2_{1}^{+}\right)}{\mathrm{B}\left(\mathrm{E} 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)}= \\
\frac{10}{7} \frac{(\mathrm{~N}-1)(\mathrm{N}+5)}{\mathrm{N}(\mathrm{~N}+4)} \underset{\mathrm{N} \rightarrow \infty}{ } \frac{10}{7} \approx 1.4  \tag{19}\\
\text { and } \\
\frac{\mathrm{B}\left(\mathrm{E} 2 ; 0_{2}^{+} \rightarrow 2_{1}^{+}\right)}{\mathrm{B}\left(\mathrm{E} 2 ; 2_{1}^{+} \rightarrow 0_{1}^{+}\right)}=0
\end{array}\right\}
$$

The $\mathrm{B}(\mathrm{E} 2)$ ratio is calculated and given in Table 4. This table includes comparison the experimental data (Sharrad et al., 2012, 2013; Casten and Warner, 1988; Yazar and Erdem, 2008; Krane and Shobaki, 1977; Hamilton, 1975; Sonzogni, 2004) with IBM-1 calculations for ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotope.

From Table 4, we can see that the theoretical values of $B(E 2)$ ratio and experimental data in a good agreement and also $\approx 1.4$. This means that the ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotope tend to show the $\mathrm{O}(6)$ limit (Iachello and Arima, 1987; Casten and Warner, 1988).

Potential Energy Surface (PES): It is one of the nuclei properties where the final shape is given by it. The PES. FOR program is applied to calculate the potential energy surface $E(N, \beta, \gamma)$. In this research, we calculate the potential energy surface from Eq. 16 and 19.

Figure 2 shows the contour plots for the ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotope in the $\gamma-\beta$ plane resulting from $\mathrm{E}(\mathrm{N}, \beta, \gamma)$. IBM
energy surface that was mapped for ${ }_{58}^{134} \mathrm{Ce}_{76}$ nucleus is triaxial shape and it is associated with range values $0<\gamma \pi / 3$. Furthermore, the transition of prolate-to-oblate shape that occurs in ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotope can be understood by the triaxial deformation. The ${ }_{58}^{134} \mathrm{Ce}_{76}$ nucleus considered here does not display any quick structural change and remains $\gamma$-soft. This evolution reflects the triaxial deformed as one approaches the neutron shell closure $\mathrm{N}=126$.

## CONCLUSION

The interacting boson model-1 was used to study the nuclear structure for ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotope where the study of the isotope in several respects and reached results was well successful. Particularly, in study the shape of the nuclei, it is prove that the ${ }_{58}^{134} \mathrm{Ce}_{76}$ isotope is deformed nucleus and has a dynamical symmetry O(6). Some energy levels have been confirmed for which the spin and/or parity are not well established experimentally for the isotope under studying. As well as the energy levels calculated and compare with the experimental values where found in good agreement. The contour plot of the potential energy surface shows ${ }_{58}^{134} \mathrm{Ce}_{76}$ nucleus is deformed and have $\gamma$-unstable-like characters. Finally Some of the reduced probability of quadrupole electric transitions $B(E 2)$ values for this isotope are in good agreement with the experimental data.

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