Study the Energy Level and Branching Ratios of ^{144}Ba, ^{144}Ce, ^{144}Nd Isotones using IBM-1

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Abstract: In this study, the most appropriate Hamiltoni an has been determined, that is needed for the present calculations of energy levels and B(E2) values of even-even Ba, Ce, Nd nuclei using the Interacting Boson Model (IBM-1). The results have been compared with a recent experimental data, it was observed that they are in good agreement. Also, the branching ratios (R, R' and R") have been calculated and the importance of studying the branching ratios is to locate the position of the isotopes relative to the dynamical symmetries U(5), SU(3) and O(6). The results of the present study have showed that the isotopes under investigation lies in the different transition regions, the results appear that Ba isotone lies in the transition region SU(3)-O(6) and the Ce, Gd, Dy isotones lies in the transition region of O(6)-U(5) and the Nd, Sm isotones lies in U(5) limit.

Key words: Determined, investigation, agreement, experimental data, dynamical symmetries, region

INTRODUCTION

The nuclei can be well described by the presence of a number of multi nuclear models, among these models that have proven widely successful is interacting boson approximation model which has been proposed by Arima and Iachello (1987) IBA or IBM (Cook, 2005).

The Interacting Boson Model (IBM) is mainly rooted in the shell model which has proved to be an excellent instrument for light nuclei (up to 50 nucleons). The larger the number of nucleons becomes the more shells have to be taken into account and the number of nuclear states soon becomes, so, colossal that the shell model will be intractable. A model of the atomic nucleus has to be able to describe nuclear properties such as spins and energies of the lowest levels, decay probabilities for the emission of gamma quantas, probabilities (spectroscopic factors) of transfer reactions, multipole moments and so forth. The interacting boson model (sometimes named Interacting Boson Approximation IBA) reduces the number of states heavily. It constitutes only 26 configurations for the 2+ state mentioned above (Casten, 1990).

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The numbers of bosons are strictly conserved, the number of protons and neutrons bosons are N_π and N_ν, respectively. Therefore, the total number of bosons (N_ρ) in the system is equal to the sum of both N_π and N_ν (Madhi, 1984). These bosons are combine to produce boson with angular momentum $L = 0$ which is called (s- bosons) or to produce boson with angular momentum $L = 2$ which is called (d-bosons).

The basis for this model is a first version of Interacting Boson Model (IBM-1) which does not distinguish between the wave functions for protons and neutrons but deals with bosons on the basis of one type, all of which are symmetrical while the second version (IBM-2) makes distinction between the wave function for proton and neutron bosons through introducing the concept of degrees of freedom.

The models IBM-1 and IBM-2 are restricted to nuclei with even numbers of protons and neutrons. So as to fix the number of bosons one takes into account that both types of nucleons frame closed shells with particle numbers 2, 8, 20, 28, 50, 82 and 126 (magic numbers). Provided that the protons fill less than half of the furthest shell the number of the corresponding active protons has to be divided by two that obtain the boson number (N_p) attributed to protons. If more than half of the shell is occupied the boson number reads (N_p = (number of holes for protons)/2). By treating the neutrons in an analogous way, one obtains their number of bosons (N_n). In the IBM-1, the boson number (N) is calculated by adding the partial numbers, i.e., ($N = N_p + N_n$). Electromagnetic transitions do not change the boson number but transfers of two identical nucleons lift or lower it by one (Pfeifer, 1998).

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MATERIALS AND METHODS

Interacting Boson Model (IBM): Arima and Iachello in 1974 suggested the nuclear model to describe the collective nuclear structure for medium and heavy nuclei called interacting boson approximation model (IBA or IBM) (Abdul Hussein, 2009). This model used to study the low lying collective states in even-even nuclei through a combination of nucleons resulting from coupling two fermion outside of the closed shell with angular momentum L = 0 and is called s boson and coupled with angular momentum L = 2 and is called d boson. The Hamiltonian operator for the system consist of s and d bosons describe by the six dimensions in Hilbert space which is formed from one dimension for s boson and five dimensions for d boson, (μ = 0, ±2), so is described by group theory U(6) (Shuaib, 2012). The first version of the Interacting Boson Model (IBM-1) is considered the fundamental basis for other versions and it does not distinguish between proton bosons and neutron bosons and it considers all bosons are symmetrical (Hussein, 2011). The Interacting Boson Model (IBM), advanced by Arima and Iachello is a nuclear pattern precedent for the characterization of collective structures. It can fit out theoretical level energies and transition ability while including anharmonicities from residual interactions (Arima and Iachello, 1987).

The spectroscopy of medium mass and heavy even-even nuclei is characterized by the occurrence of low-lying collective states. The study of the nuclear collective motion is one of the most interesting topics in nuclear physics. The basis for this was laid by Rainwater (1950).

Hamiltonian of the IBM-1: The interaction of s-bosons and d-bosons in the IBM is used to explain the collective properties of even-even nuclei (Hossain *et al.*, 2015). In IBM chara-cterizes a six-dimensional Hilbert space and is given by linear combinations of the creation and annihilation operators: s, $s^\dagger$, d and $d^\dagger$.

The allowed combinations are defined by the conservation of the finite number of bosons, N (1/2 the number of valence nucleons) as well as having up to only 2-body interactions. Therefore, terms must contain a creation and Annihilation operator and terms involving three or more operators are not allowed (Arima and Iachello, 1987).

IBM Hamiltonians have been used to suitable the experimental energy spectra and the electromagnetic transition probabilities, Casten triangle can be used to classify the experimental spectra which provides insight in terms of limiting symmetries as well as indicating phase transitions (Casten, 2006). One of the most general forms of the IBM-1 Hamiltonian is given by Eq. 1 (Abrahams *et al.*, 1981):

$$H = \varepsilon_s\left(s^\dagger.\tilde{s}\right)+\varepsilon_d\left(d^\dagger.\tilde{d}\right)$$
$$+\sum_{L=0,2,4}\tfrac{1}{2}(2L+1)^{\frac{1}{2}}C_L\left[\left[d^\dagger\times d^\dagger\right]^{(L)}\times\left[\tilde{d}\times\tilde{d}\right]^{(L)}\right]^{(0)}$$
$$+\frac{1}{\sqrt{2}}\upsilon_2\left[\left[d^\dagger\times d^\dagger\right]^{(2)}\times\left[\tilde{d}\times\tilde{s}\right]^{(2)}+\left[d^\dagger\times s^\dagger\right]^{(2)}\times\left[\tilde{d}\times\tilde{d}\right]^{(2)}\right]^{(0)}$$
$$+\frac{1}{2}\upsilon_0\left[\left[d^\dagger\times d^\dagger\right]^{(0)}\times\left[\tilde{s}\times\tilde{s}\right]^{(0)}+\left[s^\dagger\times s^\dagger\right]^{(0)}\times\left[\tilde{d}\times\tilde{d}\right]^{(0)}\right]^{(0)} \quad (1)$$
$$+\frac{1}{2}u_0\left[\left[s^\dagger\times s^\dagger\right]^{(0)}\times\left[\tilde{s}\times\tilde{s}\right]^{(0)}\right]^{(0)}$$
$$+u_2\left[\left[d^\dagger\times s^\dagger\right]^{(2)}\times\left[\tilde{d}\times\tilde{s}\right]^{(2)}\right]^{(0)}$$

where $\left(s^\dagger.\tilde{s}\right)$ and $\left(d^\dagger.\tilde{d}\right)$ are the creation and annihilation operators for s and d bosons, respectively (Casten and Warner, 1988). Two terms of one body interactions (ε_s and ε_d) and seven terms of two-body interactions [C_L(L = 0, 2, 4), υ_L (L = 0,2), u_L (L = 0, 2)] in this Hamiltonian where the single-boson energies are (ε_s) and (ε_d) and the two-boson interactions had been described by (C_L), (v_L) and (u_L), so on, it shows that for a fixed boson number (N), only one of the one-body term and five of the two body terms are independent. It can be seen by noting (N = n_s+n_d). Yet, it is more common to write the Hamiltonian of the IBM-1 as a multipole expansion, grouped into different boson-boson interactions Eq. 2 (Casten, 1990):

$$\hat{H} = \varepsilon''\hat{n}_d+a_0\widehat{P^\dagger}\cdot\hat{P}+a_1\hat{L}\cdot\hat{L}+a_2\hat{Q}\cdot\hat{Q}+a_3\hat{T}_3\cdot\hat{T}_3+a_4\hat{T}_4\cdot\hat{T}_4 \quad (2)$$

The operators are defined by the following Eq. 3:

$$\hat{n}_d = \left[d^\dagger\cdot\tilde{d}\right]$$

$$\hat{P} = \frac{1}{2}\left(\tilde{d}\cdot\tilde{d}\right)-\frac{1}{2}\left(\tilde{s}\cdot\tilde{s}\right)$$

$$\hat{L} = \sqrt{10}\left[d^\dagger\times\tilde{d}\right]^{(1)} \quad (3)$$

$$\hat{Q} = \left[d^\dagger\times\tilde{s}+s^\dagger\times\tilde{d}\right]^{(2)}+\chi\left[d^\dagger\times\tilde{d}\right]^{(2)}$$

$$\hat{T}_3 = \left[d^\dagger\times\tilde{d}\right]^{(3)}$$

$$\hat{T}_4 = \left[d^\dagger\times\tilde{d}\right]^{(4)}$$

where, χ is the quadrupole structure parameter and take the values 0 and $\pm\sqrt{7}/2$ (Casten and Warner, 1988; Arima and Iachello, 1987).

$\varepsilon = \varepsilon d-\varepsilon s$	: The boson energy
$(\hat{n}_d)$	: Operator gives the number of (d) bosons
$(\hat{P})$	: The pairing operator forthe s and d bosons
$(\hat{L})$	: The angular momentum operator

$(\hat{Q})$: The quadrupole operator

$(\hat{T}_3)$ and $(\hat{T}_4)$: The octupole and hexadecapole operators, respectively

The $(\hat{n}_d, \hat{L}, \hat{T}_3 \text{ and } \hat{T}_4)$ operators have $(\Delta n_d = 0)$ while $(\hat{P}^\dagger \cdot \hat{P})$ has $(\Delta n_d = 0, \pm 2)$ and $(\hat{Q} \cdot \hat{Q})$ has $(\Delta n_d = 0, \pm 1, \pm 2)$. The parameters a_0, a_1, a_2, a_3 and a_4 designated the strength of the pairing, angular momentum, quadrupole, octoupole and hexadecapole interaction between the bosons.

The IBM Hamiltonian has exact solutions in three dynamical symmetry limits (U(5), O(6) and SU(3)) which are geometrically similar to the an harmonic vibrator, axial rotor and γ-unstable rotor, respectively. More generally, the Hamiltonian can be expressed in terms of an invariant operator of that chain of symmetries and a shape phase transition between the dynamical symmetry limits results (Iachello, 2001; Cejnar *et al.*, 2010).

Electromagnetic transitions: The construction of operators for the various nuclear structure observables of interest is again straightforward, given the fact that they must be built from the basic elements (s, s†, d, d†). Only the lowest-order contributions to these operators have been included. Electromagnetic transition rates had been characterized by IBM as well, besides agitation energy spectra. One has to specify the transition operators in conditions of the boson operators in order to do so, (Sethi *et al.*, 1991). Another important attribute that can be deduced and calculated by using the IBM-1 called the reduced electric transition probability B(E2).

The general form of the electromagnetic transition rates operators can be written as following (Arima and Iachello, 1987):

$$\hat{T}_m^{(L)} = \alpha_2 \delta_{L2} \left[d^\dagger \tilde{s} + s^\dagger \tilde{d} \right]_m^{(2)} + \beta_L \left[d^\dagger \tilde{d} \right]_m^{(L)} + \gamma_0 \delta_{L_0} \delta_{m_0} \left[s^\dagger \tilde{s} \right] \quad (4)$$

where, L = 0, 1, 2, 3, 4, ….; m = 0, 1, 2, 3, 4, … and α_L, β_L, γ_0 represent free parameters, furthermore, equation gives transition operator for transition (E0, M1, E2, M3, E4, …). Therefore, the electric quadrupole transition operators can be written as (Arima and Iachello, 1987):

$$\hat{T}_m^{(E2)} = \alpha_2 \left[d^\dagger \widetilde{s + s^\dagger} \tilde{d} \right]_m^{(2)} + \beta_2 \left[d^\dagger \tilde{d} \right]_m^{(2)} \quad (5)$$

As for the magnetic dipole transition operator can be defined in terms of nuclear gyromagnetic factor (g_β) units nuclear magneton (μ_N) and angular momentum (Arima and Iachello, 1987):

$$\hat{T}^{(M1)} = \sqrt{\frac{3}{4\pi}} g_\beta \hat{L} \quad (6)$$

Therefore, the magnetic dipole transition operator for d-bosons is given in the following Eq. 7:

$$\hat{T}^{(M1)} = \beta_1 \left[d^\dagger \times \tilde{d} \right]_m^{(1)} \quad (7)$$

From Eq. 6 and 7 the nuclear gyromagnetic factor (g_β) can be defined as follow:

$$g_\beta = \frac{\beta_1}{\sqrt{10}} \sqrt{\frac{4\pi}{3}} \quad (8)$$

Generally, a nuclear geometric factor can be expressed by magnetic momentum (μ):

$$g_\beta = \frac{\mu_1}{I} \quad (9)$$

The general formula of the reduced transition probability for electric and magnetic transitions B(EL), B(ML) are known by the following expression (Kazem, 2010):

$$B(L, I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |< I_i \| \hat{T}^L \| I_f >|^2 \quad (10)$$

where, $|< I_i \| \hat{T}^L \| I_f >|$ is the matrix element of (E2) transition.

RESULTS AND DISCUSSION

The interacting boson approximation version one (IBM-1) has been utilized in the present work to study the several properties of the nuclear structure for even-even (Ba, Ce, Nd) (A = 144 for all) isotones by studying the low-lying positive parity states, dynamic symmetries, reduced electric transition probabilities B(E2), the electric Quadrupole moment $(Q_{2_i^+})$ values, ratios B(E2), potential energy surface. These nuclei expand from SU(3) to U(5) limits. The closed shell is adopted to be (50-82) in the calculation of number of proton boson (N_π) and neutron boson (N_ν) for every nucleus and the total number of bosons Nare shown in Table 1.

Energy levels: The checking of the experimental energy levels for the even nuclei Ba, Ce, Nd shows that they are belong to rotational limit SU(3), γ-unstable limit O(6) and its belong to vibrational limit U(5), then the Hamiltonian of the rotational, γ-unstable and vibrational been employed in the calculation by using the program PHINT. The best fit for the Hamiltonian parameters Eq. 2 utilized in the present work which gives best agreement between the calculated energy levels in the present work and their corresponding experimental data taken from Anonymous (2015) as shown in Table 1.

Table 1: The Hamiltonian parameters used in the IBM-code (PHINT) for even-even isotones

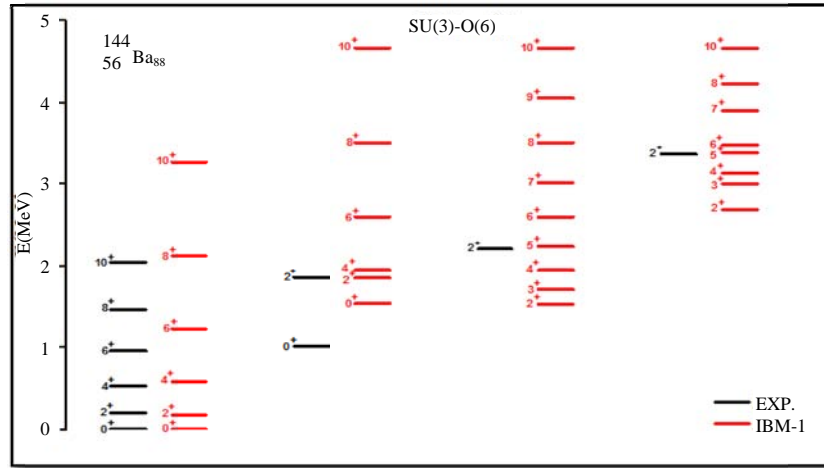| A	N	EPS (MeV)	$\hat{P} \cdot \hat{P}$ (MeV)	$\hat{L} \cdot \hat{L}$ (MeV)	$\hat{Q} \cdot \hat{Q}$ (MeV)	$\hat{T}_3 \cdot \hat{T}_3$ (MeV)	$\hat{T}_4 \cdot \hat{T}_4$ (MeV)	CHI	SO6
^{144}Ba	6	0.0000	0.1097	0.0148	-0.0422	0.0000	0.0000	-1.3228	1.0000
^{144}Ce	6	0.1970	0.0010	0.0250	0.0000	0.0246	0.0000	-1.3228	1.0000
^{144}Nd	6	0.6249	0.0000	0.0019	0.0000	0.0006	0.0071	0.0000	1.0000

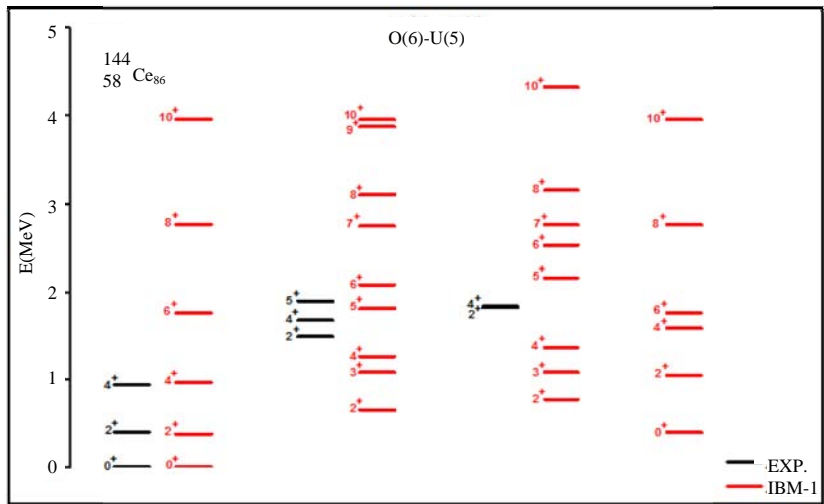Fig. 1: Comparison IBM-1 calculations with the experimental data for ^{144}Ba isoton

Fig. 2: Comparison IBM-1 calculations with the experimental data for ^{144}Ce isoton

A comparison between theoretical and experimental energy levels taken from Anonymous (2015) are shown in Fig. 1-3. In these figures, we observe that a very good agreement between our calculation for the g-band in comparison with the experimental data for all nuclei under study and a reasonable agreements for the other bands.

The B(E2) branching ratios: The B(E2) branching ratios (R, R' and R") were calculated in the present work for even-even (Ba, Ce, Nd) and it is known as the ratio between two reduced electric quadrupole transitions. The significance of studying the branching ratios is to study the shape of the nucleus and it's dynamical symmetries and to determine which dynamical symmetries:

$$R = \frac{B(E2; 4_1^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} \quad (11)$$

$$R' = \frac{B(E2; 2_2 \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} \quad (12)$$

$$R'' = \frac{B(E2; 0_2^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} \quad (13)$$

Table 2: The comparison experimental and calculated branching ratios and the typical values for the three limits

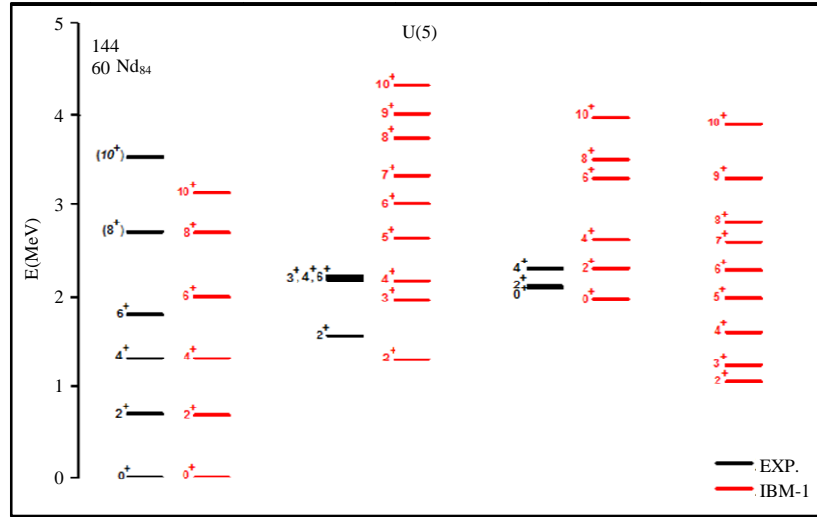| B(E2) ratios	^{144}Ba	^{144}Ce	^{144}Nd	SU(5) limit	SU(3) limit	O(6) limit
$R = \dfrac{B(E2:4_1^+ \to 2_1^+)}{B(E2:2_1^+ \to 0_1^+)}$	1.594	1.616	1.403	2	1.4	1.4
$R' = \dfrac{B(E2:2_1^+ \to 2_2^+)}{B(E2:2_1^+ \to 0_1^+)}$	1.498	0.058	0.002	2	0	1.4
$R'' = \dfrac{B(E2:0_2^+ \to 2_1^+)}{B(E2:2_1^+ \to 0_1^+)}$	1.424	0.002	0.001	2	0	0

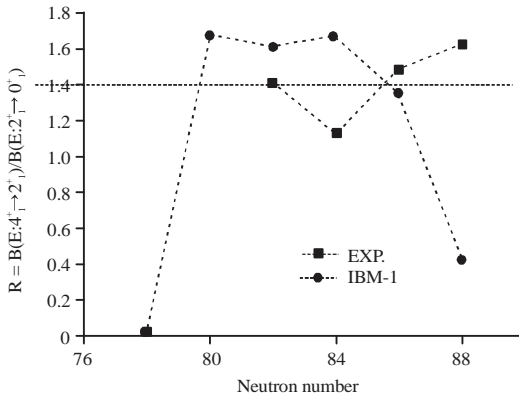Fig. 3: Comparison IBM-1 calculations with the experimental data for ^{144}Nd isoton

Fig. 4: Comparison between the experimental and calculated B(E2) branching ratios for even-even isotones with the typical values of SU(3) U(5) and O(6) limits

Branching type: For

$$R = R' = R'' = 2\left[(N-1)/N\right] = 2, ..., SU(5) \quad (14)$$

$$R = \frac{10}{7}\frac{(N-1)(2N+5)}{N(2N+3)} \approx 1.4, \ R = R' = 0, ..., SU(3) \quad (15)$$

$$R = R' = \frac{10}{7}\frac{(N-1)(N+5)}{N(N+4)} \approx 1.4, \ R'' = 0, ..., O(6) \quad (16)$$

The calculated branching ratios and their equivalent experimental values are presented in Table 2. The comparison experimental and calculated branching ratios and the typical values for the three limits are shown in Fig. 4.

CONCLUSION

The general behavior of even-even ^{144}Ba SU(3)- O(6). The Hamiltonian parameter has a large descent. This makes ^{144}Ba more nearby to rotational limit. The structure of beta and gamma bands is display up obviously and fully reproduced.

The studied structure bands of nuclei ^{144}Ce where in the transitional region O(6)-U(5). The calculated state up for L = 8 is higher than the experimental data. The level interval in beta and gamma is greater than that in the ground band point a great moment of inertia for the beta band.

The energy spectra and the spacing of these nuclei were found to fit good with experimental data. From these figures, we can see that very good reasonable agreement

between the values of energy ground state (g-band) of sequence $\left(0_1^+, 2_1^+, 4_1^+, \ldots\right)$ and their experimental state best than other bands.

The isotopes ^{144}Nd don't have experimental B(E2) values, so, calculate them by using equations and compared them with theoretical calculations, it found a good agreement.

The ratios of the reduced transition probabilities R, R' and R" have been found in agreement both experimentally and theoretically also with in consistence with their ideal corresponding limits.

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