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Modeling of Thermal Characteristics of Field Plumes and Checking the Conditions of Formation of Natural Gas Hydrates

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Abstract: The study presents a mathematical model of temperature distribution for a gas flow in a pipeline with heat exchange through the wall. Modeling is performed in a one-dimensional (hydraulic) approximation but the temperature distribution across the pipe near its wall (thermal boundary layer) is taken into account. This model includes two independent parameters that contain information about the heat-conducting properties of gas and pipe walls, taking into account thermal insulation. The resulting equation is proposed for use for field gathering plumes. The parameter that includes the heat transfer coefficient through the pipe wall, when setting the task of determining the moment of the beginning of hydrate formation in the field plumes is not known in advance and is subject to identification by comparing the calculation data with the measurement information. To solve such (inverse) problems, an analytical form of representation of the dependencies between the process parameters is important which determines the relevance of the presented result. The novelty lies in the fact that the obtained analytical dependence for the temperature distribution along the pipeline describes the heat transfer through the wall (through the thermal boundary layer), the thermal conductivity inside the gas and the convective heat transfer along the flow.

Key words: Hydraulic approximation, thermal boundary layer, diagnostics of gas hydrate formation in field plumes, parameters, dependencies

INTRODUCTION

To find the temperature distribution along the field gas gathering pipeline (plume), a mathematical model is used which takes into account the heat transfer across the stream and at the same time is one-dimensional in terms of gas movement. On the one hand, this model is quite simple and on the other hand, information about the physical properties of the processes occurring in the plume is reduced to two parameters, one of which (conditionally) is responsible for heat transfer from gas to wall and the other for thermal conductivity inside the gas and convective heat transfer. As a result, an analytical expression for the temperature distribution along the length of the plume is obtained which is suitable for determining the beginning and end of the hydrate formation process. Due to the complexity and multifactorial nature of this process (difficult to direct mathematical modeling (Buchinskii, 2009; Bondarev *et al*., 2008; Buts, 2010) and therefore, requires the solution of inverse problems), it is the analytic expression for temperature distribution that is relevant from a practical point of view and is the subject of novelty.

The construction of a mathematical model is based on the concept of the existence of a turbulent core inside a gas flow (Xu *et al*., 2009) in which intense convection heat transfer takes place and the temperature averaged over the cross section can be considered the temperature of the entire core. With a developed turbulent flow near the pipe wall, there is practically no stable velocity profile, only the boundary layer is present and it can be assumed that the gas flow velocity is characterized by one average value V. The temperature distribution over the cross section, in contrast to the velocity distribution at the same time, cannot be considered constant, if only because unlike gas, heat penetrates through the pipe wall.

MATERIALS AND METHODS

Mathematical model: The coordinates x, r are considered, respectively along and across the pipe while x counted from the entrance to the plume and r-from the axial line of the pipe. The temperature averaged over time at each point of the flow is considered to depend on these coordinates T(x, r) and is subject to a linear stationary equation of heat conductivity with a convective component (heat transfer) (Lee *et al*., 2013)

$$V\frac{\partial T}{\partial x} = \chi\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right) \quad (1)$$

Here: -V gas flow velocity, m/sec; T is the gas temperature, °C; parameter in Eq. 1 $\chi = \lambda/\rho c$, m^2/s; contains λ-the coefficient of thermal conductivity of gas, W/(deg.m); ρ- gas density, kg/m^3; c-gas heat capacity at constant volume, J/(kg.deg).

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After dimensioning x = RX (R-the radius of the pipe) and scaling the coordinates $R\text{-}r = RY\sqrt{\varepsilon}$ that is changing the variables $(x,r)\leftrightarrow(X,Y)$ as well as introducing a dimensionless parameter ε = χ/VR Eq. 1 takes the form:

$$\frac{\partial T}{\partial X}=\frac{\partial^2 T}{\partial Y^2}-\frac{\sqrt{\varepsilon}}{1\text{-}Y\sqrt{\varepsilon}}\frac{\partial T}{\partial Y}+\varepsilon\frac{\partial^2 T}{\partial X^2} \quad (2)$$

It is believed that ε = χ/VR<<1 but for field plumes, the order of this value is 10^{-5}, then the last two terms in the right part of Eq. 2 can be neglected in comparison with the first (Paranuk, 2012).

Indeed, when estimating orders of magnitude, the following conditionally is assumed: the average gas temperature over a cross section of 20°C; the wall temperature is 10°C and it is assumed that at a distance of 1 cm from the wall the temperature is equal to the average temperature across the section.

The change in temperature $\Delta_{T\,=\,10°C}$ corresponds to a change in radius $\Delta_{r\,=\,R\text{-}r\,=\,1\text{ cm}}$. Taking R = 10 cm, the following estimates of orders of magnitude are made: $Y=\frac{R\text{-}r}{R\sqrt{\varepsilon}}\approx\frac{0{,}1}{3{,}1\cdot10^{-3}}\approx30,$ respectively $\Delta Y\approx Y/10\approx3$ then $\partial T/\partial Y\approx\Delta T/\Delta Y\approx10/3\approx3{,}3$ the order of the second term on the right-hand side of Eq. 2 is estimated as $\frac{\sqrt{\varepsilon}}{1\text{-}Y\sqrt{\varepsilon}}\frac{\partial T}{\partial Y}\approx\frac{3\cdot10^{-3}}{1\text{-}30\cdot3{,}1\cdot10^{-3}}\cdot3{,}3\approx0{,}01.$

Thus, the error in dropping the summand $\frac{\sqrt{\varepsilon}}{1\text{-}Y\sqrt{\varepsilon}}\frac{\partial T}{\partial Y}$ in Eq. 2, considering the other terms of the order of unity, is about 1%.

Therefore, to simulate a change in the temperature of a gas T(X, Y) at X, Y>0 in a thermal boundary layer, a one-dimensional heat conduction equation is adopted: $\partial T/\partial X = \partial^2 T/\partial Y^2$ under boundary conditions:

$$T(X,0)=T_0;\quad T(0,Y)=T_* \quad (3)$$

Here, T_0-the temperature of the inner wall of the pipe and T_*-the temperature of the gas in the core of the flow (away from the wall), respectively, it is assumed that $T^*>T_0$. The solution of problem (Eq. 3), in particular, gives the following result (see the explanation given at the end of the main text of the article), here $y=Y\sqrt{\varepsilon}$:

$$\left.\frac{\partial T}{\partial y}\right|_{y=0}=\frac{T_*\text{-}T_0}{\sqrt{\pi\varepsilon X}} \quad (4)$$

After the transition in Eq. 4 to physically dimensional quantities, we get:

$$\left.\frac{\partial T}{\partial r}\right|_{r=R}=\text{-}(T_*\text{-}T_0)\sqrt{\frac{V}{x\pi\chi}} \quad (5)$$

In Eq. 4 and 5, the temperatures T_0 and T_* were constant (when solving the problem (Eq. 3)) and below they will be considered as changing along the pipeline -$T_0(x)$ and $T(x) = T_*$, in fact, this means that a change in these temperatures within the error of the mathematical model occurs in a segment along the pipeline with a length much greater than the characteristic thickness of the thermal boundary layer $R\sqrt{\varepsilon}$.

To write the equations of a mathematical model of the heat exchange of a gas flow with the environment is indicated: through q(t) specific (along the length of the pipeline) heat content of a single control "volume" of gas (presumably moving with speed V), internal diameter of the pipe D = 2R; through $T(x) = T_*$-gas temperature in the flow core, depending only on the coordinate along the pipeline. The temperature T_+ on the outer wall of the pipe is considered to be constant (the wall includes thermal insulation), for field plumes it is the temperature of the surrounding soil, then, on the one hand, the heat transfer equation from gas to the wall is written (Bunyakin *et al*., 2012):

$$\frac{dq}{dt}=V\frac{dq}{dx}=\pi D\left.\frac{\partial T}{\partial r}\right|_{r=R}=\lambda D(T_0\text{-}T)\sqrt{\frac{V\pi}{\chi x}} \quad (6)$$

On the other hand, the equation of heat transfer through the wall and thermal insulation to the external environment:

$$V\frac{dT}{dx}\frac{\pi D^2}{4}\rho c=\pi D\Lambda(T_+\text{-}T_0)=\frac{dq}{dt} \quad (7)$$

Here, Λ is the coefficient of heat transfer from gas to the wall, W/(degm^2). The value of this coefficient can be determined by setting the temperature in the inlet section of the pipe, finding the temperature in another section (downstream) and then selecting (variation) Λ can ensure the proximity of the last temperature to its measured value within the specified error. This is done for one of the modes, then Λ is considered constant.

In the further derivation of the final analytical expression, the temperature of the outer wall is considered to be independent of the gas temperature T_+ (constant) and the temperature of the inner wall $T_0(x)$-dependent. The temperature of the inner wall is expressed from Eq. 7:

$$T_0=T_+\text{-}\frac{DcV\rho}{4\Lambda}\frac{dT}{dx} \quad (8)$$

further by substituting this temperature into Eq. 6 and taking into account the form of the left part Eq. 7, we obtain:

$$\frac{\pi D^2}{4}cV\rho\frac{dT}{dx}=\lambda D\left(T_+\text{-}T\text{-}\frac{DcV\rho}{4\Lambda}\frac{dT}{dx}\right)\sqrt{\frac{\pi V}{\chi x}} \quad (9)$$

Hence, the derivative of the temperature in the core of the flow is expressed as:

$$\frac{dT}{dx}=\frac{4\lambda}{\rho cVD}\sqrt{\frac{\pi V}{\chi x}}\frac{T_+-T}{\pi+\frac{\lambda}{\Lambda}\sqrt{\frac{\pi V}{\chi x}}} \quad (10)$$

The differential Eq. 8, taking into account $\chi=\lambda/\rho c$ can be rewritten as follows:

$$\frac{dT}{dx}=\frac{4}{D}\sqrt{\frac{\pi\chi}{Vx}}\frac{T_+-T}{\pi+\frac{\lambda}{\Lambda}\sqrt{\frac{\pi V}{\chi x}}} \quad (11)$$

after that, the variables in it are separated:

$$\int_{T_1}^{T_2}\frac{dT}{T_+-T}=\frac{4}{D\pi}\sqrt{\frac{\pi\chi}{V}}\int_0^L\frac{dx}{\sqrt{x}+\frac{\lambda}{\Lambda}\sqrt{\frac{V}{\pi\chi}}} \quad (12)$$

by introducing coefficients $\alpha=\frac{\lambda}{\Lambda}\sqrt{\frac{V}{\pi\chi}};\ \beta=\frac{8}{D}\sqrt{\frac{\chi}{\pi V}}$ and integrating Eq. 9:

$$\int_{T_1}^{T_2}\frac{dT}{T_+-T}=\beta\int_0^L\frac{\sqrt{x}}{\sqrt{x}+\alpha}d\sqrt{x}$$

the result is the following analytical expression:

$$\ln\frac{T_+-T_1}{T_+-T_2}=\beta\left(\sqrt{L}-\alpha\ln\frac{\sqrt{L}+\alpha}{\alpha}\right) \quad (13)$$

The identical transformation of the latter leads to the equation:

$$T_2=T_++\left(T_1-T_+\right)\left(\frac{\sqrt{L}+\alpha}{\alpha}\right)^{\alpha\beta}\exp\left(-\beta\sqrt{L}\right) \quad (14)$$

which can describe not only the heat sink from the pipeline to the outside but also the reverse process (heat supply to the gas) in this case $T_1(0)$ and $T_2(L)$.

Dependence (Eq. 13) is an alternative to the well-known (effectively used for both field pipelines and wellbores) Shukhov's formula: $T_2=T_++\left(T_1-T_+\right)\exp(-\gamma L)$ also has the property of exponential tendency (at $L\to\infty$) the gas temperature to the temperature of the surrounding medium wall, however, (Eq. 13) looks more complicated it contains not one parameter "γ" but two independent parameters "α, β", since, it takes into account the presence of both a turbulent convective core and a stable thermal boundary layer.

Both parameters α, β, although, formally contain the flow rate but actually depend only on the mass flow of gas Q, since, $V\rho=4Q/\pi D^2$ и $\chi=\lambda/\rho c$:

$$\alpha=\frac{\lambda}{\Lambda}\sqrt{\frac{V}{\pi\chi}}=\frac{\lambda}{\Lambda}\sqrt{\frac{V\rho c}{\pi\lambda}}=\frac{\lambda}{\Lambda}\sqrt{\frac{4Qc}{\pi^2D^2\lambda}}=\frac{2}{\pi D\Lambda}\sqrt{cQ\lambda}$$

$$\beta=\frac{8}{D}\sqrt{\frac{\chi}{\pi V}}=\frac{8}{D}\sqrt{\frac{\lambda}{c\rho\pi V}}=\frac{8}{D}\sqrt{\frac{\pi D^2\lambda}{c\pi 4Q}}=4\sqrt{\frac{\lambda}{cQ}}$$

The physical meaning α can be characterized as a parameter inversely proportional to the heat transfer through the wall it contains a multiplier $1/D\Lambda$ that includes physical quantities that are not in the expression for β. The latter as a parameter of "competition of heat conduction with convective heat transfer" inside the gas flow, that is the greater the heat transfer along the flow (characterized by mass flow rate Q), the lower the specific mass transfer of heat from gas to the wall (through the thermal boundary layer), respectively slow asymptotic tendency of the gas temperature to the wall temperature this is reflected in direct proportion between β and $\sqrt{\lambda/Q}$.

RESULTS AND DISCUSSION

The obtained Eq. 13 was used to diagnose hydrate formation in the plumes of the field gas collection network, the graph of the corresponding temperature dependence in the current section of the plume on the length L (the path passed by the gas from the beginning of the plume) is shown in Fig. 1. The same graph shows the dependence of the hydrate formation temperature on the pressure according to the Hammerschmidt's formula (Shagapov and Musakaev, 2016): T_g°C = $20.68P(МПа)^{0.268}-17.78$.

Data on the field plume for the graphs in Fig. 1, operating under conditions of formation of gas hydrates were taken from (Paranuk, 2012): $\chi=10^{-5}$ m^2/sec; D = 0.2 m; L = 8.5 km; P_1 = 12 MPa; T_+ = -5°C; Q = 7.1.0.7 kgsec^{-1} (about 7,1 nm^3/sec).

The temperature at the beginning of the plume $T_1=20.68P_1^{0.268}-17.78$ was chosen artificially, so that, the hydrate formation condition was fulfilled already at the entrance to it. The parameter $\beta=\frac{8}{D}\sqrt{\chi/\pi V}$ was calculated by the gas flow velocity at the inlet to the plume $V=\frac{4Q\,(T_1+273{,}16)\,R_0}{\pi D^2P_1\cdot 10^6\mu}\approx 2\,{}^{M}\!/\!_{c}$ here, $R_0=8.31$ J/mol.K the universal gas constant and μ = 0.018 kg/mol the molar mass of the gas.

The parameter $\alpha\approx 279$ was selected, so that, the temperature at the exit of the plume was T_2 = 10°C, in fact it is the identification of the heat transfer coefficient through the wall Λ and finding it in another way in the

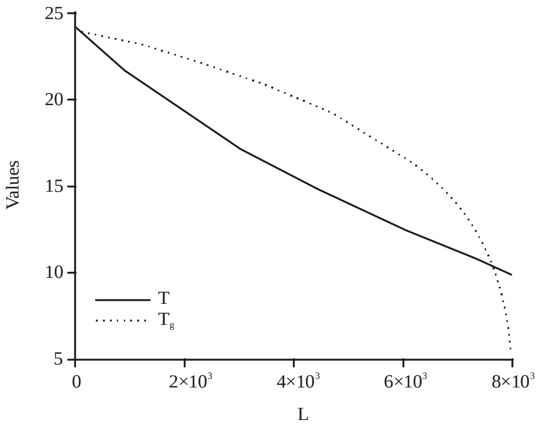

Fig. 1: Temperature dependence graphs: T°C gas temperature in the plume (according to Eq. 13)) and T_g°C hydrate formation temperature (according to the Hammerschmidt's formula), L distance from the beginning of the plume to the calculated cross section, m

field operating conditions is difficult. After that, the temperature distribution along the plume (as a dependence T(L) = T_2) was calculated by Eq. 13 and the corresponding graph was plotted in Fig. 1 as a solid line. Also in Fig. 1 (dotted line) is a graph of the temperature of hydrate formation according to the Hammerschmidt's formula, built for the plume purge mode (when the outlet pressure is specifically relieved, the gas velocity along the flow increases due to hydraulic losses, respectively while the temperature of hydrate formation decreases).

The pressure inside the field plume was calculated using a nonlinear difference scheme obtained according to the equation for hydraulic losses, taking into account changes in density and temperature:

$$p_k - p_{k+1} = \frac{16\lambda Q^2 \Delta L}{\pi^2 D^5} \frac{R_0/\mu}{p_k/T_k + p_{k+1}/T_{k+1}}$$

Coordinate increment along the pipeline in the direction of flow ΔL = L/n; k = 1, ..., n, the number of steps to the current section of the plume can change at n>50, almost without affecting the accuracy of the calculation.

An explicit expression for the pressure p_{k+1} in this difference scheme is obtained by solving a square equation for the temperature values T_k, T_{k+1} found in Eq. 10:

$$(p_k - p_{k+1})(p_{k+1}T_k + p_k T_{k+1}) = \frac{16\lambda Q^2 R_0 \Delta L}{\pi^2 D^5 \mu} T_k T_{k+1} = S_k$$

The root is selected from the natural condition of positive pressure:

$$p_{k+1} = \frac{p_k}{2T_k}\left(T_k - T_{k+1} + \sqrt{(T_k - T_{k+1})^2 + 4T_k\left(T_{k+1} - S_k/p_k^2\right)}\right)$$

For the example of calculation, a situation was selected that corresponded to a plume substantially filled with hydrates with a large hydraulic resistance λ = 1 the purge mode was modeled. From the graphs in Fig. 1 it can be seen that due to a significant pressure drop at the end of the plume, the temperature T_g drops and the hydrate formation condition is violated hydrates decompose.

Explanation of Eq. 4: The solution of the equation $\partial T/\partial X = \partial^2 T/\partial Y^2$, singular under conditions $T(X, 0) = T_0$; $T(0, Y) = T_*$ is sought by the Fourier sine transform method in the form that satisfies the equation and the first condition:

$$T(X,Y) = T_0 + \int_0^\infty \varphi(\xi)\exp(-\xi^2 X)\sin\xi Y\, d\xi$$

The function φ is from the second condition $T(0,Y) = T_0 + \int_0^\infty \varphi(\xi)\sin\xi Y\, d\xi = T_*$ the order of integration is rearranged:

$$(T_* - T_0)\int_0^\infty \sin\eta Y\, dY = \int_0^\infty \varphi(\xi)\left(\int_0^\infty \sin\xi Y \sin\eta Y\, dY\right)d\xi;$$

$$\varphi(\xi) = \frac{2}{\pi}(T_* - T_0)\int_0^\infty \sin\xi Y\, dY$$

A representation of the one-dimensional Dirac delta function is used, $\frac{2}{\pi}\int_0^\infty \sin\xi Y \sin\eta Y\, dY = \delta(\xi - \eta)$ which in turn is justified by the following calculation (it is believed that 0<σ<2η the order of integration is rearranged):

$$\int_0^\infty\left(\int_{\eta-\sigma}^{\eta+\sigma}\frac{1}{2}\left(\cos(\xi-\eta)Y - \cos(\xi+\eta)Y\right)d\xi\right)dY =$$

$$\int_0^\infty \frac{1}{2}\left(\frac{\sin(\xi-\eta)Y}{Y} - \frac{\sin(\xi+\eta)Y}{Y}\right)\Bigg|_{\xi=\eta-\sigma}^{\xi=\eta+\sigma} dY = = \int_0^\infty \frac{\sin\sigma Y}{Y}dY -$$

$$\frac{1}{2}\left(\int_0^\infty \frac{\sin(2\eta+\sigma)Y}{Y}dY - \int_0^\infty \frac{\sin(2\eta-\sigma)Y}{Y}dY\right) = \frac{\pi}{2}$$

Since, $\frac{2}{\pi}\int_0^\infty \frac{\sin\sigma x}{x}dx = \operatorname{sign}\sigma$ it is the Dirichlet integral. The solution of the heat equation under these conditions is expressed through the function:

$$\varphi(\xi)=\frac{2}{\pi}(T_*\text{-}T_0)\int_0^\infty \sin\xi\eta\, d\eta$$

$$T(X,Y)=T_0+\int_0^\infty\left(\frac{2(T_*\text{-}T_0)}{\pi}\int_0^\infty \sin\xi\eta\, d\eta\right)\exp(-\xi^2X)\sin Y\xi\, d\xi=$$

$$=T_0+\frac{2(T_*\text{-}T_0)}{\pi}\int_0^\infty\left(\int_0^\infty \exp(-\xi^2X)\sin\xi\eta\sin Y\xi\, d\xi\right)d\eta=$$

$$=T_0+\frac{T_*\text{-}T_0}{\pi}\int_0^\infty\left(\int_0^\infty \exp(-\xi^2X)\left(\cos(\eta\text{-}Y)\xi-\cos(\eta+Y)\xi\right)d\xi\right)d\eta$$

The table integral is used,

$$\int_0^\infty \exp(-a^2x^2)\cos bx\, dx=\frac{\sqrt{\pi}}{2a}\exp\left(-\frac{b^2}{4a^2}\right);\quad a>0$$

The desired solution (superposition of two fundamental solutions of the heat equation with a shift of one relative to the other) has the following form:

$$T(X,Y)=T_0+\frac{T_*\text{-}T_0}{2\sqrt{\pi X}}\int_0^\infty\left(\exp\left(-\frac{(\eta\text{-}Y)^2}{4X}\right)-\exp\left(-\frac{(\eta+Y)^2}{4X}\right)\right)d\eta$$

$$\frac{\partial T}{\partial Y}=-\frac{T_*\text{-}T_0}{2\sqrt{\pi X}}\left(\exp\left(-\frac{(\eta\text{-}Y)^2}{4X}\right)+\exp\left(-\frac{(\eta+Y)^2}{4X}\right)\right)\Bigg|_{\eta=0}^{\eta=\infty}=$$

$$\frac{T_*\text{-}T_0}{\sqrt{\pi X}}\exp\left(-\frac{Y^2}{4X}\right)$$

After the transition from a scaled variable Y to a dimensionless one $y=Y\sqrt{\varepsilon}$ we obtain an equation $\left.\frac{\partial T}{\partial y}\right|_{y=0}=\frac{T_*\text{-}T_0}{\sqrt{\pi\varepsilon X}}$ that required explanation, i.e., Eq. 4.

CONCLUSION

The obtained dependence (Eq. 13) for the temperature along the field plume, on the one hand has an analytical representation, on the other takes into account the temperature distribution across the flow in the asymptotic approximation of the thermal boundary layer. The dependence (Eq. 13) can be used not only for field gas pipelines, but also to calculate the characteristics of tubular heat exchangers it provides a fairly universal method for finding the temperature change between the inlet and outlet of one of the coils, provided that on the other this difference is set and both of them are surrounded on all sides by a receiving or giving off heat environment (with temperature T_+) this is an analogue of the soil temperature in accordance with the above.

The versatility consists in the fact that in order to identify the parameters included in Eq. 13, it is enough to select α and β only for one temperature mode of operation of the heat exchanger (for each of coils). The temperature T_+ is found from Eq. 13 for one of the coils and the conditionally unknown temperature difference between the input and output of the other coil is from a similar formula for the latter.

The reverse nature of the change in the parameters α and β when the flow rate changes leads to the fact that the dimensionless parameter $\alpha\beta=\frac{8\lambda}{\pi D\Lambda}=\frac{8}{\pi Nu}$ inversely proportional to the Nusselt number (Cuckovic-Dzodzo *et al*., 1999) does not depend on the flow rate. According to information from the literature, the dependence with such properties is new and the presence of this parameter in the exponent of the analytical expression (Eq. 13) is a distinctive feature.

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