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The Different Mathematical Modeling for Conducting Yields Growth

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Key words: Richard's equation, Blaney-Criddle equation, Mathematical model, crop yield production

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Abstract: In this study, a brief study on yield production is presented in various mathematical methods just, so, five modeling and the most significant factors affecting on crop yield growth are studied. The multiple linear regression is applied from reference studies to predict the crop yield. Richard's equation by fully implicit finite difference scheme is presented to prophesy volumetric water content in crop yield growth and the growth relationship between crop yield Y and the nitrogen N is explained. Also, the mathematical model for crop yield production is described in four main stages. The model of a crop production is converted into a system of equations that are solved numerically. Moreover, the empirical Blaney-Criddle equation for water yield is formulated at different crops such as rice, cotton and wheat.

INTRODUCTION

The crop is a plant of the same type that is grown and cultivated on a large scale in an area. Sunlight and water and nutrients from the soil are primary to grow crops. The irrigation is the process of providing water to the crops artificially in the fields and methods of irrigation are sprinkler and drip system (Onwueme and Sinhad, 1991). Crop production is the science of the genetic enhancement of crops to output new varieties with increased productivity and quality (Lenne *et al*., 2003). Crop yield production is of great significance to universal food production and the farmers profit from yield prediction to make knowing about economy management decisions (Horie *et al*., 1992).

It helps the scientist define research priority. Crop yield production is a very complex type determined by various factors such as climate (environment), soil type, genotype, irrigation water and their interactions. It is required fundamental understanding of the functional relationship between yield and these interactive factors.

Numerous studies are concentrated on characterize the phenotype as an explicit function of the Genotype (G), Environment (E) and their interactions (G×E) (DeLacy *et al*., 1996; Crossa and Cornelius, 1997; Chapman *et al*., 2000). Also, linear mixed models applied to investigate both additive and interactive effects of genotypes and environments (Crossa *et al*., 2004; Montesinos-Lopez *et al*., 2018). Crop yield production model is estimated the importance and the effect of certain parameters. Also, it has detected which factors should be more studied, thus, increasing the understanding of the real system. Agriculture meteorology, solar radiation interception and absorption, evapo-transpiration, energy and soil water balance, soil water flow, photosynthesis, respiration and crop growth development are very useful factors to crop field management modeling.

Mathematical models are approached in many disciplines such as agriculture, biology, chemistry and economy. A Mathematical model is a system of mathematical equations and constants that are usually solved to do quantitative predictions about some aspect(s) of a real system and recognized the interactions of its main components (Brun *et al*., 2006). The specific variables required as input data and generated as output predictions are important features of the model (Hodges *et al*., 2018). The equations often stem from a numerical solution to one or more differential equations and their boundary conditions.

A mathematical model is transformed into a system of equations. The solution of the equations is solved analytically or numerically means, describes how the system behaves over time t or at poise condition. The model often makes assumptions about the system. Also, the equations may make assumptions about the nature of what may happen as shown in Fig. 1. The mathematical model is essential to most computational scientific research. Computer modeling and computer simulation are used in mathematical modeling (Parry, 1985; Gause, 2019). Models may be mathematical equations, spreadsheets or computer simulations and represented as relations or functions. A model does not tell us the whole picture of the system, only the essential parts that are important to explain the system's behavior, function and purpose. The types and classifications of mathematical models are shown in Fig. 2. The models are mechanistic or empirical (statistical) models, static or dynamic models, discrete or continuous models, deterministic or stochastic models, explicit or implicit, qualitative and quantitative (Acock and Acock, 1991).

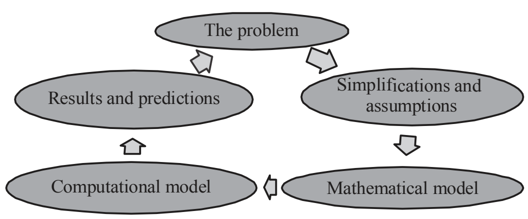

Fig. 1: The flow chart of mathematical modeling process

Linear programming, empirical (statistical) models and dynamic (mechanistic) models are three mathematical models used in agriculture. The empirical models describe relationships between variables without point to the correlated processes and use these models to fit data points, so, the variables (parameters) in empirical models do not include any biological meaning. Here, empirical models describe the crop yield behavior based on observations at the crop level. But the dynamic models represent cause-effect relationships between the variables (parameters) with biological meaning. Here, dynamical models describe the the crop yield achievement, based on the knowledge of the processes that are taking place in its growth and development (Tedeschi *et al*., 2005).

In this study, the crop yield mathematical modeling is presented to evaluate and predict crop yield and yield difference from genotype and environment data. Also, the best optimum and prediction management with recognize the basic interactions in the soil-plant-atmosphere system are presented. Crop yield prediction is introduced a function of environmental conditions. Multiple linear regressions are used to predict the crop yield and the SWAP model by Richard's equation is simulated. Also, the simple mathematical model of a crop production is converted into a system of equations. The solution of the equations are solved numerically and described how to reduce water quality degradation and improve water use efficiency over time t. Moreover, the Blaney-Criddle equation is formulated at various crops such as rice, cotton and wheat.

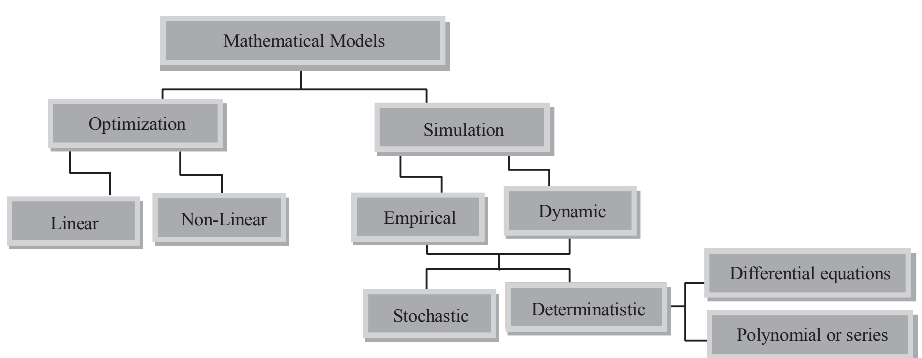

Fig. 2: Classifications of mathematical modelling

MATERIALS AND METHODS

The first method: multiple linear regression method: The model of multiple linear regression is built to establish the relationship between crop yield prediction as a dependent variable and temperature, pH, potassium, phosphorus, nitrogen, rainfall and water required as an independent variables. The multiple linear regression fits the dataset to the model is:

$$y_i = a_0 + a_1x_{1i} + a_2x_{2i} + a_3x_{3i} + a_4x_{4i} + a_5x_{5i} + a_6x_{6i} + a_7x_{7i} + \varepsilon_i$$

Where:

a_0 = Constant
a_1-a_7 = Model parameters (coefficients)
y_i (kg ha^{-1}) = The crop yield
x_{ji}, j = 1, ..., 7 = The predictors
ε_i = A random error

In matrix form:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{21} & \dots & x_{71} \\ 1 & x_{12} & x_{22} & \dots & x_{72} \\ 1 & x_{13} & x_{23} & \dots & x_{73} \\ . & . & . & \dots & . \\ . & . & ; & \dots & . \\ 1 & x_{1n} & x_{2n} & \dots & x_{7n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ . \\ a_n \end{bmatrix} + \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \\ \varepsilon_2 \\ . \\ . \\ . \\ \varepsilon_n \end{bmatrix}$$

The multiple linear regressions are applied to predict the crop yield. Characterize the important values is in Table 1 (Majumdar *et al*., 2017).

$$\text{Yield of rice} = (-0.18503)+(0.041593)\ \text{Temperature} + (0.172042)\ \text{pH}+(-8.27e-04)\ \text{Nitrogen}+(-4.28e-03)\ \text{Phosphorus}+(-0.00264)\ \text{Potassium}+(9.15e-04)\ \text{Water}$$

$$\text{Yield of cotton} = (7.149372)+(-0.14468)\ \text{pH}+(-0.00131)\ \text{Nitrogen}+(-0.00405)\ \text{Potassium}+(-0.00405)\ \text{Water}$$

$$\text{Yield of wheat} = (112)+(-4.14e-02)\ \text{Temperature} + (1.34e-04)\ \text{Rainfall}+(0.079153)\ \text{pH}+(-1.31e-03)\ \text{Nitrogen}+(-0.00167)\ \text{Potassium}+(-0.28125)\ \text{Water}$$

For rice crop, if all the independent variables are zero, the yield reduces by 0.18503.1 unit increase in temperature increase the rice yield by 0.041593 units, 1 unit increase in water will increase yield by 9.15e-04 units, 1 unit increase in pH will increase the yield by 0.172042 units, 1 unit increase in Nitrogen decreases the yield by 8.27e-04 units and 1 unit increase in potassium decreases the yield by 0.00264 units (Fig. 3 and 4).

Input for crop yield simulation models

- Weather: Soil radiation, maximum and minimum temperature (°C), rainfall (mm)

Table 1: The significant values of three crops: rice, cotton and wheat

Parameters	Rice	Cotton	Wheat
Temperature	0.003139	0.547536	0.001137
Soil pH	5.08e-07	0.011752	0.01834
Potassium K (kg/ha)	1.43e-05	2.82e-07	0.021422
Phosphorus P (kg/ha)	0.025816	0.071843	0.209524
Nitrogen N (kg/ha)	0.000841	5.85e-05	8.6e-06
Rainfall (mm)	0.105878	0.784625	0.018042
Water EC (ds/m)	1.22e-26	4.95e-05	NA

Fig. 3: Crops significant

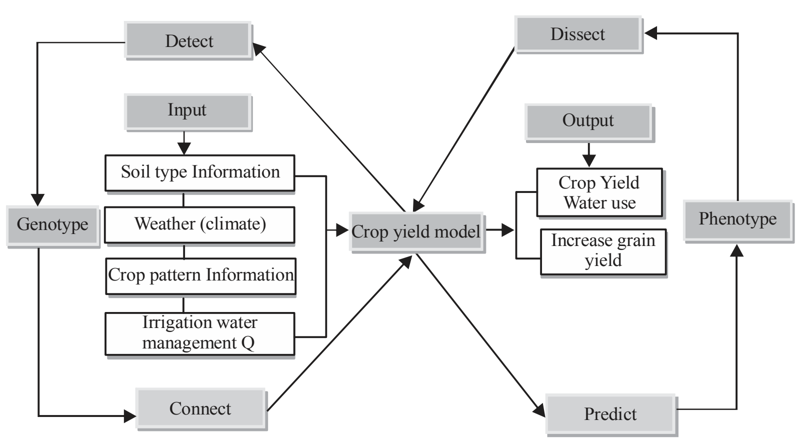

Fig. 4: Crop yield simulation models to predict crop yield growth

- Soil: Soil type, soil depth, soil texture, soil organic carbon, bulk density, soil nitrogen (kg/ha), pH value
- Initial condition of the system
- Crop and field managemen. Crop name and type, planting date and type, row space, plants per square meter, irrigation and nitrogen amount, dates of irrigation, fertilizer kind

The general equations for studying multiple regressions are these models:

$$y = a_0 + \sum_{i=1}^{n} a_i x_i + \varepsilon \quad (1)$$

$$\log y = a_0 + \sum_{i=1}^{n} a_i \log x_i + \varepsilon \quad (2)$$

$$\sqrt{y} = a_0 + \sum_{i=1}^{n} a_i \sqrt{x_i} + \varepsilon \quad (3)$$

$$\frac{1}{y} = a_0 + \sum_{i=1}^{n} a_i \frac{1}{x_i} + \varepsilon \quad (4)$$

The second method: Soil, water, atmosphere and plants model by FDM: The SWAP model is simulated the relationships between soil, water, atmosphere (weather) and plants. Darcy's law is given by:

$$q = -k(h)\frac{\partial(z+h)}{\partial z} \quad (5)$$

Where:
q (mm ha^{-1}) = The soil water flux density
k (mm ha^{-1}) = The hydraulic conductivity
h (mm) = The soil water pressure head
z (mm) = The vertical coordinate (depth of soil)

$$\frac{\partial v}{\partial t} = \frac{\partial q}{\partial z} - s(h) \quad (6)$$

Where:
v = The water content
t = A time
s = (Source term) the soil water from plant roots

From Eq. 5 and 6 Richard's equation is written as:

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial z}\left(k(h)\left(\frac{\partial h}{\partial z}+1\right)\right) - s \quad (7)$$

Equation 7 is solved numerically with to initial and boundary conditions and with known relationships between, h and k. These relationships are measured directly in the soil data (Fig. 5). The Richards equation is solved by using an implicit finite difference scheme as (Ramos, 1983).

Let $c = \frac{\partial v}{\partial h}$ is water capacity, then:

$$\frac{\partial h}{\partial z} = \frac{\partial h}{\partial v} \times \frac{\partial v}{\partial z} = \frac{1}{c}\frac{\partial v}{\partial z} \quad (8)$$

Then, we get:

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial z}\left(k(v)\left(\frac{1}{c}\frac{\partial v}{\partial z}+1\right)\right) - s \quad (9)$$

Let water soil diffusivity $(v) = \frac{k(v)}{c}$

Hence:

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial z}\left(D(v)\frac{\partial v}{\partial z}\right) + \frac{\partial k(v)}{\partial z} - s \quad (10)$$

The relationship of water content v and water pressure head h is:

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial h} \times \frac{\partial h}{\partial t} = c\frac{\partial h}{\partial t}, \quad \frac{\partial v}{\partial z} = \frac{\partial v}{\partial z} \times \frac{\partial h}{\partial z} = c\frac{\partial h}{\partial z} \quad (11)$$

Then Richard's Eq. 9 is written from Eq. 8 and 11 in water pressure head form as:

$$c\frac{\partial h}{\partial t} = \frac{\partial}{\partial z}\left(k(h)\left(\frac{\partial h}{\partial z}+1\right)\right) - s \quad (12)$$

Then Richard's Eq. 9 for general is written as:

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial z}\left(k(h)\left(\frac{\partial h}{\partial z}+\cos\theta\right)\right) - s \quad (13)$$

Richard's Eq. 13 for vertical flow $\theta = 0$ is written as:

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial z}\left(k(h)\left(\frac{\partial h}{\partial z}+1\right)\right) - s \quad (14)$$

Richard's Eq. 13 for horizontal flow $\theta = \pi/2$ is written as:

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial x}\left(k(h)\left(\frac{\partial h}{\partial x}\right)\right) - s \quad (15)$$

Then solve Richard's equation by implicit finite difference scheme as:

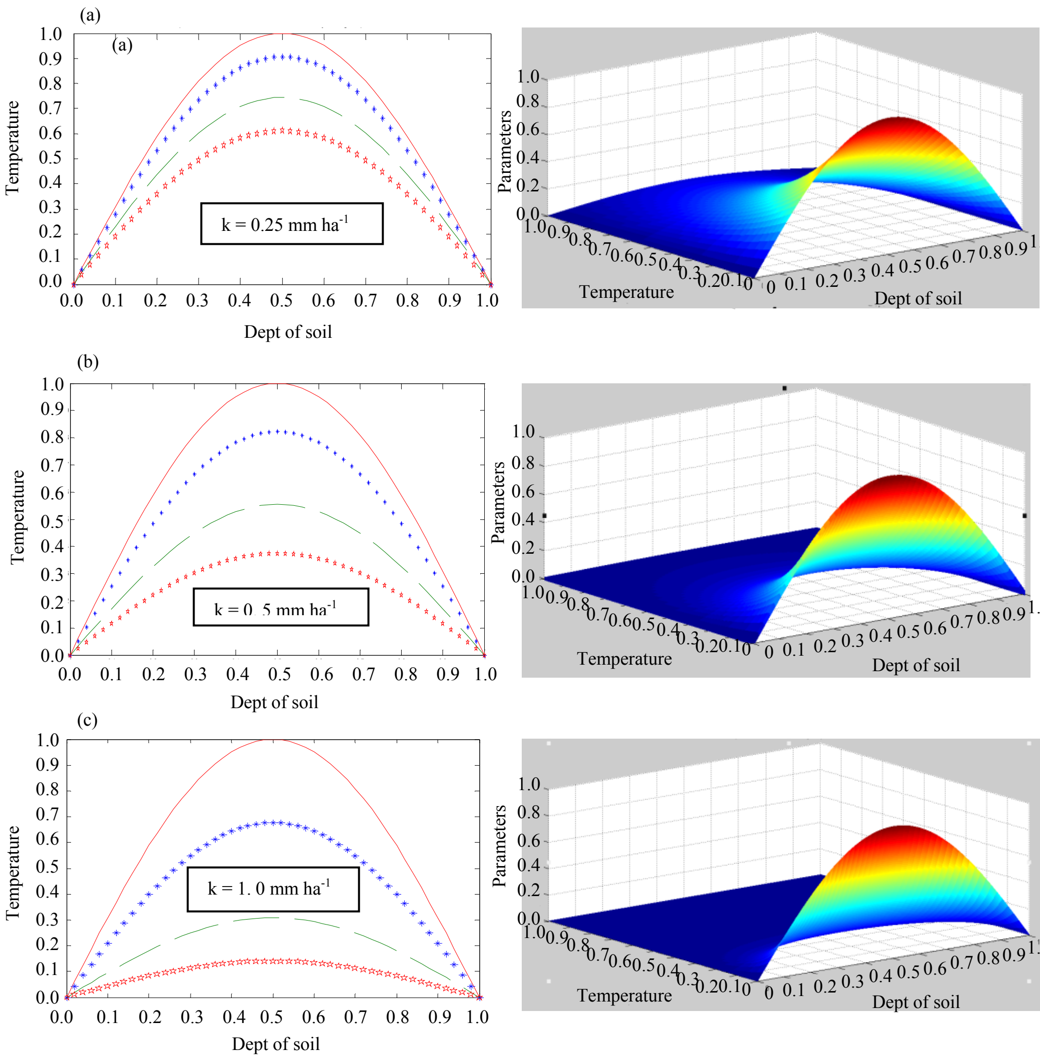

Fig. 5 (a-c): Changing the hydraulic conductivity at different times within fully implicit FDM to predict volumetric water content v (x, t)

$$\frac{u_{n,j+1}-u_{n,j}}{\Delta t} = k\frac{u_{n+1,j+1}-2u_{n,j+1}+u_{n-1,j+1}}{\Delta x^2} \quad (16)$$

$$u_{n,j} = -\beta u_{n-1,j+1}+(1-2\beta)u_{n,j+1}-\beta u_{n+1,j+1} \quad (17)$$

where, $n = 1, 2, 3, \ldots, N-1, \beta = k\Delta t/\Delta x^2$.

The third method: Mathematical relationship between crop yield and nitrogen: In a controlled field experiment by Pandey *et al*. (2000), the crop yield Y (kg ha^{-1}) responds to the nitrogen N fertilizer rate (kg ha^{-1}) by the following mathematical relationship:

$$Y = 1143+31.7\,N-0.084N^2 \quad (18)$$

Which exposes that with increasing nitrogen rates, crop yield increasing (Fig. 6).

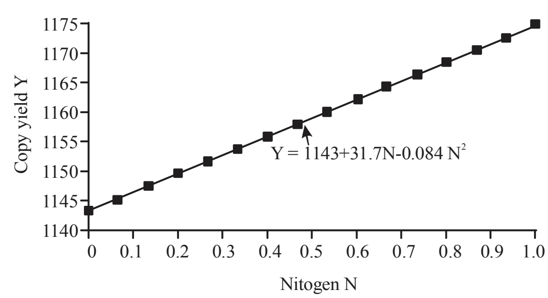

Fig. 6: The growth relationship between crop yield Y and the nitrogen N

The fourth method: Mathematical modeling for crop production: It is a simplified representation of a real system. It describes the system using mathematical principle in the form of a set of equations. The basic mathematical model for crop production is described in four main stages as shown in Fig. 7. The model of a crop production is converted into a system of equations. The solution of the equations are solved numerically and described how to reduce water quality degradation and improve water use efficiency over time t. Then, the reaction scheme is described in the following four reactions as follows (Table 2):

The dynamics of the system can be written as the following set of first order ODEs (Fig. 8):

$$d[x1]/dt = -k_1[x1],$$
$$d[x2]/dt = -k_2[x2]$$

Table 2: This table shows the modeling steps symbols

d/dt	Decay	Forward	Forward	Forward	Forward
[x1]	-k1[x1]	0	0	0	0
[x2]	-k2[x2]	0	0	0	0
[x3]	-k3[x3]	0	0	0	0
[x4]	0	k1[x1]	k2[x2]	k3[x3]	−K4[x4]

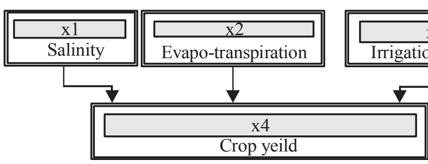

Fig. 7: Simple diagram represents the crop production

$$d[x3]/dt = -k_3[x3],$$
$$d[x4]/dt = k_1[x1] + k_2[x1] + k_3[x1] - k_4[x4]$$

The fifth method: Empirical Blaney-Criddle equation for crop water yield: The useful factors which impact the crop water requires are (Table 3-5):

- Temperature is hot or cool
- Sunshine is sunny or cloudy
- Moisture is wet or dry
- Wind speed is high or low

Blaney-Criddle (BC) equation for crop Evapo-transpiration calculation is written as (Al-Barrak, 1964):

$$ETP = 4.57\times K\times P\times (T+17.8)/100 \text{ (cm/month)}$$

Where:

ETP = Monthly evapo-transpiration
K = Empirical crop factor for the month for each crop that is depending on the kind of crop, the growth phase of the crop and the weather (Fig. 9 and 10)
P = Monthly daylight hours expressed as percent of daylight hours of the year
T = Mean monthly temperature in °C

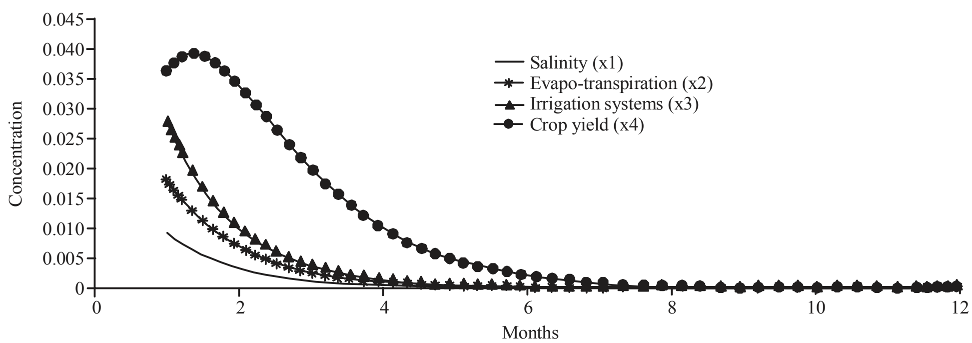

Fig. 8: Prediction of crop yield per month

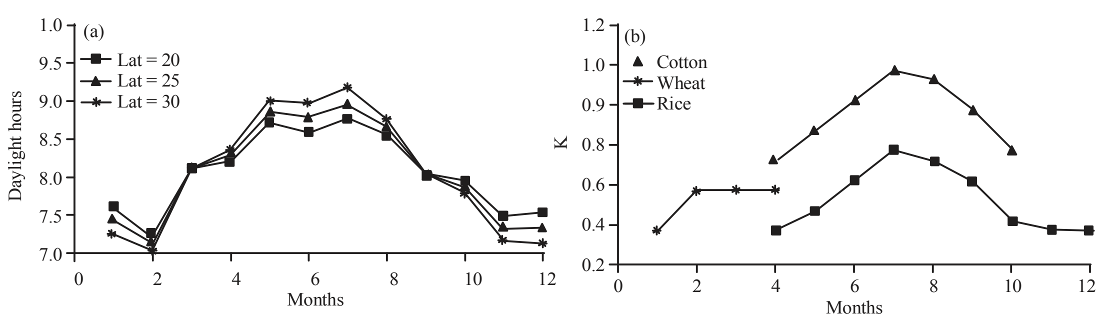

Fig. 9(a, b): Blaney-Criddle crop factor value (K) for the month and monthly daylight hours value (P)

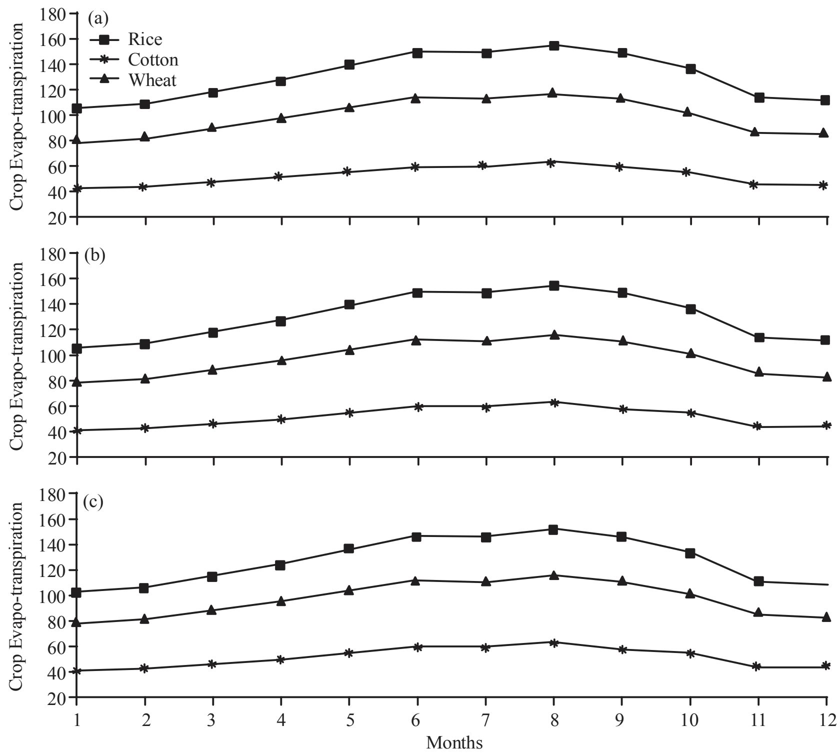

Fig. 10(a-c): Blaney-Criddle equation for crop Evapo-transpiration is predicted at different temperature (a) ETP for various crop at Lat 20 and different temperature, (b) ETP for various crop at Lat 25 and different temperature and (c) ETP for various crop at Lat 30 and different temperature

Table 3: Blaney-criddle crop factor value (K) for the month

	Months											
Crops	1	2	3	4	5	6	7	8	9	10	11	12
Rice				0.85	1.0	1.15	1.3	1.25	1.1	0.9		
Wheat	0.5	0.7	0.7	0.7								
Cotton				0.5	0.6	0.75	0.9	0.85	0.75	0.55	0.5	0.5

Table 4: Monthly daylight hours value (P)

Latitude	1	2	3	4	5	6	7	8	9	10	11	12
20	7.74	7.25	8.41	8.52	9.15	9.00	9.25	8.96	8.30	8.18	7.58	7.66
25	7.53	7.14	8.39	8.61	9.33	9.23	9.45	9.09	8.32	8.09	7.40	7.42
30	7.30	7.03	8.38	8.72	9.53	9.49	9.76	9.22	8.33	7.99	7.19	7.15

Table 5: Monthly temperature (T)

Month	1	2	3	4	5	6	7	8	9	10	11	12
T(°C)	16	17	20	23	27	30	30	32	30	26	19	18

RESULTS AND DISCUSSION

The crops are chosen by its economic significant. Crop yield prediction is demanded in agricultural planning process. In this study, the three significant crops are chosen by using the data availability as measure and the rice crop yield analysis is debated. Over here, the five different mathematical modeling is presented. In first method, the multiple linear regression model is established the relationship between crop yield prediction as a dependent variable and temperature, soil pH, potassium, phosphorus, nitrogen, rainfall and water required as an independent variables. In second method, The SWAP model is simulated the relationships between soil, water, atmosphere (weather) and plants by changing the hydraulic conductivity k at different times by using Richard's equation with fully implicit finite difference scheme to predict volumetric water content v(x, t). In third method, the mathematical relationship between crop yield and nitrogen is exposed that with increasing nitrogen rates, crop yield will be increasing. In fourth method, the mathematical model for crop production is described in four main stages with four reactions scheme and it is converted into a system of equations that are solved numerically and described how to reduce water quality degradation and improve water use efficiency over time t. In fifth and last method, the empirical Blaney-Criddle equation is used to predict crop water yield. The optimum coefficients to realize higher output, The best temperature from 23°-30°, The worst temperature higher than 30°.

CONCLUSION

The model of a crop production is converted into a system of equations that are solved numerically. Moreover, the empirical Blaney-Criddle equation for water yield is formulated at different crops such as rice, cotton and wheat.

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