Journal of Engineering and Applied Sciences
MEDWELL PUBLICATIONS SINCE 2000

On Fuzzy Discrete Exponential Distribution

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Key words: Exponential distribution, fuzzy probability, fuzzy statistic, fuzzy mean, fuzzy variance, fuzzy moment generating function

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Page No.: 2045-2049
Volume: 15, Issue 9, 2020
ISSN: 1816-949x
Journal of Engineering and Applied Sciences
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Abstract: In this study, the fuzzy discreet exponential distribution is studied. The general forms of fuzzy mean, fuzzy variance and fuzzy moment generating function of it are found.

INTRODUCTION

The uncertainty and vagueness are considered important problems in the branches of science. These problems are called of fuzzy problems. By Zadeh[1] concept of a fuzzy set (vague set). By Zadeh[2] gave the notion of fuzzy probability. By Bracquemond and Gaudoin[3] introduced the discrete exponential distribution. In this study, we study the fuzzy discrete exponential distribution.

MATERIALS AND METHODS

Preliminaries: In this section, some concepts about the fuzzy set, fuzzy number, triangular fuzzy number, fuzzy probability, fuzzy mean, fuzzy variance, fuzzy moment generating function and discrete exponential distribution are given.

Definition 1: A fuzzy set $\bar{B}$ of a nonempty set X is membership function $\beta_{\bar{B}} : X \to [0, 1]$ where [0, 1] means real numbers between 0 and 1 (including 0, 1)[2].

Definition 2: Let $\bar{B}$ be a fuzzy set of X, then the set $\{x \in X : \beta_{\bar{B}}(x) > 0\}$ is called the support of $\bar{B}$ and denoted by $S(\bar{B})$ [4].

Definition 3: Let $\alpha \in [0, 1]$. The α-cut of a fuzzy set $\bar{B}$ of X is the set $\bar{B}[\alpha] = \{x \in X \mid \bar{B}(x) \geq \alpha\}$[1].

Definition 4: Let $\bar{B}$ be a fuzzy set on the set of real numbers satisfying there[4]:

- All the α-cuts of $\bar{B}$ are not empty for $\alpha \in [0, 1]$
- All the α-cuts of $\bar{B}$ are closed intervals of R
- $S(\bar{B})$ is bounded

Then, $\bar{B}$ is called an fuzzy number.

Definition 5: A triangular fuzzy number $\bar{B}$ can be defined as a triple (c_1, c_2, c_3). Its membership function is defined as[5]:

$$\beta_{\bar{B}}(x)=\begin{cases}0, & x<c_1\\ \dfrac{x-c_1}{c_2-c_1}, & c_1\le x\le c_2\\ \dfrac{c_3-x}{c_3-c_2}, & c_2\le x\le c_3\\ 0, & x>c_3\end{cases}$$

Definition 6: The α-cuts of a triangular fuzzy number $\bar{B}$ is the interva $\bar{B}(\alpha)=[(c_2\text{-}c_1)\alpha+c_1, \text{-}(c_3\text{-}c_2)\ \alpha+c_3]$, for all $\alpha\in[0, 1]$[6].

Definition 7: Let $\bar{B} = (c_1, c_2, c_3)$. Then $\bar{B}$ is said a positive triangular fuzzy number if $c_i>0$ for all i = 1, 2, 3[4].

Remark: All the triangular fuzzy numbers in this study considered a positive triangular fuzzy numbers.

Definition 8: Suppose $B = \{y_1, ..., y_k\}$ be a subset of a set $Y = \{y_1, ..., y_m\}$ have a discrete (finitely) fuzzy probability distribution $\bar{p}(\{y_i\})=\bar{\alpha}_1, 0<\bar{\alpha}_i<1, 1\le i\le m,$ the α-cut of the fuzzy probability is given by[7]:

$$\bar{P}(B)[\alpha]=\left\{\sum\nolimits_{i=1}^{k}a_i\,|S\right\} \tag{1}$$

for $\alpha\in[0,1]$ and α_i is fuzzy number where, S is the statement $"\alpha_i\in\bar{\alpha}_i[\alpha],\ \ 1\le i\le m, \sum\nolimits_{i=1}^{m}a_i=1"$.

Definition 9: The fuzzy mean is defined by its α-cuts"[7]:

$$\bar{\mu}[\alpha]=\left\{\sum\nolimits_{i=1}^{m}y_ia_i\,|S\right\} \tag{2}$$

for $\alpha\in[0, 1]$ and $\bar{\alpha}_i$ is a fuzzy number where s is the statement $"a_i\in\bar{a}_i[\alpha], 1\le i\le m, \sum\nolimits_{i=1}^{m}a_i=1"$.

Definition 10: Buckley[3], the fuzzy variance is defined by its α-cuts as:

$$\sigma^{-2}[\alpha]=\left\{\sum\nolimits_{i=1}^{n}(y_i-\mu)^2a_i\,|S, \mu=\sum\nolimits_{i=1}^{m}y_ia_i\right\} \tag{3}$$

for $\alpha\in[0, 1]$ and $\bar{\alpha}_i$ is a fuzzy number where, S is the statement $"a_i\in\bar{a}_i[\alpha], 1\le i\le m, \sum\nolimits_{i=1}^{m}a_i=1"$.

Definition 11: Let, K be a fuzzy random variable with fuzzy probability mass function f(k; ∂) where ∂ is a vector of m parameters and $K_1, ..., K_n$ is a random sample from $f(k; \bar{\partial})$ and set $Y = K_1+, ..., +K_n$ then the α-cuts of the fuzzy moment generating function $(\bar{m}_k(t))$ is[8]:

$$\bar{m}_k(t)[\alpha]=\left\{\sum_{\forall k}e^{tk}f(k;\vartheta_i)\big|S\right\}\text{ if } k \text{ is a d.r.v} \tag{4}$$

$\bar{\vartheta}_i$ is a fuzzy number where, $\alpha\in[0, 1]$ and S is $"\vartheta_i\in\bar{\vartheta}_i[\alpha], 1\le i\le m"$.

Definition 12: The discrete exponential probability mass function is defined as[3]:

$$P(K=k)=(1-e^{-\theta})e^{-\theta k} \tag{5}$$

for k = 0, 1, 2, ... and parameter θ>0.

Theorem 1: If k have discrete exponential distribution; then[3]:

$$\mu=\frac{e^{-\theta}}{1-e^{-\theta}}$$

$$V(k)=\frac{e^{-\theta}}{(1-e^{-\theta})^2}$$

$$m_k(t)=\frac{e^{-\theta}}{1-e^{-\theta+t}}$$

RESULTS AND DISCUSSION

In this study, we study the fuzzy discrete exponential distribution and some properties of its like fuzzy probability, fuzzy mean, fuzzy variance and fuzzy moment generating function.

Fuzzy Discrete Exponential distribution (FDE): The α-cuts of a fuzzy discrete exponential distribution is given by:

$$\bar{P}(k)[\alpha]=\left\{(1-e^{-\theta})e^{-\theta k}\,|S\right\}$$

for all $\alpha\in[0,1]$, k = 0, 1, 2, … and θ>0. Where S is the statement $\theta\in\bar{\theta}[\alpha]$.

Theorem 2: Let $\bar{\theta}=(c_1, c_2, c_3), c_1<c_2<c_3$ be a fuzzy triangular number. Then fuzzy probability $(\beta_{\bar{p}(0)}(x))$ of FDE where, k = 0 is:

$$\beta_{\bar{p}(0)}(x)=\begin{cases}0, & x<c_1\\ \dfrac{\ln(1-x)+c_1}{(c_1-c_2)}, & 1-e^{-c_1}\le x\le 1-e^{-c_2}\\ \dfrac{\ln(1-x)+c_3}{(c_3-c_2)}, & 1-e^{-c_2}\le x\le 1-e^{-c_3}\\ 0, & x>c_3\end{cases}$$

Proof: Let, $\bar{\theta} = (c_1, c_2, c_3)$, $c_1<c_2<c_3$ be a fuzzy triangular number, k be random variable has a FDE and $\alpha \in [0,1]$. Then, $\bar{p}(0)[\alpha] = [p_{\theta 1}(\alpha), p_{\theta 2}(\alpha)]$ where:

$$p_{\theta 1}(\alpha) = \min\{(1-e^{-\theta})\, e^{-\theta k} \,|S\}$$
$$p_{\theta 2}(\alpha) = \max\{(1-e^{-\theta})\, e^{-\theta k} \,|S\}$$

Since, $d(1-e^{-\theta})\, e^{-\theta k}/d\theta>0$ on $\bar{p}[0]$ we obtain:

$$\bar{p}[0][\alpha] = [1-e^{\theta_1(\alpha)}, 1-e^{\theta 2(\alpha)}]$$

Where:

$$\bar{\theta}[\alpha] = [\theta_1(\alpha), \theta_2(\alpha)] = [\alpha(c_2-c_1)+c_1, c_3-\alpha(c_3-c_2)]$$

Hence:

$$\bar{p}(0)[\alpha] = [1-e^{-c_1-\alpha(c_2-c_1)}, 1-e^{-c_3+\alpha(c_3-c_2)}] \quad (6)$$

From Eq. 6, then the fuzzy probability mass function of FDE is:

$$\beta_{\bar{p}(0)}(x) = \begin{cases} 0, & x<c_1 \\ \dfrac{\ln(1-x)+c_1}{(c_1-c_2)}, & 1-e^{-c_1} \le x \le 1-e^{-c_2} \\ \dfrac{\ln(1-x)+c_3}{(c_3-c_2)}, & 1-e^{-c_2} \le x \le 1-e^{-c_3} \\ 0, & x>c_3 \end{cases}$$

Example 1: Let $\bar{\theta}$ = (1, 3, 5). Then by Theorem 2 the $\beta_{\bar{p}(0)}(x)$ of FDE is (Table 1 and Fig. 1):

$$\beta_{\bar{p}(0)}(x) = \begin{cases} 0, & x<1 \\ \dfrac{\ln(1-x)+1}{(c_1-c_2)}, & 1-e^{-1} \le x \le 1-e^{-3} \\ \dfrac{\ln(1-x)+5}{(c_3-c_2)}, & 1-e^{-3} \le x \le 1-e^{-5} \\ 0, & x>5 \end{cases}$$

Theorem 3: Let $\bar{\theta}$ = (c_1, c_2, c_3), $c_1<c_2<c_3$ be a fuzzy triangular number. Then fuzzy mean $\beta_{\bar{\mu}}(x))$ of FDE is:

$$\beta_{\bar{\mu}}(x) = \begin{cases} 0, & x<c_1 \\ \dfrac{\ln(x)-\ln(1+x)+c_3}{c_3-c_2}, & \dfrac{e^{-c_3}}{1-e^{-c_3}} \le x \le \dfrac{e^{-c_2}}{1-e^{-c2}} \\ \dfrac{\ln(x)-\ln(1+x)+c_1}{c_1-c_2}, & \dfrac{e^{-c_2}}{1-e^{-c_2}} \le x \le \dfrac{e^{-c1}}{1-e^{-c1}} \\ 0, & x>c_3 \end{cases}$$

Table 1: Some α-cuts of fuzzy probability of FDE at k = 0

α	$\bar{p}(0)[\alpha]$
0.1	[0.6988, 0.9917]
0.2	[0.7534, 0.9899]
0.3	[0.7981, 0.9877]
0.4	[0.8347, 0.9850]
0.5	[0.8646, 0.9816]
0.6	[0.8891, 0.9776]
0.7	[0.9092, 0.9726]
0.8	[0.9257, 0.9666]

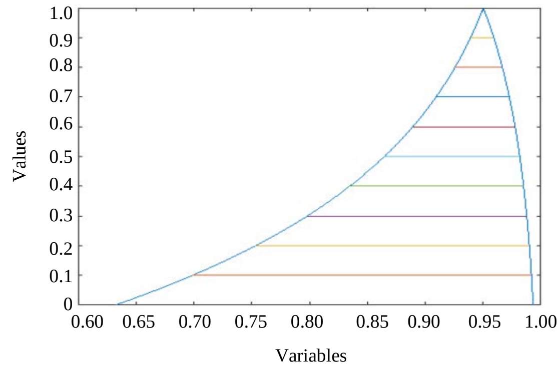

Fig. 1: $\beta_{\bar{p}(0)}(x)$ and its in Example (1)

Proof: Let $\bar{\theta}$ = (c_1, c_2, c_3), $c_1<c_2<c_3$ and $\alpha \in [0, 1]$. Now, to find $\beta_{\bar{\mu}}(x)$. From Eq. 2 and Theorem 1, we have:

$$\bar{\mu}(\alpha) = \left\{ \left(\frac{e^{-\theta}}{1-e^{-\theta}} \right) / \theta \in \bar{\theta}[\alpha] \right\}$$

where, $\bar{\theta}[\alpha] = [\theta_1(\alpha), \theta_2(\alpha)] = [\alpha(c_2-c_1)+c_1, c_3-\alpha(c_3-c_2)]$. Hence:

$$\bar{\mu}(\alpha) = \left[\frac{e^{-\theta_2(\alpha)}}{1-e^{-\theta_2(\alpha)}} \right], \left[\frac{e^{-\theta_1(\alpha)}}{1-e^{-\theta_1(\alpha)}} \right] = \left[\frac{e^{-c_3+\alpha(c_3-c_2)}}{1-e^{-c_3+\alpha(c_3-c_2)}}, \frac{e^{-c_1-\alpha(c_2-c_1)}}{1-e^{-c1-\alpha(c_2-c_1)}} \right] \quad (7)$$

From Eq. 7, then the fuzzy $\beta_{\bar{\mu}}(x)$ of FDE is:

$$\beta_{\bar{\mu}}(x) = \begin{cases} 0, & x<c_1 \\ \dfrac{\ln(x)-\ln(1+x)+c_3}{c_3-c_2}, & \dfrac{e^{-c_3}}{1-e^{-c_3}} \le x \le \dfrac{e^{-c_2}}{1-e^{-c2}} \\ \dfrac{\ln(x)-\ln(1+x)+c_1}{c_1-c_2}, & \dfrac{e^{-c_2}}{1-e^{-c_2}} \le x \le \dfrac{e^{-c1}}{1-e^{-c1}} \\ 0, & x>c_3 \end{cases}$$

Example 2: Let $\bar{\theta}$ = (1, 3, 5). Then by Theorem 3 the $\beta_{\bar{\mu}}(x)$ of FDE is (Table 2 and Fig. 2):

Table 2: Some α-cuts of the $\beta_{\bar{\mu}}(x)$ of FDE

α	$\bar{\mu}[\alpha]$
0.1	[0.0083, 0.4310]
0.2	[0.0102, 0.3273]
0.3	[0.0124, 0.2529]
0.4	[0.0152, 0.1980]
0.5	[0.0187, 0.1565]
0.6	[0.0229, 0.1246]
0.7	[0.0281, 0.0998]
0.8	[0.0345, 0.0802]

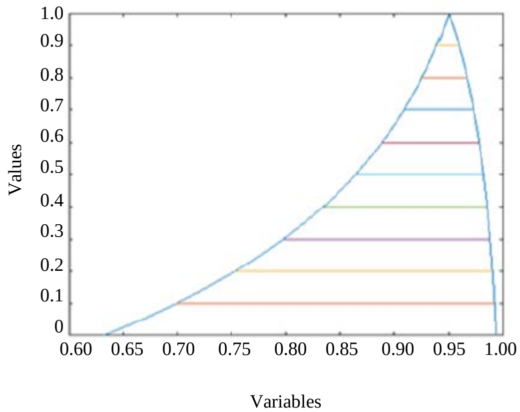

Fig. 2: $\beta_{\bar{\mu}}(x)$ and its α-cuts in Example 2

$$\beta_{\bar{\mu}}(x)=\begin{cases}0, & x<1\\ \dfrac{\ln(x)-\ln(1+x)+5}{c_3-c_2}, & \dfrac{e^{-5}}{1-e^{-5}}\le x\le\dfrac{e^{-3}}{1-e^{-3}}\\ \dfrac{\ln(x)-\ln(1+x)+1}{-2}, & \dfrac{e^{-3}}{1-e^{-3}}\le x\le\dfrac{e^{-1}}{1-e^{-1}}\\ 0, & x>5\end{cases}$$

Theorem 4: Let $\bar{\theta}$ = (c_1, c_2, c_3), $c_1<c_2<c_3$ be a fuzzy triangular number. Then fuzzy variance ($\beta\bar{v}(x)$) of FDE is:

$$\beta_{\bar{v}}(x)=\begin{cases}0, & x<c_1\\ \dfrac{\ln\left(1-\left(\dfrac{2}{1+\sqrt{1+4x}}\right)\right)+c_3}{c_3-c_2}, & \dfrac{e^{-c_3}}{(1-e^{c_3})^2}\le x\le\dfrac{e^{-c_2}}{(1-e^{-3})^2}\\ \dfrac{\ln\left(1-\left(\dfrac{2}{1+\sqrt{1+4x}}\right)\right)+c_1}{c_1-c_2}, & \dfrac{e^{-c2}}{(1-e^{c_2})^2}\le x\le\dfrac{e^{-c1}}{(1-e^{-1})^2}\\ 0, & x>c_3\end{cases}$$

Proof: Let $\bar{\theta}$ = (c_1, c_2, c_3), $c_1<c_2<c_3$ and $\alpha\in[0, 1]$. Now, to $\beta_{\bar{v}}(x)$ find from Eq. 3 and Theorem 1:

$$\bar{V}[\alpha]=\left\{\frac{e^{-\theta}}{(1-e^{-\theta})^2}\,|\,\theta\in\bar{\theta}[\alpha]\right\}$$

where, $\bar{\theta}[\alpha]=[\theta_1(\alpha), \theta_2(\alpha)]=[\alpha(c_2-c_1)+c_1, c_3-\alpha(c_3-c_2)]$. Hence:

Table 3: Some α-cuts of the $\beta_{\bar{v}}(x)$ of FDE

A	$\bar{V}[\alpha]$
0.1	[0.0083, 0.6167]
0.2	[0.0102, 0.4344]
0.3	[0.0125, 0.3169]
0.4	[0.0154, 0.2372]
0.5	[0.0190, 0.1810]
0.6	[0.0234, 0.1401]
0.7	[0.0288, 0.1097]
0.8	[0.0357, 0.0866]

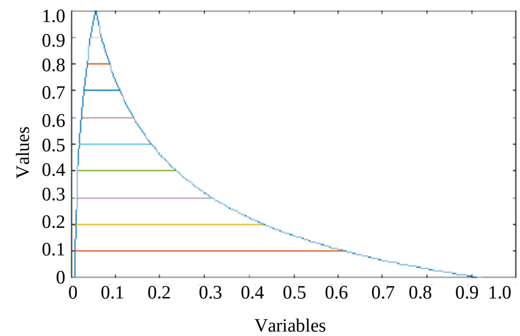

Fig. 3: $\beta_{\bar{v}}(x)$ and its α-cuts in Example 3

$$\bar{V}[\alpha]=\left[\frac{e^{-\theta_2(\alpha)}}{(1-e^{\theta_2(\alpha)})^2},\frac{e^{-\theta_1(\alpha)}}{(1-e^{\theta1(\alpha)})^2}\right]=\left[\frac{e^{-c_3+\alpha(c_3-c_2)}}{(1-e^{-c_3+\alpha(c_3-c_2)})^2},\frac{e^{-c_1-\alpha(c_2-c_1)}}{(1-e^{-c_1+\alpha(c_2-c_1)})^2}\right] \tag{8}$$

From Eq. 8 then the fuzzy $\beta_{\bar{v}}(x)$ of FDE is:

$$\beta_{\bar{v}}(x)=\begin{cases}0, & x<c_1\\ \dfrac{\ln\left(1-\left(\dfrac{2}{1+\sqrt{1+4x}}\right)\right)+c_3}{c_3-c_2}, & \dfrac{e^{-c_3}}{(1-e^{c_3})^2}\le x\le\dfrac{e^{-c_2}}{(1-e^{-3})^2}\\ \dfrac{\ln\left(1-\left(\dfrac{2}{1+\sqrt{1+4x}}\right)\right)+c_1}{c_1-c_2}, & \dfrac{e^{-c2}}{(1-e^{c_2})^2}\le x\le\dfrac{e^{-c1}}{(1-e^{-1})^2}\\ 0, & x>c_3\end{cases}$$

Example 3: Let $\bar{\theta}$ = (1, 3, 5). Then by Theorem 4 the $\beta_{\bar{v}}(x)$ of FDE is (Table 3 and Fig. 3):

$$\beta_{\bar{v}}(x)=\begin{cases}0, & x<1\\ \dfrac{\ln\left(1-\left(\dfrac{2}{1+\sqrt{1+4x}}\right)\right)+5}{2}, & \dfrac{e^{-5}}{(1-e^{-5})^2}\le x\le\dfrac{e^{-3}}{(1-e^{-3})^2}\\ \dfrac{\ln\left(1-\left(\dfrac{2}{1+\sqrt{1+4x}}\right)\right)+1}{-2}, & \dfrac{e^{-3}}{(1-e^{-3})^2}\le x\le\dfrac{e^{-1}}{(1-e^{-1})^2}\\ 0, & x>5\end{cases}$$

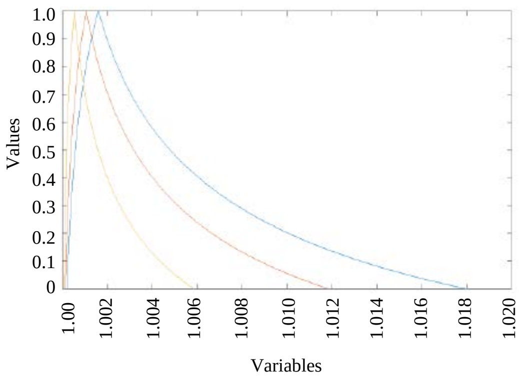

Fig. 4: Fuzzy moment generating function at t = 0.01, t = 0.03 and t = 0.02

Theorem 5: Let $\bar{\theta} = (c_1, c_2, c_3)$, $c_1<c_2<c_3$ be a fuzzy triangular number, then fuzzy moment generating function $(\beta_{\bar{m}k_{(t)}}x))$ of FDE is:

$$\beta_{\bar{m}k(t)}(x)=\begin{cases}0, & x<c_1\\ \dfrac{\ln(1-x)-\ln(1-xe^t)+c_3}{c_3-c_2}, & \dfrac{e^{-c_3}}{1-e^{-c_3+t}}\le x\le\dfrac{e^{-c_2}}{1-e^{-c_2+t}}\\ \dfrac{\ln(1-x)-\ln(1-xe^t)+c_1}{c_1-c_2}, & \dfrac{e^{-c_2}}{1-e^{-c_2}+^t}\le x\le\dfrac{e^{-c_1}}{1-e^{-c_1+t}}\\ 0, & x>c_3\end{cases}$$

Proof: Let $\bar{\theta} = (c_1, c_2, c_3)$, $c_1<c_2<c_3$ $\alpha\in[0, 1]$ and $0<t<1$. To find $\beta_{\bar{m}k_{(t)}}x)$ from Eq. 4 and theorem (1).

$$\bar{m}_k(t)[\alpha]=\left\{\frac{1-e^{-\theta}}{1-e^{-\theta+t}}\middle|\theta\in\bar{\theta}[\alpha]\right\}$$

Where $\bar{\theta}[\alpha] = [\theta_1(\alpha), \theta_2(\alpha)] = [\alpha(c_2-c_1)+c_1, c_3-\alpha(c_3-c_2)]$. Hence:

$$\bar{m}_k(t)[\alpha]=\left[\frac{1-e^{-\theta_2(\alpha)}}{1-e^{-\theta_2(\alpha)+t}}, \frac{1-e^{-\theta_1(\alpha)}}{1-e^{-\theta_1(\alpha)+t}}\right]=$$

$$\left[\frac{1-e^{-c_3+\alpha(c_3-c_2)}}{1-e^{-}c_3+\alpha(c_3-c_2)+t}, \frac{1-e^{-c_1-\alpha(c_2-c_1)}}{1-e^{-c_1-\alpha(c_2-c_1)+t}}\right]$$

So that, the fuzzy $\beta_{\bar{M}k_{(t)}}X)$ of FDE is:

$$\beta_{\bar{m}k(t)}(x)=\begin{cases}0, & x<c_1\\ \dfrac{\ln(1-x)-\ln(1-xe^t)+c_3}{c_3-c_2}, & \dfrac{e^{-c_3}}{1-e^{-c_3+t}}\le x\le\dfrac{e^{-c_2}}{1-e^{-c_2+t}}\\ \dfrac{\ln(1-x)-\ln(1-xe^t)+c_1}{c_1-c_2}, & \dfrac{e^{-c_2}}{1-e^{-c_2+t}}\le x\le\dfrac{e^{-c_1}}{1-e^{-c_1+t}}\end{cases}$$

Example 4: Let $\bar{\theta} = (1, 3, 5)$. Then by theorem (5) the $\beta_{\bar{m}k_{(t)}}x)$ of FDE is (Fig. 4):

$$\beta_{\bar{m}k(t)}(x)=\begin{cases}0, & x<1\\ \dfrac{\ln(1-x)-\ln(1-xe^t)+5}{2}, & \dfrac{e^{-5}}{1-e^{-5+t}}\le x\le\dfrac{e^{-3}}{1-e^{-3+t}}\\ \dfrac{\ln(1-x)-\ln(1-xe^t)+1}{-2}, & \dfrac{e^{-3}}{1-e^{-3+t}}\le x\le\dfrac{e^{-1}}{1-e^{-1+t}}\\ 0, & x>5\end{cases}$$

CONCLUSION

In this study, we find the fuzzy discrete exponential distribution, its mean, variance and its moment generating function. Therefore, we conform the work by examples, tables of some α-cuts and figures.

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