Journal of Engineering and Applied Sciences 15 (2): 581-585, 2020
ISSN: 1816-949X

Introduction to Cartesian, Tensor and Lexicographic Product of Bipolar Interval Valued Fuzzy Graph

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Abstract: In this study, we discuss the product of bipolar, interval valued fuzzy graphs concept and we give some properties for them. We will study some corollaries on Cartesian product and define some properties on it. Tensor product, lexicographic product of bipolar interval valued fuzzy graph will also be defined.

Key words: Interval valued fuzzy set, bipolar interval valued fuzzy set, bipolar interval valued fuzzy graph, cartesian product, tensor product, lexicographic product

INTRODUCTION

A fuzzy set theory was introduced by Zadeh (1965). Fuzzy set theory has become a vigorous area of research in different disciplines including mathematics, physics, statistics, engineering and computer networks. In mathematics fuzzy groups, rings and graphs have been discussed by Massa'deh (2010a, b), Massa'deh and Ba'arah (2013) and Massa'deh and Gharaibeh (2011). Bipolar fuzzy set's concepts defined by Zhang (1998) is a generalization of fuzzy sets (Muthuraj *et al*., 2016; Muthuraj and Sridharan, 2012) discussed the concepts of bipolar fuzzy normal-subgraph (Massa'deh, 2017) introduced and studied bipolar fuzzy cosets. Ramya and Lavanya (2017) studied the edge contraction on bipolar fuzzy graphs (Massa'deh and Ba'arah, 2013) introduced the concept of degrees types in bipolar fuzzy graphs. Akram and Dudek (2011) extended the fuzzy set theory to interval-valued fuzzy sets, in addition (Ramprasad *et al*., 2016) discussed interval valued fuzzy graphs, also (Massa'deh, 2017) discussed regular, degree of vertex, strong, complete interval valued fuzzy graphs (Rashmanlou and Pal, 2013a-c; Rashmanlou and Jun, 2013). Mishra *et al*. studied bipolar interval valued fuzzy graphs (Mishra and Pal, 2016). In this study, we gave and studied the product of two bipolar interval-valued fuzzy graph concepts and discussed some of their properties, we defined also tensor product and lexicographic of bipolar interval-valued fuzzy graphs.

MATERIALS AND METHODS

Preliminaries

Definition 2.1 (Zadeh 1965): A fuzzy set μ is a mapping from X to [0, 1].

Definition 2.2 (Massa'deh and Gharaibeh (2011): A fuzzy graph G is a pair of functions G = (λ, μ) where, λ is a fuzzy subset of a non-empty set X and μ is a symmetric fuzzy relation on λ this means that μ (xy)$\leq$max {λ (x), λ (y)}. The underlying crisp graph of G = (λ, μ) is denoted by G* = (V, E) where E$\subseteq$V X V.

Definition 2.3 (Massa'deh and Ba'arah (2013): Let G = (λ, μ) be a fuzzy graph, the degree of a vertex a$\in$G is defined by:

$$D_G(a) = \sum a \neq b\, \mu(ab) = \sum ab \in E\ \mu(ab)$$

Definition 2.4 (Massa'deh and Ba'arah (2013): The order of a fuzzy graph G is defined by O (G) = $\Sigma_{ab\in E}$ λ (a):

Definition 2.5 Zhang (1998): Let X be a non-empty set. A bipolar fuzzy set μ in X is an object having the form μ = {(x, μ^+ (x), μ^- (x)); x$\in$X} where μ^+(x): X→[0, 1] and μ^- (x): X→[-1, 0] are mapping. Here, μ^+ (x) is the positive membership value which denotes the satisfaction degree of an element x$\in\mu$ and μ^-(x) is the negative membership value which denotes the satisfaction degree to some implicit counter property of an element x$\in\mu$. If for any

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$x \in \mu$, $\mu^+(x) \neq 0$ and $\mu^-(x) = 0$, it is the situation that x has only positive satisfaction for μ, if for any $x \in \mu$, $\mu^+(x) = 0$ and $\mu^-(x) \neq 0$ then the situation that x does not satisfy the property of μ but somewhat satisfies the counter property of μ. It is possible for an element x for which $\mu^+(x) = 0$ and $\mu^-(x) = 0$ then we say that the satisfaction property of an element overlaps with its counter satisfaction property over some portion, we shall use the symbol $\mu(\mu^+, \mu^-)$ for the bipolar fuzzy set $\mu = \{(x, \mu^+(x), \mu^-(x)); x \in \mu\}$.

Definition 2.6 (Zhang, 1998): For every two bipolar fuzzy sets $\mu = (\mu^+, \mu^-)$ and $\lambda = (\lambda^+, \lambda^-)$ in A, we define:

$$(\mu \cap \lambda)(a) = (\min(\mu^+(a), \lambda^+(a)), \max(\mu^-(a), \lambda^-(a))$$
$$(\mu \cup \lambda)(a) = (\max(\mu^+(a), \lambda^+(a)), \min(\mu^-(a), \lambda^-(a))$$

For all $a \in A$:

Definition 2.7 (Zhang, 1998): Let $\mu = (\mu^+, \mu^-)$ and $\lambda = (\lambda^+, \lambda^-)$ be two bipolar fuzzy sets on A. If $\delta = \mu \times \lambda$ is any relation on A then $\delta = (\delta^+, \delta^-)$ is called a bipolar fuzzy relation from $\mu = (\mu^+, \mu^-)$ on $\lambda = (\lambda^+, \lambda^-)$ where $\delta^+(a, b) \leq \min\{\mu^+(a), \lambda^+(b)\}$ and $\delta^-(a, b) \geq \max\{\mu^-(a), \lambda^-(b)\}$ for all $a \in \mu$ and $b \in \lambda$.

Throughout this study, G is a crisp graph, F_G is a fuzzy graph, BF_G is a bipolar fuzzy graph, IVF_G is an interval-valued fuzzy graph and $BIVF_G$ is a bipolar interval-valued fuzzy graph.

Let $A = \{a_1, a_2, ..., a_n\}$ be any set and [0, 1] be the set of all closed sub-intervals of the interval [0, 1], [-1, 0] be the set of all closed sub-intervals of the interval [-1, 0] and elements of these sets are denoted by uppercase letters. If $\mu \in C[0, 1]$ or $K[-1, 0]$ then it can be represented as $\mu = [\mu_L, \mu_u]$ where μ_L and μ_u are the lower and upper limit of μ.

Definition 2.8 (Mishra and Pal, 2016) BIVF subset is given by $\mu = \{<a, \mu^+(a), \mu^-(a)>, a \in A\}$ where $\mu^+: A \to C[0, 1]$, $\mu^-: A \to K[-1, 0]$. The intervals $\mu^+(a)$ and $\mu^-(a)$ denote the degree of membership and the degree of non-membership of the element a to the set, where $\mu^+(a) = [\mu^+_L(a), \mu^+_U(a)]$ and $\mu^-(a) = [\mu^-_L(a), \mu^-_U(a)]$.

Definition 2.9 (Mishra and Pal, 2016): By bipolar interval-valued fuzzy set on V and $\lambda = [\lambda^-, \lambda^+]$ is a bipolar interval-valued fuzzy relation on E such that:

- $\lambda^-(ab) \leq \min\{\mu^-(a), \mu^-(b)\}$
- $\lambda^+(ab) \leq \min\{\mu^+(a), \mu^+(b)\}$ for all $ab \in E$

Definition 2.10 (Ramya and Lavanya, 2017): By a bipolar fuzzy graph BF_G of G = (V, E) we main a pair (δ, γ) where $\delta = (\delta^+, \delta^-)$ is a bipolar fuzzy set on V and $\gamma = (\gamma^+, \gamma^-)$ is a bipolar fuzzy relation on $E \subseteq V \times V$ such that $\gamma^+(ab) \leq \min\{\delta^+(a), \delta^+(b)\}$ and $\gamma^-(ab) \leq \max\{\delta^-(a), \delta^-(b)\}$ for all $a, b \in V$ and $ab \in E$.

Definition 2.11 (Rashmanlou and Pal 2013): Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two simple graphs, we can construct several new graphs. The first construction called the Cartesian product of G_1 and G_2 gives a graph $G_1.G_2 = (V, E)$ with $V = V_1 \times V_2$ and $E = \{(a, b)(a, c); a \in V_1, bc \in E_2\} \cup \{(d_1, e)(d_2, l); d_1d_2 \in E_1, e \in V_2\}$.

Throughout this study, we assume C [0, 1] be the set of all closed sub-intervals of the interval [0, 1] and K [-1, 0] is the set of all closed sub-intervals of the interval [-1, 0].

RESULTS AND DISCUSSION

Definition 3.1: A Bipolar Interval-valued Fuzzy Graph $BIVF_G$ with underlying graph G = (V, E) is defined to be a pair (λ, μ) where:

The function $\lambda^+: V \to C[0, 1]$ and $\lambda^-: V \to K[-1, 0]$ denote satisfaction degree interval and the satisfaction degree interval to some implicit counter-property of an element $a \in \lambda$, respectively.

The function $\mu^+: E \subset V \times V \to C[0, 1]$ and $\mu^+: E \subset V \times V \to K[-1, 0]$ are defined by $\mu^+_L(a, b) \leq \min\{\lambda^+_L(a), \lambda^+_L(b)\}$ and $\mu^-_L(a, b) \geq \max\{\lambda^-_L(a), \lambda^-_L(b)\}$ $\mu^+_U(a, b) \leq \min\{\mu^+_U(a), \mu^+_U(b)\}$ and $\mu^-_U(a, b) \geq \max\{\mu^-_L(a), \mu^-_U(b)\}$ for all $ab \in E$.

Example 3.2: Consider a Bipolar Interval-valued Fuzzy graph BIF_G where, λ = (a, [0.2, 0.6], [-0.7, -0.5]), (b, [0.3, 0.5], [-0.9, -0.3]), (c, [0.5, 0.4], [-0.6, -0.5]), (d, [0.1, 0.7], [-0.8, -0.2]). Then the corresponding $BIVF_G$ is shown in Fig. 1.

Definition 3.3: Let $G_1 = (\lambda_1, \mu_1)$ and $G_2 = (\lambda_2, \mu_2)$ be two bipolar interval valued fuzzy graph of the graph is $G^*_1 = (V_1, E_1)$ and $G^*_2 = (V_2, E_2)$ then the Cartesian product $G = (G_1 \times G_2)$ is defined as pain $(\lambda_1, \lambda_2, \mu_1 \times \mu_2)$ such that:

$$(\lambda^+_{1L} \times \lambda^+_{2L})(a, b) = \min\{\lambda^+_{1L}(a), \lambda^+_{2L}(b)\}$$
$$(\lambda^+_{1U} \times \lambda^+_{2U})(a, b) = \min\{\lambda^+_{1U}(a), \lambda^+_{2U}(b)\}$$
$$(\lambda^-_{1L} \times \lambda^-_{2L})(a, b) = \max\{\lambda^-_{1L}(a), \lambda^-_{2L}(b)\}$$
$$(\lambda^-_{1U} \times \lambda^-_{2U})(a, b) = \max\{\lambda^-_{1U}(a), \lambda^-_{2U}(b)\}$$
$$\text{For all } (a, b) \in V$$

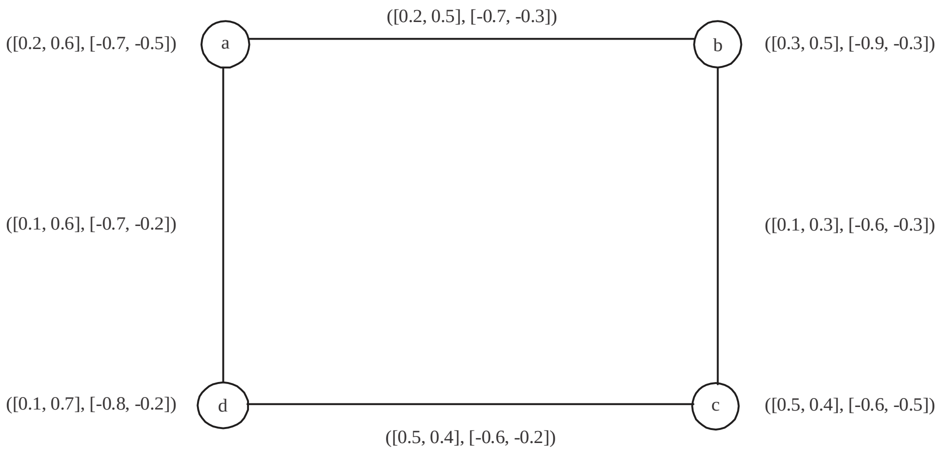

Fig. 1: Bipolar interval-valued fuzzy graph

$$\left(\mu_{1L}^{+}\times\mu_{2L}^{+}\right)\left((a,b)(a,c)\right) = \min\left\{\lambda_{1L}^{+}(a),\mu_{2L}^{+}(bc)\right\}$$
$$\left(\mu_{1U}^{+}\times\mu_{2U}^{+}\right)\left((a,b)(a,c)\right) = \min\left\{\lambda_{1L}^{+}(a),\mu_{2U}^{+}(bc)\right\}$$
$$\left(\mu_{1L}^{-}\times\mu_{2L}^{-}\right)\left((a,b)(a,c)\right) = \max\left\{\lambda_{1L}^{-}(a),\mu_{2L}^{-}(bc)\right\}$$
$$\left(\mu_{1U}^{-}\times\mu_{2U}^{-}\right)\left((a,b)(a,c)\right) = \max\left\{\lambda_{1U}^{-}(a),\mu_{2U}^{-}(bc)\right\}$$

For all $a \in V$ and $bc \in E_2$

$$\left(\mu_{1L}^{+}\times\mu_{2L}^{+}\right)\left((a,b)(c,b)\right) = \min\left\{\mu_{1L}^{+}(ac),\lambda_{2L}^{+}(b)\right\}$$
$$\left(\mu_{1U}^{+}\times\mu_{2U}^{+}\right)\left((a,b)(c,b)\right) = \min\left\{\mu_{1U}^{+}(ac),\lambda_{2L}^{+}(b)\right\}$$
$$\left(\mu_{1L}^{-}\times\mu_{2L}^{-}\right)\left((a,b)(c,b)\right) = \max\left\{\mu_{1L}^{-}(ac),\lambda_{2L}^{-}(b)\right\}$$
$$\left(\mu_{1U}^{-}\times\mu_{2U}^{-}\right)\left((a,b)(c,b)\right) = \max\left\{\mu_{1U}^{-}(a),\lambda_{2U}^{-}(b)\right\}$$

For all $b \in V$ and $ac \in E_1$

Definition 3.4: If $G_1 = (\lambda_1, \mu_1)$ and $G_2 = (\lambda_2, \mu_2)$ are two bipolar interval valued fuzzy graph of $G^*_1 = (V_1, E_1)$ and $G^*_2 = (V_2, E_2)$, respectively then the lexicographic product $G_1{*}G_2$ is defined as a pair (λ, μ) where $\lambda = (\lambda^+, \lambda^-)$ and $\mu = (\mu^+, \mu^-)$ are bipolar interval valued fuzzy sets on $V = V_1\times V_2$ and $E = \{(a, b)\ (a, c);\ a\in V_1,\ (b, c) \in E_2\} \cup \{(x, y)\ (z, w);\ xz\in E_1,\ yw\in E_2\}$, respectively which satisfies the following conditions:

$$\left(\lambda_{1L}^{+}*\lambda_{2L}^{+}\right)(x,y) = \min\left\{\lambda_{1L}^{+}(x),\lambda_{2L}^{+}(y)\right\}$$
$$\left(\lambda_{1U}^{+}*\lambda_{2U}^{+}\right)(x,y) = \min\left\{\lambda_{1U}^{+}(x),\lambda_{2U}^{+}(y)\right\}$$
$$\left(\lambda_{1L}^{-}*\lambda_{2L}^{-}\right)(x,y) = \max\left\{\lambda_{1L}^{-}(x),\lambda_{2L}^{-}(y)\right\}$$
$$\left(\lambda_{1U}^{-}*\lambda_{2U}^{-}\right)(x,y) = \max\left\{\lambda_{1U}^{-}(x),\lambda_{2U}^{-}(y)\right\}$$

For all $(x, y) \in V_1\times V_2$

$$\left(\mu_{1L}^{+}*\mu_{2L}^{+}\right)\left((a,b)(a,c)\right) = \min\left\{\lambda_{1L}^{+}(a),\mu_{2L}^{+}(bc)\right\}$$
$$\left(\mu_{1U}^{+}*\mu_{2U}^{+}\right)\left((a,b)(a,c)\right) = \min\left\{\lambda_{1U}^{+}(a),\mu_{2U}^{+}(ac)\right\}$$
$$\left(\mu_{1L}^{-}*\mu_{2L}^{-}\right)\left((a,b)(a,c)\right) = \max\left\{\lambda_{1L}^{-}(a),\mu_{2L}^{-}(ac)\right\}$$
$$\left(\mu_{1U}^{-}*\mu_{2U}^{-}\right)\left((a,b)(a,c)\right) = \max\left\{\lambda_{1L}^{-}(a),\mu_{2U}^{-}(ac)\right\}$$

For all $a \in V_1$ and $bc \in E_2$

$$\left(\mu_{1L}^{+}*\mu_{2L}^{+}\right)\left((x,y)(z,w)\right) = \min\left\{\mu_{1L}^{+}(xz),\mu_{2L}^{+}(yw)\right\}$$
$$\left(\mu_{1U}^{+}*\mu_{2U}^{+}\right)\left((x,y)(z,w)\right) = \min\left\{\mu_{1U}^{+}(xz),\mu_{2U}^{+}(yw)\right\}$$
$$\left(\mu_{1L}^{-}*\mu_{2L}^{-}\right)\left((x,y)(z,w)\right) = \max\left\{\mu_{1L}^{-}(xz),\mu_{2L}^{-}(yw)\right\}$$
$$\left(\mu_{1U}^{-}*\mu_{2U}^{-}\right)\left((x,y)(z,w)\right) = \max\left\{\mu_{1U}^{-}(xz),\mu_{2U}^{-}(yw)\right\}$$

For all $xz \in E_1$ and $yw \in E_2$

Definition 3.5: Let $G_1 = (\lambda_1, \mu_1)$ and $G_2 = (\lambda_2, \mu_2)$ be two bipolar interval valued fuzzy graph of $G^*_1 = (V_1, E_1)$ and $G^*_2 = (V_2, E_2)$, respectively then the tensor product $G_1\otimes G_2$ is defined as a pain (λ, μ) where λ and μ are bipolar interval valued fuzzy sets on 1 2 V = V ´V and $V = V_1\times V_2$ and $E = \{(a, b)\ (c, d);\ (a, c) \in E_1,\ (b, d) \in E_2\}$, respectively which satisfies the following axioms:

$$\left(\lambda_{1L}^{+}\otimes\lambda_{2L}^{+}\right)(a,b) = \min\left\{\lambda_{1L}^{+}(a),\lambda_{2L}^{+}(b)\right\}$$
$$\left(\lambda_{1U}^{+}\otimes\lambda_{2U}^{+}\right)(a,b) = \min\left\{\lambda_{1U}^{+}(a),\lambda_{2U}^{+}(b)\right\}$$
$$\left(\lambda_{1L}^{-}\otimes\lambda_{2L}^{-}\right)(a,b) = \max\left\{\lambda_{1L}^{-}(a),\lambda_{2L}^{-}(b)\right\}$$
$$\left(\lambda_{1U}^{-}\otimes\lambda_{2U}^{-}\right)(a,b) = \max\left\{\lambda_{1U}^{-}(a),\lambda_{2U}^{-}(b)\right\}$$

For all $(a, b) \in V_1\times V_2$

$$\left(\mu_{1L}^{+}\otimes\mu_{2L}^{+}\right)\left((a,b)(c,d)\right) = \min\left\{\mu_{1L}^{+}(ac),\mu_{2L}^{+}(bd)\right\}$$
$$\left(\mu_{1U}^{+}\otimes\mu_{2U}^{+}\right)\left((a,b)(c,d)\right) = \min\left\{\mu_{1U}^{+}(ac),\mu_{2U}^{+}(bd)\right\}$$
$$\left(\mu_{1L}^{-}\otimes\mu_{2L}^{-}\right)\left((a,b)(c,d)\right) = \max\left\{\mu_{1L}^{-}(ac),\mu_{2L}^{-}(bd)\right\}$$
$$\left(\mu_{1U}^{-}\otimes\mu_{2U}^{-}\right)\left((a,b)(c,d)\right) = \max\left\{\mu_{1U}^{-}(ac),\mu_{2U}^{-}(bd)\right\}$$

For all $ac \in E_1$ and $bd \in E_2$

Definition 3.6: If G_1, G_2 are two bipolar interval valued fuzzy graph and $G = G_1\times G_2$ is the Cartesian product of G_1 and G_2 then for any vertex $(a, b)\in V_1\times V_2$, we define the degree of (a, b) as:

$$D_G^+(a,b) = \sum_{(a,b)(c,d)\in E} \left(\mu_{1U}^+ \times \mu_{2U}^+\right)\left((a,b)(c,d)\right) - \sum_{(a,b)(c,d)\in E} \left(\mu_{1U}^- \times \mu_{2U}^-\right)\left((a,b)(c,d)\right) =$$
$$\sum_{a=c,bd\in E_2} \min\left\{\lambda_{1U}^+(a), \mu_{2U}^+(bd)\right\} + \sum_{b=d,ac\in E_1} \min\left\{\lambda_{2U}^+(b), \mu_{1U}^+(ac)\right\} -$$
$$\left\{\sum_{a=c,bd\in E_2} \max\left\{\begin{matrix}\lambda_{1U}^-(a),\\ \mu_{2U}^-(bd)\end{matrix}\right\} + \sum_{b=d,ac\in E_1} \max\left\{\begin{matrix}\lambda_{2U}^-(b),\\ \mu_{1U}^-(ac)\end{matrix}\right\}\right\}$$

Similarly:

$$D_G^-(a,b) = \sum_{(a,b)(c,d)\in E} \left(\mu_{1L}^+ \times \mu_{2L}^+\right)\left((a,b)(c,d)\right) - \sum_{(a,b)(c,d)\in E} \left(\mu_{1L}^- \times \mu_{2L}^-\right)\left((a,b)(c,d)\right) =$$
$$\sum_{a=c,bd\in E_2} \min\left\{\begin{matrix}\lambda_{1L}^+(a),\\ \mu_{2L}^+(bd)\end{matrix}\right\} + \sum_{b=d,ac\in E_1} \min\left\{\begin{matrix}\lambda_{2L}^+(b),\\ \mu_{1L}^+(ab)\end{matrix}\right\} -$$
$$\left\{\sum_{a=c,bd\in E_2} \max\left\{\begin{matrix}\lambda_{1L}^-(a),\\ \mu_{2L}^-(bd)\end{matrix}\right\} + \sum_{b=d,ac\in E_1} \max\left\{\begin{matrix}\lambda_{2L}^-(b),\\ \mu_{1L}^-(ac)\end{matrix}\right\}\right\}$$

Example 3.7: Consider a bipolar interval valued fuzzy graphs G_1, G_2 and $G_1 \times G_2$ in Fig. 2.

Definition 3.7: A Cartesian product of graph is:

- Strong, if $D^-_G \le 0$ and $D^+_G > 0$
- Week, if $D^-_G < 0$ and $D^+_G \ge 0$
- Super strong, if $D^-_G < 0$ and $D^+_G > 0$
- Very week, if $D^-_G < 0$ and $D^+_G > 0$

Theorem 3.8: Let G be the Cartesian product of two bipolar interval valued fuzzy graphs then G is super strong if:

- Min $\{\lambda^+_{iL}(a_i)\} \ge$ max $\{\lambda^-_{iL}(a_i)\}$
- Min $\{\lambda^+_{iU}(a_i)\} \ge$ max $\{\lambda^-_{iU}(a_i)\}$

Proof SinG G is super strong if and only if $D^-_G < 0$ and $D^+_G > 0$:

$$\sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu_{1L}^+ \times \mu_{2L}^+\right)\left((a_1,a_2)(b_1,b_2)\right) - \sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu_{1L}^- \times \mu_{2L}^-\right)\left((a_1,a_2)(b_1,b_2)\right) < 0$$
$$\sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu_{1L}^+ \times \mu_{2L}^+\right)\left((a_1,a_2)(b_1,b_2)\right) < \sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu_{1L}^- \times \mu_{2L}^-\right)\left((a_1,a_2)(b_1,b_2)\right) *$$

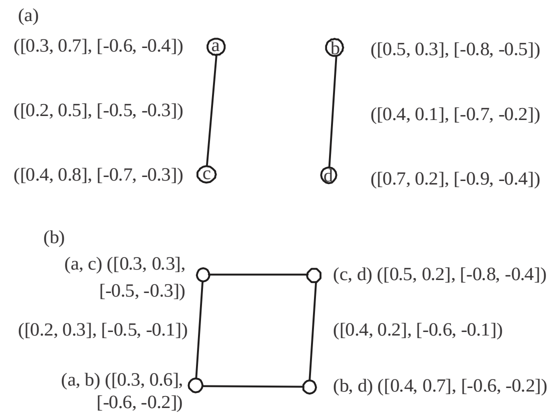

Fig. 2: Cartesian product of G_1, G_2

Now:

$$\sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu_{1L}^+ \times \mu_{2L}^+\right)\left((a_1,a_2)(b_1,b_2)\right) - \sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu_{1L}^- \times \mu_{2L}^-\right)\left((a_1,a_2)(b_1,b_2)\right) < 0$$
$$\sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu_{1L}^+ \times \mu_{2L}^+\right)\left((a_1,a_2)(b_1,b_2)\right) < \sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu_{1L}^- \times \mu_{2L}^-\right)\left((a_1,a_2)(b_1,b_2)\right) *$$

Such that n = number of edges and i = 1, 2, 3, ..., n by the same case we get:

$$\sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu_{1L}^- \times \mu_{2L}^-\right)\left((a_1,a_2)(b_1,b_2)\right) = n\left(\max\left\{\lambda_{iL}^-(a_i)\right\}\right)$$

By *, we get min $\{\lambda^-_{iL}(a_i)\} \ge$ max $\{\lambda^-_{iL}(a_i)\}$. Similarly, we can show $\{\lambda^-_{iU}(a_i)\} \ge$ max $\{\lambda^-_{iU}(a_i)\}$.

Corollary 3.9: If G_1, G_2 are two super strong Cartesian product graph then the Cartesian product is always super strong.

Proof: Since, G_1, G_2 are two super strong Cartesian product graph then $D^-_{G_1} < 0$ and $D^+_{G_1} > 0$ and $D^-_{G_2} < 0$ and $D^+_{G_2} > 0$ we know $D^-_{G_1 \times G_2} = D^-_{G_1} + D^-_{G_2}$ and $D^+_{G_1 \times G_2} = D^+_{G_1} + D^+_{G_2}$ for the Cartesian product of bipolar interval valued fuzzy graph. By theorem 3.7 we get . $D^-_{G_1 \times G_2} < 0$ and $D^+_{G_1 \times G_2} > 0$.

Lemma 3.10: If G_1, G_2 are two very week Cartesian product graph then $G_1 \times G_2$ is always very week.

Proof: Straight forward

Definition 3.11: For any vertex $(a_1, a_2) \in V_1 \otimes V_2$ then the degree of a vertex in tensor product is:

$$\begin{aligned}
D^{-}_{G_1 \otimes G_2}\begin{pmatrix} a_1, \\ a_2 \end{pmatrix} &= \sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu^{+}_{1L} \otimes \mu^{+}_{2L}\right)\left((a_1,a_2)(b_1,b_2)\right) - \\
&\sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu^{-}_{1L} \otimes \mu^{-}_{2L}\right)\left((a_1,a_2)(b_1,b_2)\right) = \\
&\sum_{a_1,b_1\in E_1,a_2,b_2\in E_2} \min\left\{\mu^{+}_{1L}(a_1b_1), \mu^{+}_{2L}(a_2b_2)\right\} - \\
&\sum_{a_1,b_1\in E_1,a_2,b_2\in E_2} \max\left\{\mu^{-}_{1L}(a_1b_1), \mu^{-}_{2L}(a_2b_2)\right\}
\end{aligned}$$

Definition 3.12: For any vertex $(a_1, a_2) \in V_1 * V_2$ then the degree of a vertex in lexicographic product is:

$$\begin{aligned}
D^{-}_{G_1 * G_2} &= \sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu^{+}_{1L} \times \mu^{+}_{2L}\right)\left((a_1,a_2)(b_1,b_2)\right) - \\
&\sum_{(a_1,a_2)(b_1,b_2)\in E} \left(\mu^{-}_{1L} \times \mu^{-}_{2L}\right)\left((a_1,a_2)(b_1,b_2)\right) = \\
&\max\left\{ \sum_{\substack{a_1 = b_1, \\ a_2b_2\in E_2}} \min\begin{Bmatrix} \lambda^{+}_{1L}(a_1), \\ \mu^{+}_{2L}(a_2b_2) \end{Bmatrix}, \sum_{\substack{a_2 = b_2, \\ a_1b_1\in E_1}} \min\begin{Bmatrix} \lambda^{+}_{1L}(a_2), \\ \mu^{+}_{1L}(a_1b_1) \end{Bmatrix} \right\} - \\
&\max\left\{ \sum_{\substack{a_1 = b_1, \\ a_2b_2\in E_2}} \max\begin{Bmatrix} \lambda^{-}_{1L}(a_1), \\ \mu^{-}_{2L}(a_2b_2) \end{Bmatrix}, \sum_{\substack{a_2 = b_2, \\ a_1b_1\in E_1}} \max\begin{Bmatrix} \lambda^{-}_{1L}(a_2), \\ \mu^{-}_{1L}(a_1b_1) \end{Bmatrix} \right\}
\end{aligned}$$

CONCLUSION

In this study, we introduce the degree and discuss it for Cartesian product, tensor product and lexicographic product of two bipolar interval valued of fuzzy graphs also we can generalized it to like strong product, week product. We use this concept in homomorphism bipolar interval valued fuzzy graph and in bipolar intuitionistic interval valued fuzzy graph on the other hand the concept of fuzzy sets, bipolar fuzzy sets and intuitionistic fuzzy sets will be applied to the following topics listed by Alnaser (2014a, b, 2017, 2018).

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